MATHEMATICAL SCIENCES RESEARCH INSTITUTE

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Notes from the Director

David Eisenbud

Congressman Vernon D. Ehlers and MSRI Director David Eisenbud at a Congressional briefing (see page 5)

The Spring programs on *Infinite Dimensional Algebras and Mathematical Physics* and *Stacks, Intersection Theory, and Non-Abelian Hodge Theory* are drawing to a close. Once again I hear from departing members of all the work they did here, all the mathematical contacts they made, the excitement of the program, the kindness of Jackie Blue and the rest of the staff... The pleasure that people take in participating in MSRI's events makes my job a pleasure too.

Plans for Next Year

Although the mathematical life swirls around me here, my duties often prevent me from direct participation. Next year I'll be able to do much more of this: I'll be on sabbatical, and take part in the Commutative Algebra program that will be at MSRI. Through this time I'll

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Prime Possibilities and Quantum Chaos

Andrew Granville

This article is derived from a lecture given before an audience of friends of mathematics on February 6, 2002. The event is mentioned by David Eisenbud at the end of his article, on page 7. Andrew Granville is a well-known expert and enthusiast on the distribution of primes.

The distribution of prime numbers is one of the oldest topics in mathematics and has been intensively studied with the most modern methods of the time for one hundred and fifty years. Despite this and the high quality of researchers, we still know depressingly little about many of the most fundamental questions. As one might expect in such an old and dignified field, progress in recent decades has been slow, and since the subject has been so well studied, even seemingly minor advances require deep, tough ideas and often great technical virtuosity.

But recently there has been extraordinary progress in our understanding from an unexpected direction. The ideas come from an area that seemed to have absolutely nothing to do with prime numbers — the mathematics of quantum physics.

I am a specialist in analytic number theory, untrained in physics; indeed, my undergraduate course in quantum physics left me more puzzled than enlightened. Due to the recent breakthrough in my own subject, I have had to go back and try to get a basic feel for the key developments in quantum physics. Here I try to share my limited understanding with you, discussing the disturbing consequences of quantum mechanics and the origins of Einstein's famous quote:

> *God does not play dice with the universe.*

For a good layman's introduction, see *The Ghost in the Atom* [5].

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Notes from the Director

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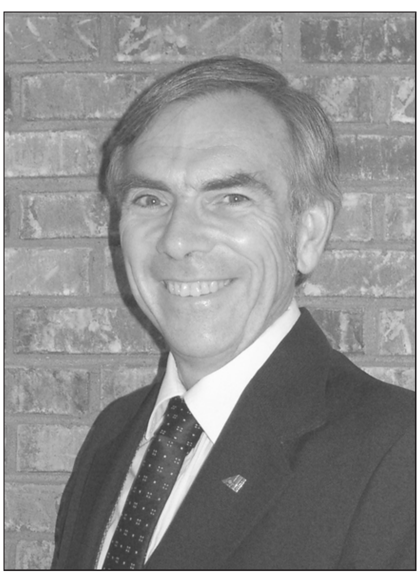
keep some of my responsibility for fund-raising, and I'll return to be Director for another four years starting in July, 2003. I'm excited about moving out of my office and into a member office in some far corner of the building. Part of the reason I can look forward to this whole-heartedly is that MSRI will be in terrific hands: Michael Singer will be Acting Director and Bob Megginson, from the University of Michigan, will be Deputy Director.

Michael Singer

I first met Michael when he was a program organizer for *Symbolic Computation* a few years ago. He subsequently returned to be Acting Deputy Director when Joe Buhler was helping run the Algorithmic Number Theory program in Fall 2000, and finally returned to be Deputy Director starting in July of 2001. I've been impressed again and again by his excellent sense and the quality of his ideas about the directions in which MSRI should go; I think he'll do a wonderful job!



Bob Megginson

Bob Megginson is a functional analyst whom I first met in his capacity as Chair of MSRI's Human Resources Advisory Committee; his own background is part Native American. I've since learned that Bob is well-known across the country as a champion of programs for minorities. For example, he is a winner of the U.S. Presidential Award for Excellence in Science, Mathematics, and Engineering Mentoring. Bob has done a great deal of work, formally and informally, for MSRI since well before I came on the scene; he's now a member of our Board of Trustees. Bob has always impressed me as original and effective in his work for minorities, highly diplomatic in his dealings with others, and extraordinarily hard-working on the projects he undertakes. I expect to enjoy taking direction from him next year, and having him as my deputy director the year after!

Sixty-Six Academic Sponsors

MSRI could not be what it is without a base of support extending far beyond Berkeley. The proposal for the Institute written in 1979 listed eight Academic Sponsors, and I believe that this unheard of breadth of involvement was a big reason for the success of the original proposal to NSF. By the time I came in 1997, the number had increased to twenty-eight. Now it is sixty-six. The list includes top schools like Berkeley, Chicago, Harvard, MIT and Princeton and a wide range of other institutions — the whole list is proudly displayed on our web site and above our tea tables, as any recent visitor to MSRI will know. Ten new schools have joined since the beginning of 2001: Brandeis, Columbia, Georgia Tech, SUNY at Stony Brook, University of Georgia, University of Illinois at Chicago, University of Maryland at College Park, University of Pennsylvania, Washington University in St. Louis, and Wayne State University. Welcome to them all!

The Academic Sponsors continue to be the core of MSRI's success, and a powerful argument for its continuation. Schools in the group receive substantial benefits from our pure and applied Summer Graduate Programs for their students (this summer: *Arithmetic of Polynomials* and *Fluid Mechanics of Blood Flow*), from our contributions to conferences that they run, and from other programs. The dues are still only $3000. Is your University a member yet?

Pleasures of the Popular

One category of activity I've particularly enjoyed at MSRI is what you might call "Mathematics and (the rest of) Culture". Our programs in this area are primarily the work of Bob Osserman and David Hoffman. The first extravaganza of this type, under previous Director Bill Thurston, was the *Fermat Fest*, celebrating Wiles' proof with a program including Tom Lehrer's songs as well as the background of the mathematics. The public's hunger for this was such that the 1000 seat Palace of Fine Arts was completely sold out, and scalpers were getting $50 per ticket at the door! Most of our recent programs have dealt with mathematics in literature and theater: Tom Stoppard on his play *Arcadia*, Father George Coyne, the

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Mathematics in the News: It Does Not Have to Be the Empty Set

Erica Klarreich

Open up a newspaper or even a science magazine, and the chances are pretty slim that you'll come upon any stories about mathematics. This is rather odd, because the recent success of popular mathematics books and movies like *A Beautiful Mind* seems to suggest that a large part of the general public is willing or even eager to hear about mathematics and the people who create it. Given that people appear to have an appetite for stories about mathematics, why does the popular press not provide more of them?

Some would say that it's simply hard to find a math story that will interest readers, because mathematics is more abstract than the other sciences and its applications seem less immediate. Yet areas of science such as string theory and cosmology have captured the public's imagination, even though they have no obvious applications and stretch intuition to its limits.

Others would argue that most mathematical research is incremental, of interest only to people working in the field, and that only rarely does a new theorem warrant a cry of Eureka! It is true that most mathematics results don't lend themselves to popularization. Yet this is true of all the sciences.

Nor is the scarcity of math stories due to lack of interest on the part of editors. On the contrary, I've found that editors of science magazines receive math story ideas with enthusiasm, well aware of the holes in their coverage.

Although writing math stories is certainly not always easy, I believe the shortage of mathematics articles in the popular press stems not from some inherent difficulty in writing about mathematics but simply from the lack of direct avenues of communication between mathematicians and journalists.

This communication gap is nowhere near as wide for the other sciences. Physics, biology and chemistry are far too dependent on public funding to leave their public image to chance. All of them have well-tended systems for churning out press releases and informing journalists of the latest developments in their fields. To explain why these efforts are crowding out mathematics so thoroughly, I'd like to discuss briefly the day-to-day workings of Nature Science Update, the online daily news publication of the journal Nature, for which I wrote before coming to MSRI.

Nature Science Update posts about 13 or 14 news stories a week on its website. While I was there, it employed four writers, so each was responsible for three or sometimes four stories per week. This allowed at most two days to turn a story around, from conception through reporting, writing and editing. From the perspective of research mathematics, such a pace seems ridiculously frantic. But it is typical of daily news coverage.

Given such requirements, reporters can't afford to look far for ideas. While I was at Nature Science Update, our main sources of stories fell into four categories. The major research journals, Nature, Science, and Proceedings of the National Academy of Sciences, each of which produces a weekly press release summarizing the upcoming issue, routinely accounted for at least a third of our stories. We also often found stories in the press releases that university public information officers emailed to us regularly, and on websites such as Eurekalert! (www.eurekalert.com) and its European counterpart, AlphaGalileo (www.alphagalileo.com), which gather press releases and post them daily. We always looked at the short digests of noteworthy new research articles issued by professional societies; a good example is Physical Review Focus (focus.aps.org), put out by the American Physical Society. Lastly, we scanned preprint servers and journal tables of contents for appealing new research papers.

None of these are particularly good sources of mathematics stories. Journals like Nature and Science seldom publish mathematics papers; nor do Eurekalert! and AlphaGalileo post many press releases about mathematics. And the mathematical societies don't publish any tip sheets for reporters, or many press releases. That leaves journal tables of contents — but the titles of math papers tend to be, if possible, even more impenetrable to non-experts than those of the other sciences, making it hard to tell which articles are good story prospects and which are not.

Unlike most journalists, I actively sought to write mathematics stories while with Nature. But given the stringent requirements of daily journalism, I found it nearly impossible to take the time to figure out where they were hiding. Of the 30 or 40 news stories I wrote for Nature Science Update, only one — about randomness in the digits of pi — was a mathematics story, and it came from the single math press release I received during my six months with Nature.

