Spring 2012

EMISSARY

Mathematical Sciences Research Institute

www.msri.org

Randomness, Change, and All That

A word from Director Robert Bryant

statistical mechanics
conformal invariance
evolution equation
mixing times
dimer model
random sampling
lattice models
random clusters
Schramm–Loewner
critical exponent
Markov chains
percolation
scaling limits
Ising model

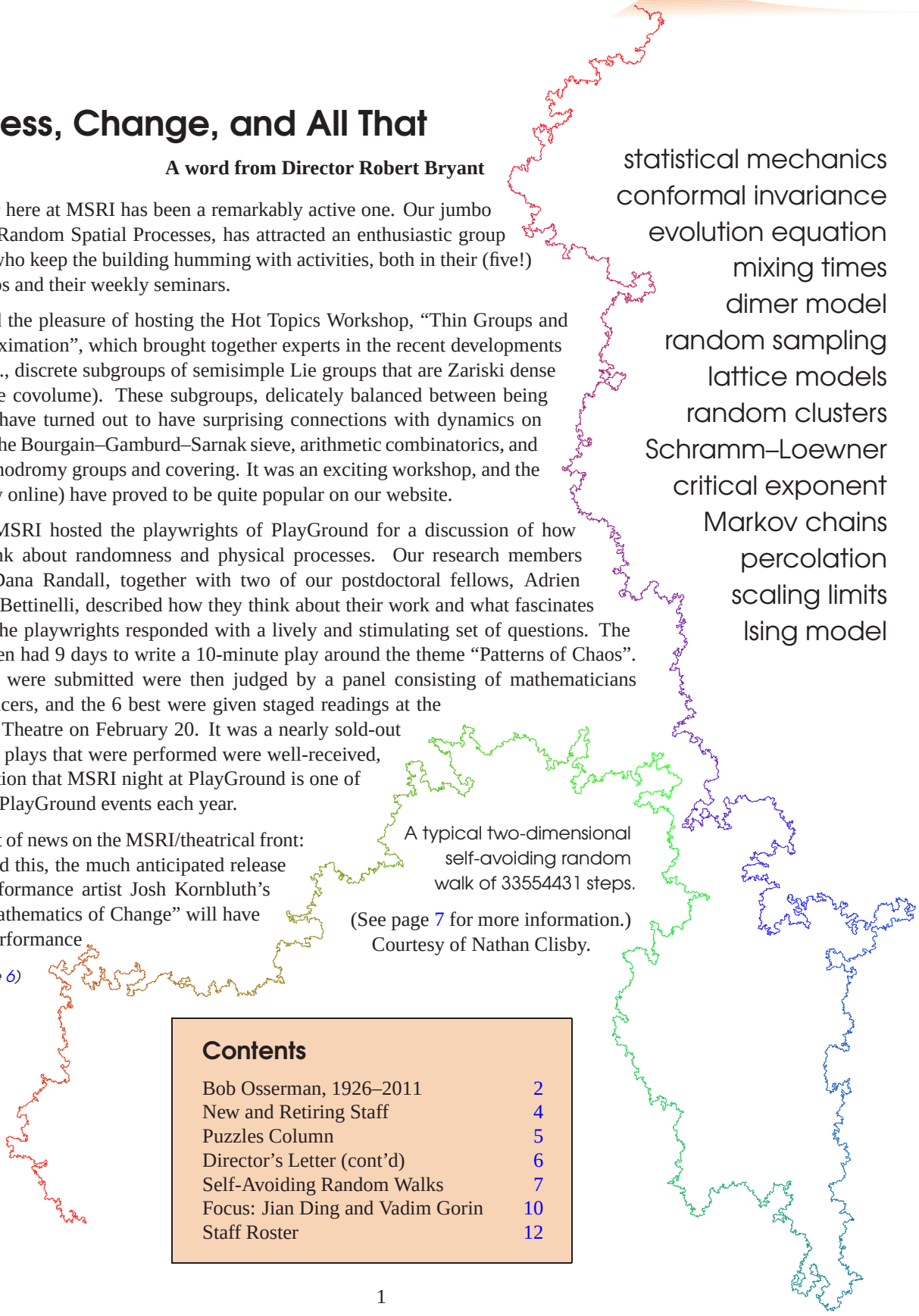
The spring semester here at MSRI has been a remarkably active one. Our jumbo scientific program, Random Spatial Processes, has attracted an enthusiastic group of mathematicians who keep the building humming with activities, both in their (five!) scheduled workshops and their weekly seminars.

