Julia Robinson Mathematics Festival

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Kakuro Counting

In a Kakuro puzzle, the goal is to fill in "words" made of numbers. The given clues are the sums of each "word".

In a usual Kakuro puzzle, the rules are that each number in the word must be a single digit 1 through 9, but here we'll also consider some examples where 0 or negative numbers or numbers greater than 10 are allowed. We'll stick with integers, though.

Another usual Kakuro rule is that the numbers in each "word" must all be different. Here we will have occasional exceptions to that rule as well.

1. First of all, perhaps you'd like to try some of the usual Kakuro puzzles, where the numbers must be digits 1 through 9 and they must all be distinct. There are several available, ranging from easy to extremely difficult.

For each of the following problems, fill in a copy of the Kakuro Counting chart, showing the number of ways to make each total *n* by adding up exactly *k* numbers, using the restrictions given.

2. Positive integers only, repetitions allowed, order matters. So for instance, to make 3 by adding up exactly 2 numbers, there are two ways: 1+2 and 2+1. These are called "compositions" of 3 into 2 parts.

3. Positive integers only, repetitions allowed, order doesn't matter. So for instance, to make 3 by adding up exactly 2 numbers, there is only one way: 1+2. As a hint, think about ways of determining one (*n*, *k*) entry in your table in terms of earlier ones. Also, of course, for the first few it's really not that hard to list them all out. These are called "partitions" of *n* into *k* parts.

4. Positive integers only, repetitions not allowed.

5. Digits 1 and 2 only, repetitions allowed. [You might enjoy visiting Fibonacci Flips too!]

6. Digits 1 through 9 only, repetitions allowed.

7. Digits 1 through 9 only, repetitions not allowed. [This gives some great clues when working with Kakuro puzzles: the combinations that have only 1 or 2 ways are great places to start, because there are fewer options to check.]

8. For some more fun with partitions, list all the partitions of 6 into any number of parts. How many are there? If you count the total number of parts, how many are there? If you add up the largest number in each partition, what is that total? Explain why they are the same.

9. And more with partitions: Instead of counting the number of partitions of 6 into *k* parts, count the number of partitions of 6 in which the largest part is less than or equal to *k*. Or where the largest part is greater than or equal to *k*. Or where the smallest part is greater than or equal to *k*. Which ones of these lend themselves to nice formulas?