I've chosen to focus on daily news, not on feature articles, following the time-honored mathematical technique of looking at the extreme case. Certainly features, which are longer than news stories and have looser deadlines, are more amenable to most mathematical topics. But journalists' feature ideas often emerge out of the news they cover, and as long as mathematics is not part of their frame of reference, it will be as roundly ignored in features as in news.

As long as the other sciences keep delivering ready-made story ideas through press releases and news summaries, journalists will have small motivation to go looking far afield for mathematics stories. If mathematics is to appear more in the popular press, mathematicians can't wait for journalists to come to them.

What can mathematicians do to improve this situation? Although most mathematics research projects probably don't make promising material for news stories, the minority that do, if properly publicized, would provide journalists with a wealth of material. Individual mathematicians, or department chairs, could make a start by acquainting their university's public information officers with their department's research activities. And the mathematics professional societies could start issuing weekly or monthly summaries of interesting research papers. Such activities, if performed energetically, would result in an immediate increase in coverage of mathematics (as the Mad Hatter says, you can always have more than nothing).

In a perfect world, perhaps, mathematics would receive the attention it deserves without such efforts. Mathematicians may well feel that attempts to reach out to journalists are beneath them, yet there are good reasons why they should care about engaging with journalists. Science journalism has many flaws: it tends to be superficial and simplistic, and often falls into error. Nevertheless, it is virtually the only source of information about the sciences for lay readers (with the honorable exception of popular articles written by scientists themselves, which usually have a fairly narrow distribution).

Informing the public about what is going on in mathematics is certainly in the interest of mathematicians. To me, a still more compelling reason for pursuing mathematics coverage in the media is the simple fact that much of the general public thinks mathematics was all figured out and written down centuries ago. When I try to explain to people that mathematics is still an active and vibrant field of research, I'm often taken aback by the surprise I generate. Most people have little idea what mathematicians actually do.

Although journalism may not be the ideal vehicle for correcting this misapprehension, it is the vehicle we have. If the occasional mathematics article can convey to the general public some sense of the beauty of mathematics and the excitement of plumbing its depths, it will surely be worth the effort.

Erica Klarreich is the Journalist-in-Residence at MSRI from February to September 2002. She got her math Ph.D. at SUNY Stony Brook, was a postdoc at the University of Michigan and did a one-year graduate program in science writing at the University of California, Santa Cruz.

AfterMath: Tax-Wise Giving

Finding the time, energy, and expertise to do personal financial planning can be tough. And, estate planning can be the most emotionally difficult part of financial planning. It is good to remember though that there are very significant advantages to you in creating a good personal estate plan. And through planning, you may well have an opportunity to help the non-profit of your choice. For obvious reasons, we'll use MSRI as an example.

The difficulty of planning your estate has not been made any easier by enactment of The Economic Growth and Tax Relief Reconciliation Act of 2001. As you have no doubt heard, estate taxes are phased out over years, only to reappear in 2010. Experts are advising special planning to deal with the uncertainties.

Including the Mathematical Sciences Research Institute in your will, Living Trust, or other planned giving can make a real impact on the lives of future mathematicians. The independence, possibly even the long-term survival of the Institute will be influenced by the generosity of those who have been affected by it, and the decisions they make regarding their estates. MSRI's Development Director, Jim Sotiros, is a Certified Specialist in Planned Giving who can help answer any questions you have and, working with your financial advisors, help you plan your estate.

Here are some tips that might prove useful. As you review your assets and their distribution, it can be very helpful to specify who gets what because assets are taxed differently according to both the type of asset and who receives it. Three types of taxes may be due: income, gift, and estate. They each treat assets differently.

For example, a gift of appreciated stock to MSRI is an excellent choice while you are alive, because the capital gains taxes due on the transfer are avoided. The same gift of stock to an individual during your lifetime would be received by him/her with your cost basis, so when s/he sells it capital gains will be realized and taxed. The gift is substantially reduced in value. At your death, things are different. S/he will receive a stock gift with a stepped-up basis, and pay no capital gains tax-excellent planning.

Likewise, an IRA, 401(k), 403(b) or other retirement plan when left to an individual at your death will be taxed on all of the appreciation that took place during your lifetime in addition to any estate taxes due. These two taxes can require that as much as 90% of your retirement account go to the government! The same retirement plan left to MSRI at your death will be received with no deferred income tax nor estate taxes due. The impact of the gift is much greater.

So, consider leaving your IRA, 401(k), or 403(b) to MSRI.

And, also remember, you usually can't direct a 401(k) 403(b) or other retirement plan through your will or Living Trust. Instead, you contact the retirement plan trust administrator, and change the beneficiary of the plan with him or her. Contact Jim Sotiros, C.S.P.G. in the MSRI Development Office at 510-643-6056 or JSotiros@msri.org and we can help with contact information for some of the more popular plan administrators, and with any questions you may have.

AfterMath is not intended as tax advice. Know that individual financial circumstances require specific professional guidance. Please contact your legal and financial advisors to see how these general issues apply to you.

Making Wavelets in Congress

Professor Ingrid Daubechies, of Princeton University, spoke on "Mathematics, Patterns and Homeland Security" at this year's Congressional lunch briefing on Capitol Hill for Members of Congress and their staffs, held February 28, 2002. The Mathematical Sciences Research Institute (MSRI), Berkeley, California, joined AMS to sponsor the event, one in a series intended to bring mathematicians to Washington to discuss federally-funded research that affects sensitive issues currently before Congress.

Professor Daubechies described how mathematicians use wavelet analysis in several of these areas, for example, the FBI uses wavelets to compress its vast library of fingerprint data. Wavelets are also a key ingredient in the analysis of sonar data.

David Eisenbud welcomed guests on behalf of AMS and MSRI. James Schatz, of the National Security Agency, introduced the speaker and gave a short overview of the types of security issues encountered by NSA.

Congressman Rush Holt and Congressman Vernon J. Ehlers, long-time champions of science, were our Congressional hosts, and both attended the briefing.

Ingrid Daubechies lectures at a Congressional briefing.

Photo and text courtesy of the American Mathematical Society

Current Programs

Infinite-Dimensional Algebras and Mathematical Physics, January 7, 2002 to May 17, 2002; organized by E. Frenkel, V. Kac, I. Penkov, V. Serganova, and G. Zuckerman.

Algebraic Stacks, Intersection Theory, and Non-Abelian Hodge Theory, January 7, 2002 to May 17, 2002; organized by W. Fulton, L. Katzarkov, M. Kontsevich, Y. Manin, R. Pandharipande, T. Pantev, C. Simpson, and A. Vistoli.

Summer Graduate Program: Excursions in Computational Number Theory – Polynomials with Integer Coefficients, June 17 to June 28, 2002; organized by Peter Borwein and Michael Filaseta.

International School on Biomathematics, Bioengineering and Clinical Aspects of Blood Flow, July 23 to August 9, 2002; organized by Stanley A. Berger, Giovanni P. Galdi, Charles S. Peskin, Alfio Quarteroni, Anne M. Robertson, Adélia Sequeira, and Howard Yonas.

Commutative Algebra, August 19, 2002 to May 16, 2003; organized by Luchezar Avramov, Mark Green, Craig Huneke, Karen E. Smith, and Bernd Sturmfels.

Quantum Computation, August 19 to December 20, 2002; organized by Dorit Aharonov, Charles Bennett, Richard Jozsa, Yuri Manin, Peter Shor, and Umesh Vazirani.

See http://www.msri.org/calendar for details. See also page 18 for a list of workshops, including those belonging to these programs.

Notes from the Director

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Director of the Vatican Observatory, on Brecht's *Galileo*, David Auburn on *Proof*, Michael Frayn on *Copenhagen*, and most recently Sylvia Nasar and Dave Bayer on the book and movie *A Beautiful Mind* (see photos and descriptions of these last three events on page 10 of this *Emissary*). The last two will be broadcast on national radio by The Commonwealth Club of San Francisco; both the Fermat Fest and the Conversation *Mathematics in Arcadia* with Tom Stoppard are available from us on video tape.

It seems, from the enthusiastic public response to these events, that the public awareness of mathematical research and its importance is on the rise. A small sign of this: the bottom picture on page 11 of this Emissary is from the Oakland Tribune — the Theater section, no less. This public awareness goes hand in hand with an increasing range of books, plays, movies, and articles about our mathematical world. What began this change? I think that the hero's status of Wiles, encouraged by the Nova show *The Proof* about him, may have produced the discontinuity; we and other institutions have probably helped fan the flames, both with cultural programs like the ones above and with events explaining mathematical applications (like our *Mathematics, Magic, and Coincidences* program a few years ago.) I've enjoyed every one of them (once the event was actually happening!)

MSRI Will Be Twenty

The Fall will hold an important landmark (and a great celebration) for MSRI: the twentieth anniversary of our first programs. Mathematical programming takes pride of place as usual, with a series of special Colloquia. But there will be some hoopla too: a mathematical Film Festival at the Pacific Film Archive, a special Sponsors and Trustees meeting October 25, talks for the general public, and, if all goes well, even an event with comedian Steve Martin, are some of the things planned.

Thanks, Cal!

Continuing tradition, MSRI and UC Berkeley have had excellent collaboration in recent years. Calvin Moore, one of MSRI's founders, has been Chair of the Mathematics Department throughout this period, and has been a wonderful partner, teaching me a great deal about the local scene and doing great things for MSRI within the University. He's stepping down as Chair this year, but his successors Ted Slaman and (for an "acting" year) Hugh Woodin seem highly sympathetic to MSRI as well. I'm confident that our good relations and mutual support will continue. Cal will still be around too, and he and I are laying plans for the future. . .

Banff Follow-up

The Banff International Research Station, www.pims.math.ca/birs/, is MSRI's collaboration with the Pacific Institute of Mathematics and several other partners. It is now poised to begin full operation in March, 2003 with 40 weeks of workshops, chosen from a rich field of 105 proposals (the choice was hard!) First we'll run a workshop on *Quantum Algorithms and Complexity* there (September 23 to 27, 2002). I look forward to seeing you there sometime!