In February, we had the pleasure of hosting the Hot Topics Workshop, "Thin Groups and Super-strong Approximation", which brought together experts in the recent developments in "thin groups" (i.e., discrete subgroups of semisimple Lie groups that are Zariski dense and yet have infinite covolume). These subgroups, delicately balanced between being "big" and "small", have turned out to have surprising connections with dynamics on Teichmüller space, the Bourgain–Gamburd–Sarnak sieve, arithmetic combinatorics, and the geometry of monodromy groups and covering. It was an exciting workshop, and the talks and notes (now online) have proved to be quite popular on our website.

Also in February, MSRI hosted the playwrights of PlayGround for a discussion of how mathematicians think about randomness and physical processes. Our research members James Propp and Dana Randall, together with two of our postdoctoral fellows, Adrien Kassel and Jérémie Bettinelli, described how they think about their work and what fascinates them about it, and the playwrights responded with a lively and stimulating set of questions. The playwrights each then had 9 days to write a 10-minute play around the theme "Patterns of Chaos". The 20+ plays that were submitted were then judged by a panel consisting of mathematicians and theatrical producers, and the 6 best were given staged readings at the Berkeley Repertory Theatre on February 20. It was a nearly sold-out evening, and the six plays that were performed were well-received, continuing the tradition that MSRI night at PlayGround is one of the most successful PlayGround events each year.

There's one more bit of news on the MSRI/theatrical front: By the time you read this, the much anticipated release of the DVD of performance artist Josh Kornbluth's monologue "The Mathematics of Change" will have taken place. This performance

(continued on page 6)



A typical two-dimensional self-avoiding random walk of 33554431 steps.

(See page 7 for more information.)
Courtesy of Nathan Clisby.

Contents

Robert Osserman, 1926–2011

Bob Osserman, our esteemed and beloved friend and colleague, died at home on the evening of November 30th, surrounded by his family and loved ones.

Bob is remembered as not only a first-class mathematician, teacher, and mentor, but as an enormously cultured, thoughtful, and wise man who generously shared his insights into how mathematical ways of thought interact with and illuminate our culture. We are extraordinarily fortunate to have had Bob's participation in MSRI as both a Deputy Director and, for so many years, as our Special Projects Director. MSRI's public outreach efforts set a high standard under his leadership that will long be remembered and admired; indeed, his legacy of public conversations on cultural matters related to mathematics is a jewel in MSRI's treasury.

* * *

(A version of this obituary with links to all the activities mentioned can be found at www.msri.org/~osserman.)

Bob Osserman wrote his Ph.D. thesis on the subject of Riemann surfaces under the direction of Lars V. Ahlfors at Harvard University. He continued to work on geometric function theory and later on differential geometry, combining the two in a new global theory of minimal surfaces. Bob also worked on the isoperimetric inequality and related geometric questions. After obtaining his PhD, he joined the faculty of Stanford University where he remained until his retirement, with periods of leave to serve as Head of the Mathematics Branch at the Office of Naval Research, Fulbright Lecturer at the University of Paris and Guggenheim Fellow at the University of Warwick. Some later papers were on astronomy, geometry, cartography and complex function theory. Most recently, he wrote on geometric and other aspects of Saarinen's "Gateway Arch" in St. Louis.

In 1987, he was named Mellon Professor for Interdisciplinary Studies, and, in 1990, he joined MSRI as half-time Deputy Director.