Archimedes Society and Museion

The NSF wants us to demonstrate increasing broad-based financial support and become more independent. This means increased private fundraising. Our Development Director Jim Sotiros has been on the job for a little more than a year, and has started several interesting programs, including the Archimedes Society (for annual

MSRI Is Molting!

Paul Halmos says somewhere that when he sat on a plane next to a stranger who asked "And what do *you* do for a living?" he was always tempted to stop the conversation dead by saying "roofing and siding". The need for such defenses has lessened as the population has become more aware of mathematics, but at least I can now make the "roofing and siding" reply honestly: the University of California is engaged in making needed repairs on our siding (the roofing is already done) and windows. This will result in a certain amount of disturbance at MSRI over the Summer (so far not as bad as I thought it might be). All for a good cause.

donors) and the Museion Dinners for those in the Archimedes society who can afford to give $5041 a year or more. (The Museion was one of the two famous institutions of higher learning in ancient Alexandria, the other being the Library.) Jim is arranging special perks for members, particularly at our public events. See www.msri.org/development for information about donating (and the easy possibility of doing it with a credit card) or just for an explanation of the nonstandard giving levels (this does not mean we deal in infinitesimals.) You can also join by enclosing a check in the envelope that accompanies this Emissary. MSRI is still a newcomer in fundraising and your comments (and of course your donations!) would be most welcome.

As everyone involved in this sort of thing seems to know (I've been learning!), large donors and foundations pay a lot of attention to how much grass-roots support an institution receives. Small gifts are actually important, way out of proportion to their size.

One of the fruits of this activity was a gorgeous lecture by Andrew Granville at a Museion dinner at Jim Simons' Fifth Avenue mansion last fall. Andrew agreed to write it up, and you can follow his stroll through "Prime Possibilities and Quantum Chaos" on the front page of this Emissary.

Tondeur Retires from NSF

Philippe Tondeur headed the NSF's Mathematics Division from 1999 to 2002 and oversaw an increase in the Division's budget from US$106 million in FY 2000 to (probably) US$182 million in FY 2003. These are his remarks at his Retirement Reception, held May 15, 2002 at the National Academy of Sciences in Washington.

The overriding sense I want to express today is my sense of gratitude. First and foremost gratitude for the support by Rita Colwell and Robert Eisenstein. At no time was there more enthusiastic support in the Federal Government for the Mathematical Sciences than displayed by the current NSF leadership, as well as the officers in the Office of Management and Budget. I also want to include in my thanks Adriaan DeGraaf and the MPS staff. I want to take this occasion to thank my predecessor, Donald Lewis, who prepared the groundwork for the current situation. During his tenure, General William Odom, former head of the National Security Agency, chaired an outstanding International Assessment Effort of the Mathematical Sciences, which amounted to compelling marching orders for my task. Bill Odom has continued to be a steady supporter of the Mathematical Sciences throughout these years at NSF. Thank you all.

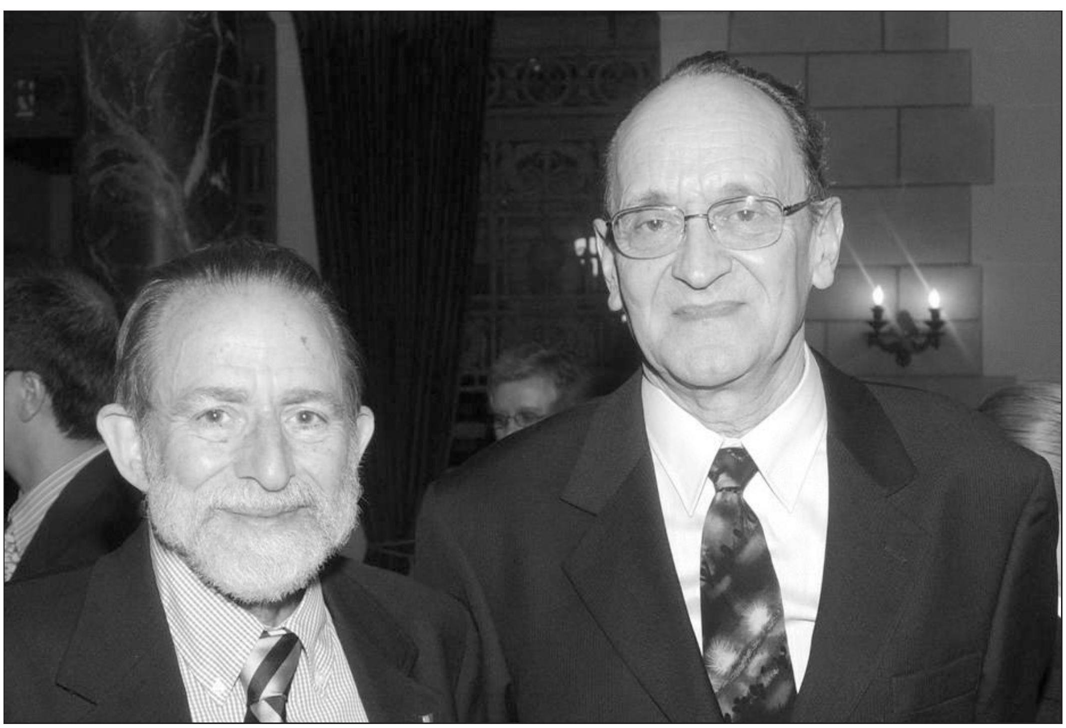
This support of the mathematical sciences became reality because of our increased attention to the multiple environments in which we act: within our discipline; as enablers of science and engineering; and as supporters of the next generation.

It has been said by one of our street philosophers: luck is what happens when preparation meets opportunity. I am indeed a lucky person, and grateful for the opportunity to serve.

I feel deep gratitude for an incredibly dedicated staff at the Division of the Mathematical Sciences, starting with Bernie McDonald and Tidy Henson, and extending throughout the Program Directors and support staff. They have a large part in the progress the Division has made during the past few years. They also provided me with a splendid education about the many facets of this amazing organization. I would be remiss if I did not acknowledge the outstandingly competent help the professional societies provide for our work at NSF. These are the American Mathematical Society, the Society for Industrial and Applied Mathematics, the American Statistical Association, and the Mathematical Association of America. As one concrete example, I have no doubt that our staff of 21 Program Directors is the group of the worlds most intense users of MathSciNet, which owes so much to the outstanding executive leadership of John Ewing at the AMS.

I wish to express my gratitude and admiration to the many people throughout the National Science Foundation, who exercise their public service with such a remarkable sense of purpose. Even as severe a critic as the OMB judged this agency to be one of the best run in the federal government. Rita and all NSFers, you make us proud to serve.

I owe much more to NSF. Here is a relevant piece of my life story. I got a Ph.D. at the University of Zurich, quite a while ago, and started roaming the world as a postdoctoral fellow — the roaming



being an adolescent's dream — and a scientific career the unexpected realization of that dream. One day I got a letter from a US educational establishment inviting me to spend a year as its guest. I walked into a US Consulate, and asked for a visa to this country. The Consul looked at my letter, smiled, and gave me the visa I asked for on the spot. How does one explain this miracle? The secret may be that the Consul was wearing the tie of the establishment inviting me, a Harvard tie. This is how I came to the US, the visa became a green card, then a certificate of citizenship. My heart is overflowing with gratitude for all the opportunities this generous country offered to a young man from a far away land.

I found out much later that my Harvard salary was actually paid by NSF , through a PI grant to Raoul Bott, and the Program Director was Ralph Krause. Thank you both, Raoul and Ralph. [The photo on the previous page shows Krause and Tondeur. –Editor]

And the accent is still there, after all these years, as my freshmen at Illinois invariably pointed out with no small amount of disapproval.

Three years later I was on a regular faculty position, towards the end of the academic year, and my colleagues said: if you want to be paid in the summer, you have to get a grant. And how do you do that?, the ignorant young man asked. You write to NSF, explain your research, and the deadline is tomorrow. I did write that day, and a few weeks later I got my first PI grant allowing me to indulge in geometry and topology to my heart's content. I have to report that grant approvals take a bit longer today. This research was my life's focus for many happy years — 36 years in fact.

Six years ago I was invited to chair my department at the U of Illinois. This brought a substantial change of professional perspective, from single minded research absorption to a public service function. With this change in perspective, I found it impossible not to respond to the invitation to come to NSF three years ago, and to contribute to this agency, which had been such an essential part of my life, which has done so much for the US sciences, and which is destined to do much more for the sciences world-wide. Here my thanks go to Margaret Wright, for the role she played in that decision, and to Bob Eisenstein, who appointed me.

I also wish to express my deep gratitude to Claire, my cherished life-long companion, who willingly shared in this as so many other complicated and exhilarating adventures throughout our happy life together.

I am grateful to have been able to repay in some small measure the wonderful things NSF did for my generation. I leave the place with the conviction that my successor will be able to build on the work begun by his predecessors and take another big step in the progress of the Mathematical Sciences, the ultimate enabler of science and engineering. I have the deep conviction that the progress of the Mathematical Sciences in this century will exceed by far what has been achieved in the past twenty-five hundred years, and that this will contribute mightily to the Science and Engineering enterprise of this country and the world. And I expect that this country of unprecedented opportunity will continue to be a beacon to the world in large measure because of its faith in science and the opportunities it offers to pursue science.

Thank you.

The Three-Century Mathematician

Silvio Levy

Leopold Vietoris was the oldest Austrian citizen and probably the oldest mathematician anywhere when he died on April 9, 2002. He was going to be eleventy-one, 111, a rather curious number, and a very respectable age for a — oops, that's from another story. Let's start over: He was born June 4, 1891 in Radkersburg and did an amazing amount of mathematics, having published his last paper at the sprightly old age of 103. He attended the Technical University in Vienna and proved theorems right through the First World War in spite of having been a combatant and a prisoner of war. (This amateur obituarist has not been able to ascertain what he did during the Second.) After obtaining his degree and holding appointments at Innsbruck and TU Vienna, he settled at Innsbruck in 1930, where he remained a professor until retiring in 1961.

Vietoris is known to every mathematician from — among other things — the Mayer–Vietoris sequence (the same Mayer who was Einstein's assistant and collaborator at the Institute for Advanced

$$\cdots \to H_q(K_1 \cap K_2) \to H_q(K_1) \oplus H_q(K_2) \to \\ \to H_q(K_1 \cup K_2) \to H_{q-1}(K_1 \cap K_2) \to \cdots$$

Study). He made many other contributions to algebraic topology. Less well-known are the results and ideas he developed in general topology, which were in many cases reinvented later and are now generally associated with other mathematicians. According to Heinrich Reitberger, in "The contributions of L. Vietoris and H. Tietze to the foundations of general topology" (see http://math1.uibk.ac.at/vietoris.pdf), Vietoris invented the notion of filters (1916–1919), topologized the p-adic numbers, topologized the space of closed subsets of a topological space (1922), thus generalizing the notion of Hausdorff distance, and introduced the notion of directed sets and nets (1921), through which he proves, for example, that any compact space is normal (in modern terminology). Reitberger points out that the proof given by Bourbaki in *General Topology*, §9.2, proposition 2 is the same. (Reitberger's article appeared in *Handbook of the History of General Topology*, vol. 1, pp. 31–40, Kluwer, Dordrecht, 1997; the link above is to a free version. Other references are given below.)

Vietoris was a mountain climber and skier, and combined his scientific inclinations with these hobbies in producing papers such as "Der Schi im Licht der Festigkeitslehre".

Leopold was married to Maria Vietoris for 66 years. She died a few weeks before him.

R. Liedl and H. Reitberger, "Leopold Vietoris — 90 Jahre", pp. 169–170 in *Yearbook: Surveys of mathematics* 1982, Bibliographisches Institut, Mannheim, 1982.

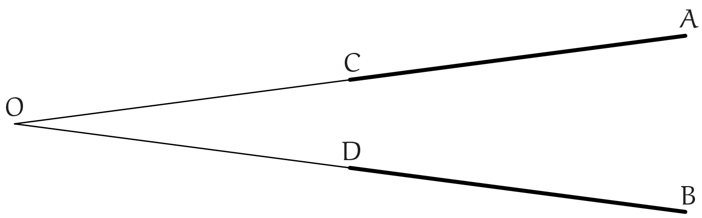
Gilbert Helmberg and Karl Sigmund, "Nestor of mathematicians: Leopold Vietoris turns 105", *Math. Intelligencer* **18**:4 (1996), 47–50.

Puzzles Column

Elwyn R. Berlekamp and Joe P. Buhler

1. Line segments OA and OB meet at O with an angle of 1 degree. Points C and D bisect OA and OB respectively. Imagine that the segments CA and DB act as reflective mirrors, and that a light ray in the plane enters the larger opening AB, bounces back and forth between the two mirrors for a while and then exits. What is the maximum number of reflections that are possible?

Remark: This problem appears in *50 Mathematical Puzzles and Problems, Red Collection* published by the Key Curriculum Press.



2. (Just when you thought you were safe from hat problems…) Let n be a positive integer. A team of n people has a strategy session. After that a judge randomly places red and blue hats on their heads. Each person can see all hats except their own (and sees who is wearing which hat, in addition to the color of the hat). *Simultaneously, and with no communication whatsoever*, each player must vote on their hat color (abstention is not allowed). However, they follow "Chicago" rules: each player can place as many votes as they desire. If a majority of the players are correct, the team shares one million dollars.

Find a strategy that maximizes the team's chances of winning the million dollars.

Remark: Our column in the previous Emissary (Fall 2001) had two hat problems. Like the second one there, this one first occurred in 1993 in *The Expressive Power of Voting Polynomials* by James Aspnes, Richard Beigel, Merrick Furst, and Steven Rudich. In some sense this is easier than the earlier problems in that there is a clean answer for all n.

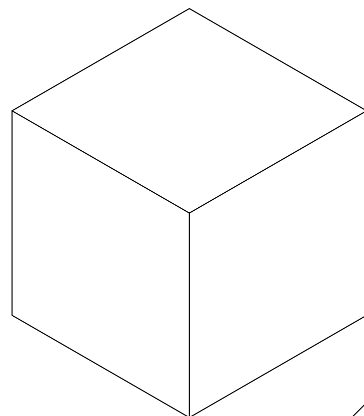
3. In the diagrams to the right, a regular hexagon and a regular octagon of side 1 are tiled by rhombuses whose sides have length 1. Show that for any positive integer n the regular $2n$-gon of side 1 can be tiled by rhombuses of side 1. How many rhombuses did you use?

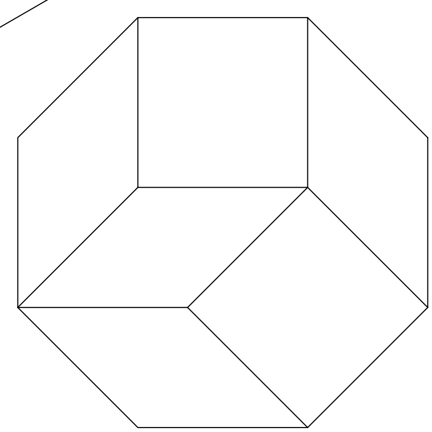
Remark: This is derived from a problem on this year's Bay Area Mathematical Olympiad (see the November 2001 Emissary at http://www.msri.org/publications/emissary/index.html for an article on the Berkeley Math Circles and BAMO). The question of uniqueness (i.e., whether or not tilings were possible using different number of rhombuses) was left unstated, perhaps in the hopes that someone would answer the question and maybe get the brilliancy prize, which is sometimes offered for especially ingenious solutions.

4. You are a playwright and want to write a play, for a company of n actors, in which actors will enter and exit one at a time, subject to the following constraint: any actor that exits is the one that has been on stage the longest.

For $n = 3$, $n = 4$, and $n = 5$, can you devise a sequence of entrances and exits in which each set of actors appears once and only once? The play begins with an empty stage, so the initial subset is the empty set. You get extra credit if the final set contains a single actor, because then the sequence could cycle, like a Beckett play.

Remark: This problem, and its cute formulation, are due to Brett Stevens. Little is known about this in general. For $n = 3, 4, 5, 6$ and 8 solutions are known, though cyclic solutions are impossible for $n = 3$ and $n = 4$. Amusingly, this problem about Beckett–Gray codes also appears in the sections of volume 4 of *The Art of Computer Programming* that Donald Knuth has made available on his web page.





Scary Math

In a March 19 Reuters newspiece about a press release from the British Antarctic Survey, one could read: *The Antarctic Peninsula has warmed by 36 degrees Fahrenheit over the past half century, far faster than elsewhere on the ice-bound continent or the rest of the world.*

Here is what the British Antarctic Survey actually said: *During the last 50 years the Antarctic Peninsula has warmed by 2.5°C, much faster than mean global warming.*

Now, $36 = \frac{9}{5} \times 2.5 + 32$. It sure looks like Reuters misapplied the temperature conversion formula…

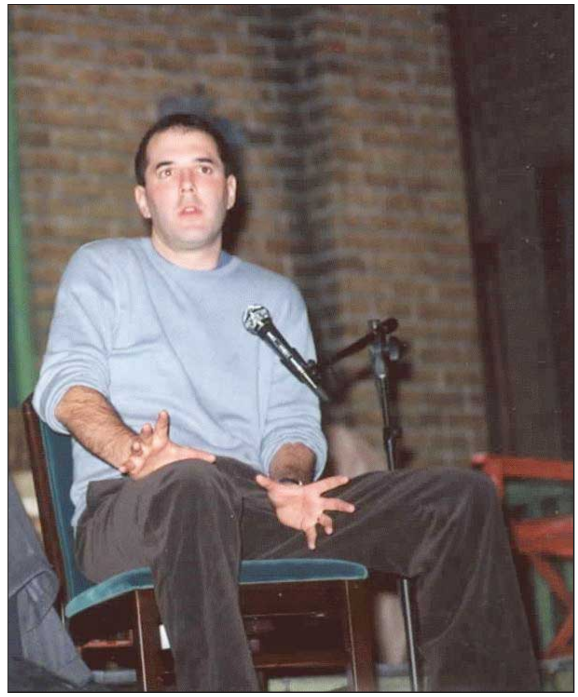
(Reported by John Shonder of Oak Ridge National Laboratory, via Frank Morgan's Math Chat web site.)

Conversations with Bob Osserman

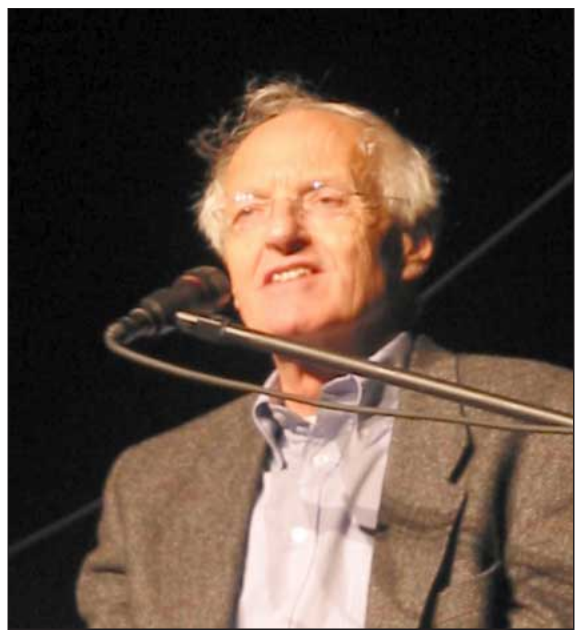
Bob Osserman, MSRI's Special Projects Director, has had an especially eventful year.