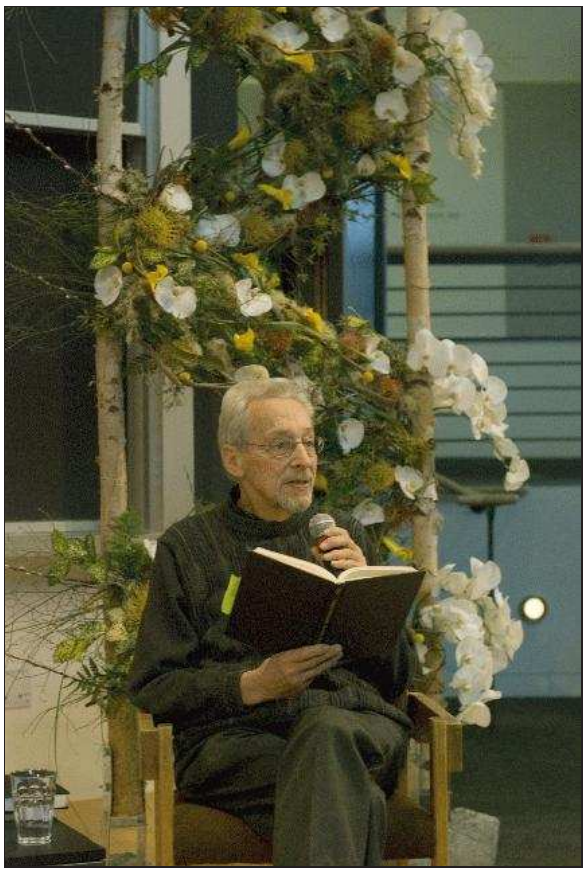
In 1995, he took on the new position of Special Projects Director at MSRI. He became increasingly involved with outreach activities to the general public, starting with the "Fermat Fest" in 1993 and the subsequent production of a videotape with accompanying booklet. In 1999, he engaged in a public conversation with playwright Tom Stoppard on "Mathematics in Arcadia", which is also available on videotape, as is another public event, "Dialog on Galileo: Science, Mathematics, History and Drama", in association with the Berkeley Repertory Theatre, and their production of the play *Galileo*, by Bertolt Brecht. A public conversation with Steve Martin entitled "Funny Numbers" is available on DVD through the AMS. Later public events included "The John Cage Legacy: Chance in Music and Mathematics," with Persi Diaconis, Merce Cunningham, and the musicians of the Merce Cunningham Dance Company, and "M*A*T*H*" with Alan Alda, which may be viewed on DVD, or directly on the MSRI website via VMath.

While at Stanford, he developed and taught a new course — jointly with a physicist and an engineer — designed to present mathematics, science, and technology to a non-technical audience. A portion of the course was elaborated in the book *Poetry of the Universe: A Mathematical Exploration of the Cosmos*, intended to provide the general public with an introduction to cosmology — focusing on a number of mathematical ideas that have played a key role. This book is also available in foreign editions.

During 2004–05, he chaired the committee charged with preparing materials for the April 2005 Mathematics Awareness Month, whose theme was "Mathematics and the Cosmos." Among those materials is a film called "The Right Spin" with astronaut Michael Foale, available on DVD from the AMS.

* * *

In celebration of Bob's life and work, a one-day mathematical conference took place at Stanford on April 21, and on April 22 a reception and memorial service were held at MSRI. At the MSRI memorial, a number of Bob's friends and colleagues spoke about the many ways in which he profoundly enriched our community. All three of Bob's children were present: Paul, from an earlier marriage to Maria Osserman, and two younger sons, Brian (a mathematician) and Stephen, who played the guitar (see photo). Bob's wife of 33 years, Janet Adelman, died in April 2010; she had been a highly distinguished professor in UC Berkeley's English Department until her retirement in 2007.



David Eisenbud

Bob at an MSRI public event in January 2008.

David Eisenbud

David Eisenbud

David Eisenbud

Clockwise from top left:

Stephen Osserman and Amy Schaner perform at the memorial service for Bob at MSRI, on April 22.

With Brian Osserman at MSRI's panel discussion (on October 16, 2005) with writers from the animated show *The Simpsons*.

With actor and director Steve Martin on December 15, 2002.

Bob and Janet at their son Stephen's wedding.

Three New Staff Members Join MSRI. . .

Stefanie Yurus

Stefanie Yurus joins MSRI as Controller bringing more than 20 years of accounting experience working in various industries in the Bay Area including 10 years working for nonprofit organizations. She graduated from U.C. Berkeley with a B.S. in Business Administration and from Golden Gate University with an MBA in Finance. She obtained her CPA while at KPMG in San Francisco, CA. She has served in various leadership roles including national president of the American Women's Society of CPAs and is currently on committee of the AICPA. She is a California native and is married and has a daughter.