On November 29, 2001, MSRI held its first San Francisco event devoted to mathematics and theater. Entitled *Math Takes Center Stage*, it featured David Auburn, Pulitzer Prize-winning author of the play *Proof*, in conversation with Bob. The public conversation took place at the Curran Theatre as a curtain opener prior to an opening-week performance of the Tony Award winning play. David Auburn's early experience writing Second City style reviews and sketches was evident in an understated humor that was clearly appreciated by the audience. The success of the play itself is partly due to some of the same brand of humor, sprinkled in with a very serious subject: whether 25 year-old Catherine has inherited her father's mathematical brilliance and/or his susceptibility to serious mental illness.

On April 17, 2002, MSRI in association with the Berkeley Repertory Theater and the Commonwealth Club of California presented Michael Frayn in a conversation with Bob entitled *Theater, Science and History in 'Copenhagen'*. Frayn's play *Copenhagen*, winner of a Tony Award, is a fascinating attempt to reconstruct what happened during a World War II visit of Werner Heisenberg to Neils Bohr in German-occupied Denmark. The conversation centered around the scientific and moral issues raised in the play as well as the more general theme of the recent surge of science and mathematics in books, plays and movies.



Playwrights David Auburn (above)
and Michael Frayn (below)



On May 2, 2002, MSRI and the Commonwealth Club of California presented Sylvia Nasar, author of the book *A Beautiful Mind*, and Dave Bayer, mathematical consultant to the film, in conversation with Bob. The discussion reviewed the main events in the life of the real John Nash including some of his research contributions to mathematics, his long mental illness and his gradual recovery, and the making of the movie as well as the decision making process of what to include and how to present it.

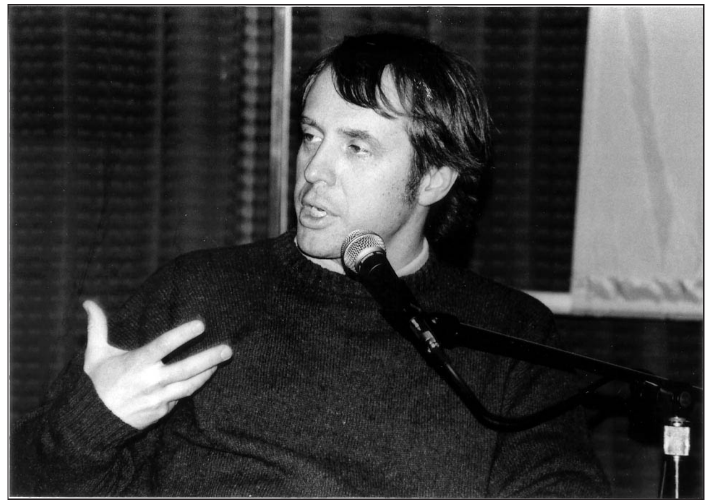
The conversation with Frayn and the one with Nasar and Bayer will be broadcast nationally by The Commonwealth Club via National Public Radio.

MSRI Archimedians Enjoy Art and Mathematics Conversations

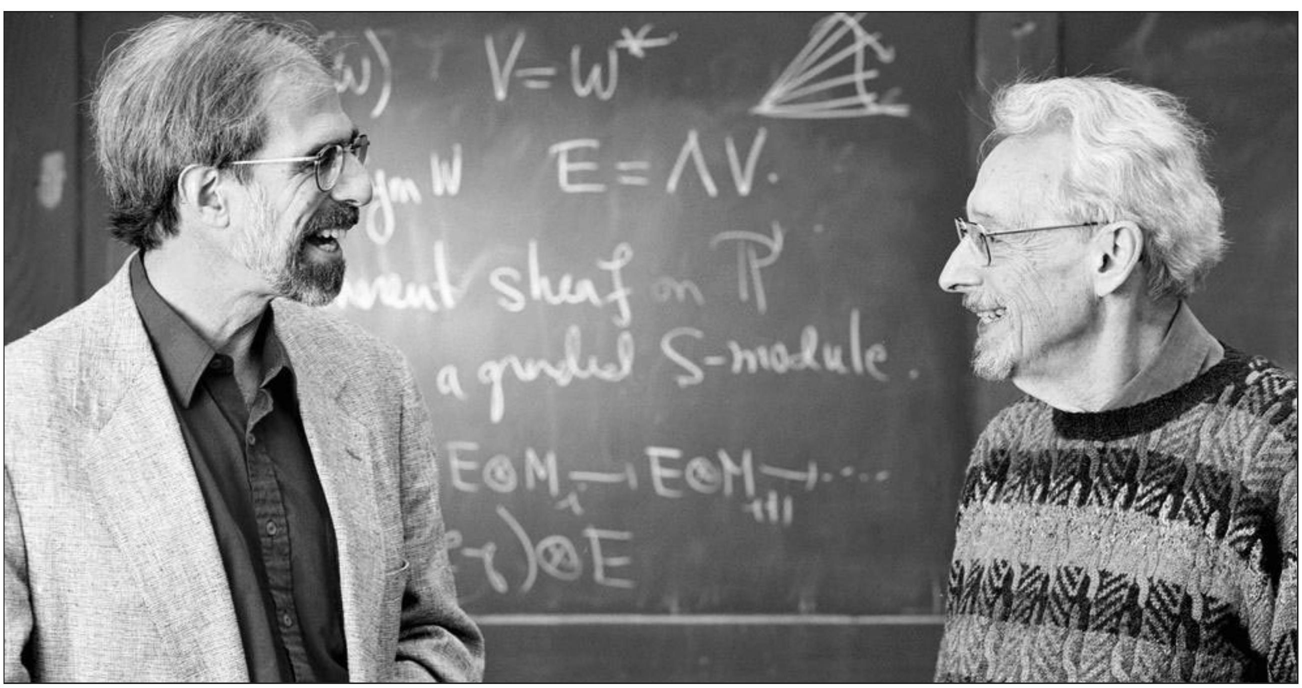
Members of the MSRI Archimedes Society enjoyed special receptions with playwright Michael Frayn and author Silvia Nasar and reserved complimentary seating at the recent "Conversation with Bob Osserman" events sponsored by MSRI.

MSRI strives to facilitate conversations for mathematicians and the general public that focus on the intersection of art and mathematics. Several Conversation events are currently in the planning phase. Joining the Archimedes Society assures that you will be notified of the event, given free or discounted tickets, and often provided reserved seating. Join the Archimedes Society online at www.msri.org/development, or by contacting Jim Sotiros, Development Director at 510-643-6056 or JSotiros@msri.org.

Your support of the MSRI Archimedes Society assists the operations of one of the world's finest centers of mathematics research, and will score you some cool tickets! Join today.



Author Silvia Nasar (top left) and Mathematician Dave Bayer (above) discussed John Nash's life and *A Beautiful Mind* with Bob Osserman. On April 16, 2002, several newspapers, including the Oakland Tribune, ran an article about this event and other MSRI initiatives, adorned with a large photo of Osserman and David Eisenbud (below).



Prime Possibilities and Quantum Chaos

(continued from page 1)

But let me begin with a subject I am much more at home in:

What are primes and why do we need them?

This year, 2002, factors into primes as $2 \times 7 \times 11 \times 13$. Next year has a different factorization, and indeed, each whole number has its own way of being broken down into primes. "So what?" you might ask; but everyone uses the fact that factoring large numbers is difficult, in their everyday life… Have you ever bought something on the web and been deluged by little screens that go on and on about "RSA cryptosystems"? That's number theory at work — what you input is held secure from unauthorized prying by the difficulty of factoring large numbers!

So primes are indeed worthy of careful study. Moreover, just as atoms are the building blocks of nature, so primes are the building blocks of numbers; therefore to study the theory of numbers, we must get a grip on primes. Next, I want to move on to a fundamental question in understanding primes.

How many primes are there up to a million? Up to a trillion? Up to any given point?

Carl Friedrich Gauss wrote in 1849:

> *I pondered this problem as a boy, in 1792 or 1793, and found that the density of primes around* t *is* $1/\log t$*, so that the number of primes up to a given bound* x *is approximately* $\int_2^x dt/\log t$.

Gauss, who was just 15 years old at the time of this finding, made his prediction by studying tables of primes up to three million. An extraordinary guess, as it turns out, which is amazingly accurate.

x	# of primes up to x	Overcount in Gauss's guess
10^8	5761455	754
10^9	50847534	1701
10^{10}	455052511	3104
10^{11}	4118054813	11588
10^{12}	37607912018	38263
10^{13}	346065536839	108971
10^{14}	3204941750802	314890
10^{15}	29844570422669	1052619
10^{16}	279238341033925	3214632
10^{17}	2623557157654233	7956589
10^{18}	24739954287740860	21949555
10^{19}	234057667276344607	99877775
10^{20}	2220819602560918840	223744644

Primes up to various x, and Gauss's prediction

Can we predict the size of the error in Gauss's guess? After a brief perusal of the table above we see that the error term is about half the length of the number of primes; that is, about the square root. In other words, we might guess that

$$\int_2^x \frac{dt}{\log t} - \text{\# of primes up to } x$$

is bounded above by some function like $\sqrt{x}$. Another surprising feature of this data is that the error term is always positive, indicating that, at least in the data computed to date, Gauss's prediction is too large. This might lead us to think that we can introduce a secondary term which will give an even more accurate prediction, but this is not the case: The error term changes sign infinitely often, as was shown by Littlewood in 1914.

Trying to find out when the error term first goes negative is not easy. The first bound on the first such x (call it x_0) was given in 1933 by Skewes,

$$x_0 < 10^{10^{10^{34}}}.$$

For a long time this was recorded, in several places, as the largest number to have any significant meaning. This bound has been gradually whittled down to $x_0 < 1.39822 \times 10^{316}$, and persuasive arguments are given in [1] that this is indeed about the correct value of x_0! (See my forthcoming article [8] for more details.)

Gauss's statement can easily be modified to provide a probabilistic model for the primes, as was done in 1936 by Cramér [4]: let $X_3, X_4, \ldots$ be a sequence of independent random variables with

$$\text{Prob}(X_n = 1) = \frac{1}{\log n} \quad \text{and} \quad \text{Prob}(X_n = 0) = 1 - \frac{1}{\log n}.$$

The sequence $\pi_3, \pi_4, \ldots$, etc., where $\pi_n = 1$ if and only if n is prime, might be supposed to be a "typical" element of this probability space; and if a statement can be made about this space with probability 1, then it might be expected to be true of the primes (that is, for the sequence $\pi_3, \pi_4, \ldots$). Certainly

$$\sum_{n \le x} X_n \sim \int_2^x \frac{dt}{\log t} \text{ as } x \to \infty \text{ with probability } 1;$$

that is, the expected value for the count of primes as given by the Gauss–Cramér model conforms with reality. Moreover in short intervals (which is more what Gauss was looking at)

$$\sum_{x<n\le x+y} X_n \sim \int_x^{x+y} \frac{dt}{\log t} \text{ as } x \to \infty \text{ with probability } 1$$

when y is a fixed small power of x.

The second statistic people usually like to look at is the variance:

$$\text{mean} \left| \sum_{x<n\le x+y} X_n - \int_x^{x+y} \frac{dt}{\log t} \right|^2.$$

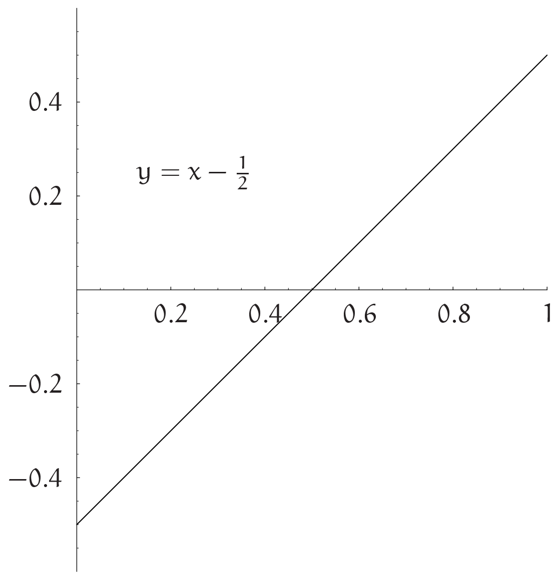
Here we have a big surprise; it can be proved that the value predicted by the Gauss–Cramér model cannot be the variance for the counted primes! After Gauss's model worked so well before, the breakdown of the model for this question is quite unexpected, as noted by Paul Erdős:

God may not play dice with the universe, but something strange is going on with the primes.

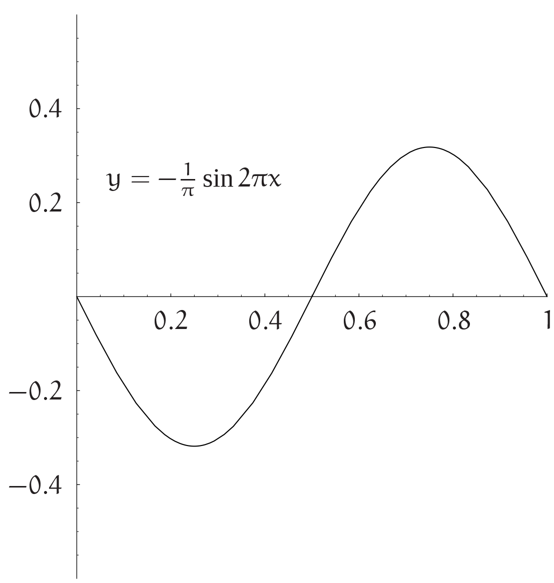
Gauss's prediction is just that, it's not a proof of anything, and one would like a proof, after all. It turned out to be very difficult to find a method that would give a prediction for primes that can be proved. When a method did come, it came from a quite unexpected direction.

Primes and music

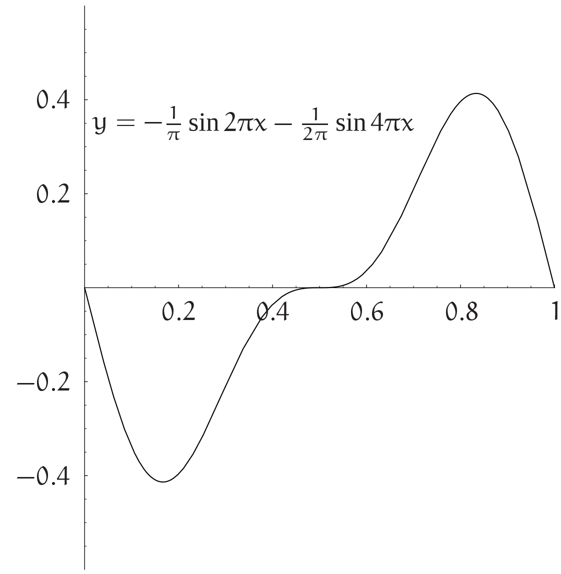
We start with a seemingly unrelated topic — how does one transmit signals that are not "waves"? We've all heard the words radio waves and sound waves, and indeed sound is transmitted in wave form, but the sound one makes doesn't seem very wavy to me; instead it sounds fractured, broken up, stopping and starting. How does that get converted into waves? As an example, we'll look at a gradually ascending line:



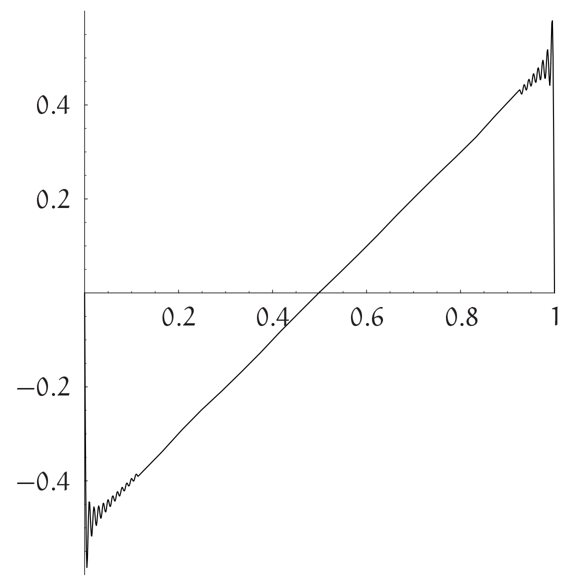
If we approximate it with a wave, the closest we can come is something like this:



The middle part is a good approximation to a straight line, but the approximation for $x < 1/4$, and $x > 3/4$ is poor. How can we fix that? The idea is to "add" a second wave to the first, this second wave going through two complete cycles in our interval rather than just one. By adding such a wave to that in the previous figure, we get an improved approximation:



We can proceed like this, adding more and more waves, getting increasingly better approximations to the original straight line. Here is the sum of one hundred carefully chosen sine waves:



This is a good approximation to the original, though one can see, at the end points, that the approximation is not quite so good (this is an annoying and persistent problem known as the "Gibbs phenomenon").

As one might guess from the pictures above, the more waves one allows, the better approximation one gets. For transmitting sound, maybe 100 sine waves will do; for data transfer, perhaps more.

However, to get "perfect" transmission one would need infinitely many sine waves, which one gets by using the formula

$$x - \frac{1}{2} = -2 \sum_{n=1,2,3\ldots} \frac{\sin(2\pi n x)}{2\pi n} \qquad \text{for } 0 \le x \le 1.$$

Not of practical use (since we can't, in practice, add up infinitely many terms), but a gorgeous formula!

Riemann's revolutionary formula

The great geometer Riemann only wrote one paper that could be called number theory, but that one short memoir has had an impact lasting 140 years, and its ideas today define the subject we call analytic number theory. In our terms, Riemann's idea is simple, albeit rather surprising: *Try counting the primes as a sum of waves.* His precise formula is a bit too technical for this talk, but we can get a good sense of it from the approximation

$$\frac{\#\text{ of primes up to } x \;-\; \int_2^x \frac{dt}{\log t}}{\sqrt{x}/\log x} \approx -1 - 2 \sum_{\substack{\gamma > 0 \text{ and } \frac{1}{2}+i\gamma \\ \text{is a zero of } \zeta(s)}} \frac{\sin(\gamma \log x)}{\gamma}. \qquad (\dagger)$$

Notice that the left side of this formula is suggested by Gauss's guess: it is the error term when comparing Gauss's guess to the actual count for the number of primes up to x, divided by what appeared from our data to be about the size of the error term, namely $\sqrt{x}/\log x$ (which is close to the size of our first guess, $\sqrt{x}$).

The right side of the formula bears much in common with our formula for $x - 1/2$. It involves a sum of sine functions with γ employed in two different ways in place of $2\pi n$: namely, inside the sine (the reciprocal of the "wavelength") and dividing into the sine (the reciprocal of the "amplitude"). We also get the -2 factor in both formulae. However, the definition of the γ's here is much more subtle than just $2\pi n$ and needs some explanation:

The Riemann zeta function $\zeta(s)$ is defined as

$$\zeta(s) = \frac{1}{1^s} + \frac{1}{2^s} + \frac{1}{3^s} + \cdots$$

for complex number $s = \sigma + it$. The series only converges for sure when $\sigma > 1$; and it is not clear at first sight whether or not this is a genuine limitation on where we can define such a function. In fact, the beautiful theory of "analytic continuation" tells us that often there is a sensible definition of a function on all $s \in \mathbb{C}$ provided there is in part of $\mathbb{C}$; and this does apply to $\zeta(s)$. In other words $\zeta(s)$ can be defined in the whole complex plane (see [17] for details).

We are going to be interested in the "zeros of $\zeta(s)$"; that is, the values of s for which $\zeta(s) = 0$. One can show

$$\zeta(-2) = \zeta(-4) = \zeta(-6) = \ldots = 0$$

(which are called the "trivial zeros"), and that all other $s = \sigma + it$ with $\zeta(\sigma + it) = 0$ satisfy $0 \le \sigma \le 1$.