Alex Gonzalez, our new Director of Development, comes to MSRI from California State University, Sacramento where he served as a Director of Development. He managed a portfolio that included fund-raising for various academic and student programs as well as leading all foundation and corporate development efforts for the University. Previously, Alex was a Senior Grant Writer and Development Associate for a public radio network in San Francisco. Alex holds a B.A. in Psychology from the University of California, San Diego and an M.A. in Communication Studies from California State University, Sacramento.

Lisa Jacobs is thrilled to rejoin MSRI as the Executive Assistant to the Director and Development Associate. Prior to joining MSRI, Lisa worked in a wide range of academic institutions — including The Art Institute of California, Parsons The New School for Design, and New York University — as an academic advisor to undergraduate students. In earlier years, Lisa worked as a properties artisan at The Juilliard School and Spaeth Design. She received a B.A. in English Literature from Columbia University and an M.A. from NYU's Gallatin School.

Alex Gonzalez

Lisa Jacobs

. . . and One Retires

Marsha Borg, MSRI's Facilities and Administrative Coordinator, retired last November after 17 years working for the Institute in a variety of capacities. Resourceful and always helpful, it was she in the first instance who ensured that untold thousands of MSRI members and visitors could settle in, find what they needed, and be productive.

From 2001 through 2006, she met and gracefully overcame new challenges daily, as MSRI planned and carried out its building expansion. Organizing meetings, hiring contractors, sorting out a mountain of details, interfacing with dozens of campus entities ranging from the Comptroller's Office to the Fire Marshal, finding and remodeling MSRI's temporary quarters, supervising the move to them, making frequent visits to the construction site, and finally ushering us back into the promised land of Chern Hall — Marsha made it all look easy, but those of us who were there know that it was anything but.

Marsha and her husband Garry are spending their well-earned retirement in Hesperia, Southern California, growing their own vegetables and planting an orchard. As Marsha's former colleagues, we miss her daily, and we wish her and Garry many enjoyable years in peace and good health.

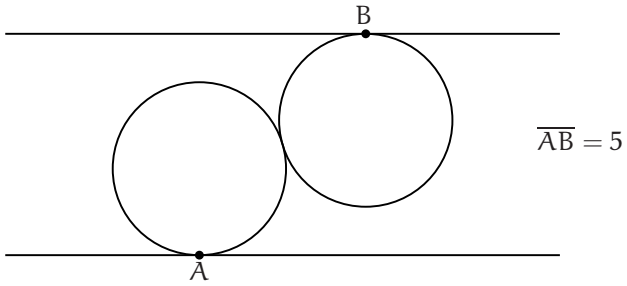
Puzzles Column

Joe P. Buhler and Elwyn Berlekamp

1. Several people sit at a round table and eat raisins from a basket that originally contained 1001 raisins. At some moment, each person has eaten either twice as many raisins as his or her neighbor on the right, or 10 raisins fewer than that neighbor. Prove that the basket still contains some raisins.

Comment: This problem was one of the easier ones on the 2012 Bay Area Mathematical Olympiad (BAMO) exam given in February.

2. Two circles of the same size sit in a 4-inch wide strip of the plane. Each circle is tangent to one edge of the strip, the tangency points being 5 inches apart. The circles are also tangent to each other. What is their radius?



Comment: As recounted in the New York Times "Numberplay" online blog, Will Shortz hosted a "MoMath Masters Tournament" as a fundraiser for MoMath — the Museum of Mathematics (which will open in New York sometime next year). This problem is derived from one of those questions. In the original version, solvers knew the radius can be expressed as an irreducible fraction a/b with $a-b$ a one-digit number, and had two minutes to find that number!

3. Consider an arbitrary horizontal segment AB of length 1 located above the x-axis, and consider all (infinitely many) angles AnB with the vertex n, where n runs over all the integers along the x-axis from $-\infty$ to ∞. What is the sum of these angles over all $n \in (-\infty, \infty)$: is it finite or infinite? If it is finite, does it depend on the y-coordinate of the segment AB? What are possible values for this sum?

Comment: The problem is due to Gregory Galperin.