In Riemann's memoir he stated a remarkable hypothesis:

$$\text{If } \zeta(\sigma + i\gamma) = 0 \text{ with } 0 \le \sigma \le 1 \text{ then } \sigma = \tfrac{1}{2};$$

that is, the non-trivial zeros all lie on the line $\operatorname{Re} s = 1/2$. This leads to the definition of the γ's in our formula; these are the values of γ for which $\zeta(\frac{1}{2} + i\gamma) = 0$. It has been proven that there are infinitely many such zeros, so you might ask how we add up this infinite sum? Simple, add up by order of ascending $|\gamma|$ values and it will work out.

Formula (†) above holds if and only if Riemann's hypothesis holds. If it doesn't hold then there is a similar formula, but it is rather complicated and technically far less pleasant, since the coefficients $1/\gamma$, which are constants, get replaced by functions of x.

So we want Riemann's hypothesis to hold because it gives the formula above, and that formula is a delight. Enrico Bombieri, one of the great prime number specialists of our time, notes:

> *That the distribution of primes can be so accurately represented in* [this way] *is absolutely amazing and incredibly beautiful. It tells of an arcane music and a secret harmony composed by the prime numbers.*

That is, it is like the formula for breaking sound down into sine waves. Thus, one can paraphrase the Riemann Hypothesis as:

> *The primes have music in them.*

The Riemann Hypothesis: The evidence

The Riemann Hypothesis. *All zeros of* $\zeta(s)$ *with* $0 \le \operatorname{Re} s \le 1$ *satisfy* $\operatorname{Re} s = \frac{1}{2}$.

Riemann's memoir (1859) did not contain any hints as to how he made this remarkable conjecture. He merely wrote: "It is very probable that all [$\operatorname{Re} s = 1/2$]. Certainly one could wish for a stricter proof here; I have temporarily set aside the search after some fleeting futile attempts." For many years this conjecture was held up as evidence of the heights one could attain by sheer intellect alone. It seemed as if Riemann had come to this very numerical prediction on the basis of some profound undisclosed intuition, rather than pedestrian calculation — the ultimate conclusion of the power of pure thought alone.

In 1929, many years after Riemann's death, the prominent number theorist Siegel heard that Riemann's widow had donated his scratch paper to the Göttingen University library. It was quite an undertaking, deciphering Riemann's old notes, but Siegel uncovered several jewels. First he found a tremendously useful formula not quite fully developed by Riemann (so not included in his published memoir) which he now brought to flower (though it took him three years to produce a proof despite having the formula in front of him). Second, Siegel discovered pages of substantial calculations, including several of the lowest zeros calculated to several decimal places. So much for "pure thought alone".

There is a long history of computing zeros of $\zeta(s)$, and the very question is synonymous with several great events in the history of science. When the earliest computers were up and running, what was one of the first tasks set for them? Computing zeros of the Riemann zeta function. (This computation, done on the Manchester University Mark 1 Computer, was Alan Turing's last publication.) When the Clay Math Institute established seven million dollar prizes for problems to be solved in the new millennium, the Riemann Hypothesis headed the list (though, rest assured, there are far easier ways of earning a million dollars). By November of last year, the lowest ten billion zeros had been computed (by Stephan Wedeniwski of IBM Deutschland) and every last one of them lies on the $1/2$-line (that is, is of the form $1/2 + i\gamma$). This seems to be pretty good evidence for the truth of the Riemann hypothesis, but who knows? Perhaps the ten billion and first zero does not lie on the half line. Am I being too cautious? Maybe, but maybe not… Remember Gauss's prediction for the count of primes doesn't get smaller than the actual count until we get out beyond 10^{316}, which is a lot further out than 10^{10} (ten billion).

My own view is that Riemann's formula, as discussed above, is far too beautiful not to be true; yes, I believe the primes have music in them.

The variance for the count of primes: A new beginning

In her 1976 Ph.D. thesis Julia Mueller, following up on a suggestion of her advisor, Pat Gallagher, revisited the old question of the variance for the count of the number of primes (compared to Gauss's prediction). Remembering that the Gauss–Cramér model did not give a prediction that could possibly be correct, Mueller developed Riemann's approach to get a better idea, looking at the slightly more refined question of the distribution of primes in short intervals around x (for example, between x and $x + x^{\delta}$ for values of δ between 0 and 1 — this is, in fact, closer to Gauss's original assertion), and established an important link. Building on her work, Goldston and Montgomery made the remarkable discovery that a good understanding for the variance is *equivalent* to a proper understanding of the spacing between pairs of zeros of $\zeta(s)$.

Riemann showed that understanding the count of primes is equivalent to knowledge of the zeros of $\zeta(s)$; and that the count is predictable from a beautiful natural formula if all the non-trivial zeros lie on the half-line. These new ideas went one big step further if the Riemann Hypothesis is true. A basic understanding of the size of the variation in the count of primes can be obtained by looking at pairs of zeros and the distance between them.

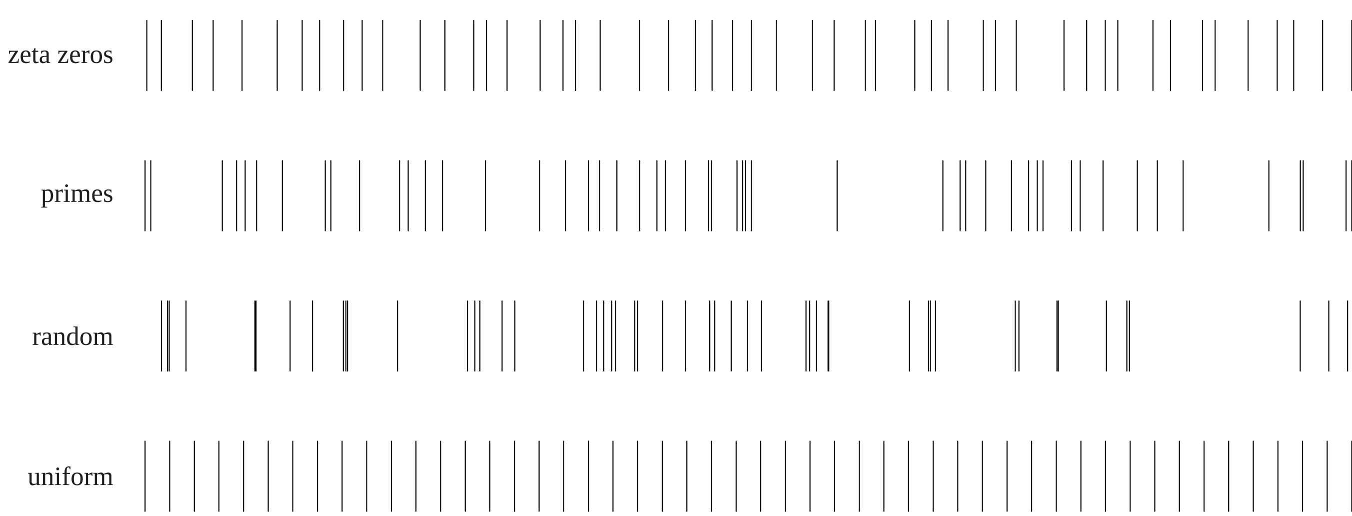
Assuming the Riemann Hypothesis, the zeros are of the form $\frac{1}{2} \pm i\gamma_1, \frac{1}{2} \pm i\gamma_2, \ldots$ with

$$0 \leq \gamma_1 \leq \gamma_2 \leq \gamma_3 \leq \cdots$$

up to height T (that is, the γ_n with $0 \leq \gamma_n \leq T$). We might ask how those γ_j are distributed on the line segment $[0, T]$. Do they look like randomly selected numbers on that interval? Or do they seem to adhere to some other pattern? In the diagram below we compare the data for some zeros of $\zeta(s)$ with set of points from distributions corresponding to other mathematical phenomena.

It's not hard to see that the data for the γ_j's doesn't much look like randomly chosen numbers (a Poisson process). Indeed, in the distribution for randomly chosen values we see that the points do occasionally clump together (making them more-or-less indistinguishable), whereas the γ_j's do not seem to clump together anywhere like as often, and seem to be better spread out than random. If anything the γ_j's seem almost to repel one another.

Just a couple of years earlier, motivated by an entirely different question in number theory, Montgomery had wanted to understand this distribution, and so made a precise conjecture for understanding gaps between zeros.



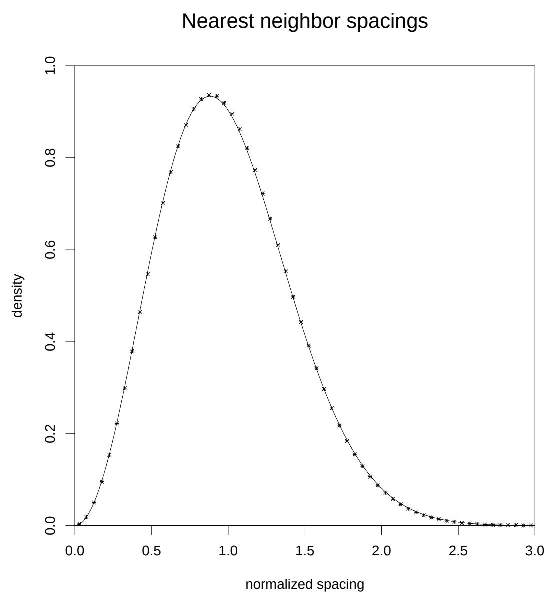
Fifty consecutive zeros of the zeta function; fifty consecutive primes starting with the one-millionth; fifty random numbers; fifty equally spaced numbers. The data on the top row and the general layout are borrowed from "Chaotic motion and random matrix theories", by O. Bohigas and M. J. Giannoni, in *Mathematical and Computational Methods in Nuclear Physics*, Springer-Verlag, 1984.

Montgomery's Conjecture (1973)**.** *The expected number of zeros in a gap of length* T *times the average gap, following a zero, is*

$$\int_0^T \left(1 - \left(\frac{\sin \pi u}{\pi u}\right)^2\right) du.$$

If the zeros were like a random distribution then this value would simply be T; in fact, a careful examination of this conjecture reveals that it does affirm the repulsion of zeros we observed in the limited data of the previous figure. For example, we expect a zero within $1/100$ of a given zero, 1 in 100 times for randomly chosen zeros, but only 1 in 911963 times for zeta function zeros if Montgomery's prediction holds true.

Let us see how Montgomery's prediction compares with data collected by Andrew Odlyzko over the last fifteen years. The graph below, taken from [14], actually measures "nearest neighbor spacings", that is the distribution of $(\gamma_{n+1} - \gamma_n)$/average spacing. The continuous line is Montgomery's prediction; the dots represent data collected by Odlyzko (a scatterplot). The data is based on a billion zeros near the 1.3×10^{16}-th zero.



What a good fit! One surely believes Montgomery's prediction. In fact, Montgomery even proved his prediction is true in part (technically he showed that the "Fourier transform" of his distribution function is correct in a small range if the Riemann Hypothesis is true).

Quantum Mechanics enters the picture

Scientific Progress can happen in strange ways. Sometimes it seems like the same revolutionary idea occured to two people at the same time, though they have not been in contact, without any obvious recent changes in the landscape of the subject. So why simultaneously? Chance meetings sometimes stimulate great new ideas, and so it was in our subject. Soon after developing his new outlook on the count of zeros, Montgomery passed through Princeton wanting, in particular, to discuss his idea with the two great analytic number theorists, Selberg and Bombieri, both at the Institute for Advanced Study. The Institute has a communal tea each working day, where people from different disciplines can and do socialize and discuss ideas of mutual interest. Freeman Dyson, the great mathematical physicist (though originally a number theorist), was at tea, and Montgomery explained to him what he was up to. Montgomery was taken aback to discover that Dyson knew very well the rather complicated function appearing in Montgomery's conjecture, and even knew it in the context of comparing gaps between points with the average gap. However — here's the amazing thing: It wasn't from number theory that Dyson knew this function but from quantum mechanics. It is precisely the function that Dyson himself had found a decade earlier when modelling energy levels in complex dynamical systems when taking a quantum physics viewpoint. It is now believed that the same statistics describe the energy levels of chaotic systems; in other words, quantum chaos!

Could this be a coincidence? Surely not. Is it an indication of something lying much deeper? The questions beg answers and provide the starting point for much of the recent progress.

Mathematically speaking, the equations of quantum chaos are relatively simple to develop compared to those of prime number theory, and so much more was (and is) known about them. The Montgomery–Dyson observation cries out for mathematicians to develop further formulae for zeros of the Riemann zeta function and to compare them with those of quantum chaos. The first things to look at were the models physicists had developed for comparing close-by zeros, not just two at a time, but also three at a time, four at a time, or even n at a time (the so-called "n-level correlations").

Although these led to obvious predictions for the zeros of $\zeta(s)$, showing these predictions to be to some extent correct was a major barrier, attempted by many but frustratingly difficult to accomplish ... It was more than twenty years until Rudnick and Sarnak in 1996 made the breakthrough and proved the analogy of Montgomery's result (assuming the Riemann Hypothesis, the Fourier transform of the predicted n-level correlation function is correct in a small range; in fact, the range directly analogous to Montgomery's range). Now number theorists had to believe that at least some of the predictions that could be made by analogy to quantum chaos had to be correct, and a flood of research ensued.

Also in 1996, two mathematical physicists, Bogolmony and Keating, re-derived the Montgomery–Dyson prediction (for n-level correlations without any restriction on the range) from a new angle (which was anticipated for the $n = 2$ case by [12]). They took a classic conjecture of analytic number theory, the Hardy–Littlewood version of the prime k-tuplets conjecture, and showed that this also led to the same conclusion. Now there was no room for doubt — these predictions have to be correct!

Mathematicians at play: The Sarnak School

The Montgomery–Dyson predictions tell us that zeros of the Riemann zeta function "behave" much like numbers predicted in certain questions in quantum chaos. Although different chaotic systems have different quantum energy levels, one remarkable observation is that the energy levels that arise are distributed in one of only a handful of ways.

There are many different types of "zeta functions" that appear in number theory; not only in counting primes, but also in a fundamental way in algebraic, arithmetic, and analytical problems. For example, Wiles's proof of Fermat's Last Theorem is all about a certain kind of zeta function. All of these zeta functions share various properties with the original one: they have some easily identifiable "trivial" zeros; otherwise all other zeros lie in a "critical strip" (like $0 \leq \mathrm{Re}\, s \leq 1$). To name one more property, the most important, we believe that all of their non-trivial zeros lie on some critical line (like $\mathrm{Re}\, s = 1/2$), a "Riemann Hypothesis". Sarnak became intrigued with determining whether the spacings between the zeros of other zeta functions were also predictable by these same handful of distributions from quantum chaos.

Together with Rubinstein they did large scale calculations and found excellent experimental agreement between the distributions of zeros of various zeta functions and the energy levels of various quantum chaotic systems. Then Katz and Sarnak thought to experiment with other, rather different, data of interest to number theorists; for example, how about the lowest zero for each zeta function? Should that be distributed according to one of these magical distributions? Experimental data implied a quantum chaotic model for this question and several others (see [9]). Finally they looked at analogies of the zeta functions that appear in algebraic geometry, a field far removed from quantum chaos. These have finitely many zeros, and the appropriate analogy to the Riemann Hypothesis is true for them (some optimists feel the proof(s) of this might point the way to a proof of the real Riemann Hypothesis; many of us have our doubts). Katz and Sarnak reasoned that since the Riemann Hypothesis is known to be true for these zeta functions (due to Deligne), perhaps they could go one step forward and actually prove the analogy to Montgomery's pair correlation conjecture, or even the Montgomery–Dyson predictions.

In one of the most remarkable works of recent number theory, Katz and Sarnak did what they set out to do [10], using the results of Deligne in a perhaps unexpected and highly ingenious manner. Their four hundred page book is a landmark achievement: motivated by dubious forecasts from quantum chaos they proved a deep and profound result for zeta functions in an entirely unrelated field — lovely!

Physicists at play: The Berry School

Just as a new generation of number theorists, led by Peter Sarnak, have learned to exploit these connections in new, exciting ways, so the next generation of mathematical physicists, led by Sir Michael Berry and his collaborators at the University of Bristol, have been taking new and more aggressive approaches to developing analogies between the two fields. Their basic attitude has been to go out on a limb, stretching analogies in a way that mathematicians would never dare. It's a quite different attitude, one that I find very appealing and a little shocking. They look at equations that they know can't really be formally justified and yet glean much useful information nonetheless.

The key developments come in a series of papers by Berry and Jon Keating and contain perhaps a road map for the future of the study of primes. Some of what they say may not be quite correct, but I'm sure it is close to the truth, and in several problems they make predictions where we number theorists had no idea how to proceed.

The ideas don't flow in just one direction either. The more cautious development in prime number theory allows for several rather precise formulae (such as the Riemann–Siegel formula alluded to earlier), which have helped to correct and modify less pedantically obtained (though analogous) formulae of quantum chaos.

The latest generation of mathematical physicists, led by Jon Keating and Nina Snaith are going one step further, and perhaps most usefully. They are targeting some of the biggest mysteries in the study of the Riemann zeta function; for instance, what's the largest it gets on a large interval of the half line? Proceeding with great care, they are making predictions about important problems on which prime number experts had had no idea how to proceed.

In summary, the more intuitive development of quantum chaos allows more fruitful predictions about the distribution of primes (and beyond). On the other hand the more cautious development of prime number theory leads to more accurate predictions in quantum chaos. This mutually beneficial interaction between two previously unrelated fields is an exciting new development and many researchers in both fields are now turning their attention to such questions. This is exactly what an institution like MSRI is perfect for: the time being ripe, we have the opportunity to bring these two communities together, physically, which would not be possible otherwise, to accelerate these developments.

Much ado about nothing?

Put aside all these developments for a moment, these generalizations, these exciting new formulae. What about the million dollar problem? Do any of these new ideas help us to get a better grip on the Riemann Hypothesis? Is there much chance now that this problem will finally succumb? Just a few years ago (1995), the great analytic number theorist Atle Selberg said:

> *There have been very few attempts at proving the Riemann Hypothesis because nobody has had a really good idea about how to do it.*

An old idea of Hilbert and Pólya for proving the Riemann Hypothesis is to find a quantum chaotic system in which every zero of the Riemann zeta function corresponds to an energy level of the system (they put this is in a somewhat different language). If such a quantum chaotic system exists, it must have several very special properties. Recently Berry and Keating gave even more restrictions on such a system (associating the primes to the periodic orbits of such a chaotic system), arguably pointing the way to finding it. This perhaps provoked Berry — a bold knight if there ever was one — to say in 2000:

> *I have a feeling that the Riemann Hypothesis will be cracked in the next few years. I see the strands coming together.*

It could be that he is correct, though I suspect that the proof is still a long way off. Nonetheless these new findings are the most exciting in many years and promise, at the very least, a much better understanding of the Riemann zeta function.

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Current and Forthcoming Workshops

Most of these workshops are offered under the auspices of one of the current programs (page 5). For more information about the programs and workshops, see http://www.msri.org/calendar.

June 3 to 7, 2002: *Finsler Geometry*, organized by David Bao, Robert Bryant, S. S. Chern, and Zhongmin Shen.

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October 7 to 11, 2002: *Semidefinite Programming and Applications*, organized by Dimitris Bertsimas, Stephen Boyd, Laurent El Ghaoui (chair), and Bernd Sturmfels.

November 4 to 8, 2002: *Quantum Information and Error Correction*, organized by Richard Jozsa (chair).

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December 2 to 6, 2002: *Commutative Algebra: Local and Birational Theory*, organized by Craig Huneke (chair), Paul Roberts, Karen Smith, and Bernd Ulrich.

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