4. Find a tiling of the unit square by 7 isosceles right triangles, no two of which are congruent.

Comment: This problem is due to Bob Jewett, and we learned of it from Al Hales. There are a variety of additional related questions that will appear on this column's MSRI web site in due course.

5. A quiz is given to a class, and the teacher notes that the answered quizzes have the following properties:

(a) No student got all questions right.

(b) Every question was answered correctly by exactly 4 students.

(c) For every pair of questions, exactly one student got both of them right.

What is the largest possible number of questions on the quiz?

Comment: This is taken from a recent edition of the contest organized monthly by the Berkeley Math Circle. The curious reader will find past contests, with solutions and lists of winners, on the BMC site, mathcircle.berkeley.edu.

6. Are there any nonzero polynomials $g(x, y)$ in two variables such that $g(F_n, F_{n+1}) = 0$ for all integers n? Here F_n denotes the n-th Fibonacci number.

Comment: This is due to Richard Stong.

7. Find all polynomials $f(X)$ with integer coefficients with the property that $f(x)$ and $f(y)$ are coprime whenever x and y are coprime integers.

Comment: This problem was one of the harder ones on the 2012 BAMO.

Named Positions at MSRI (2012–2013)

Both the Simons Professorships and the Eisenbud Professorship are funded by the Simons Foundation. MSRI is grateful for the Foundation's support for these critical positions.

Academic Year 2012-13 – Commutative Algebra

Simons Professorships:

Luchezar Avramov, University of Nebraska-Lincoln
Alicia Dickenstein, Universidad de Buenos Aires
Craig Huneke, University of Kansas
Frank-Olaf Schreyer, Universität des Saarlandes
Jerzy Weyman, Northeastern University

Fall 2012 – Cluster Algebras

Eisenbud Professorship:

Andrei Zelevinsky, Northeastern University

Simons Professorships:

Bernhard Keller, Université de Paris VII (Denis Diderot)
Idun Reiten, Norwegian University of Science and Technology (NTNU)

Spring 2013 – Noncommutative Algebraic Geometry and Representation Theory

Simons Professorships:

Bill Crawley-Boevey, University of Leeds
Toby Stafford, University of Manchester
Michel Van den Bergh, Universiteit Hasselt

A Word from the Director

(continued from page 1)

(to a sell-out crowd) was filmed in April 2011 over two nights here at MSRI, and we are excited that this concert video will soon be available. Josh's piece is a humorous and touching account of his first semester at Princeton, when his dreams of being a mathematician met the subject of calculus, the mathematics of change. Anyone who has ever taught or taken calculus will be able to relate.

Our plans for our collaboration with other mathematics institutes around the world in the Mathematics of Planet Earth 2013 project have seen some exciting developments. This activity will focus attention on the mathematical challenges inherent in addressing the global problems of sustainability, managing diseases and epidemics, management of resources, and studies of climate and its effect on life on earth. You can find out more about MPE2013 and MSRI's involvement at www.mpe2013.org. We are also supporting travel by junior faculty at our academic sponsoring institutions to the first Mathematical Congress of the Americas, to take place August 5–9, 2013, in Guanajuato, Mexico (www.mca2013.org).

Although it was too late to make it into our Fall 2011 Emissary, we had an enormously successful conference on the centennial of the birth of our founding director, Shiing-Shen Chern. We marked the occasion with the dedication of a statue of Professor Chern, installed in the newly renovated entry to Chern Hall, and the premiere of a film, *Taking the Long View: The Life of S.-S. Chern*. The statue, commissioned with the support of the Simons Foundation, is by the leading Chinese-American sculptor Wei Li ("Willy") Wang, whose work graces major museums in both China and the US. The film, which contains much archival material from Chern's life and features extensive interviews with many of Chern's family, friends, and collaborators, is by George Csicsery, famous for his film biographies of other mathematical figures, such as Paul Erdős and Julia Robinson, and his interviews with a wide range of mathematicians. (DVDs of this remarkable film can be obtained by going to the web site http://www.takingthelongviewfilm.com.)

This has also been a semester of considerable change of personnel here at MSRI. We are pleased to be welcoming new members of the Board of Trustees: Karen Uhlenbeck (University of Texas at Austin) and Mark Kleiman (Factorial Partners of New York). At the Trustees meeting in March, the Board elected three new members of the Human Resources Advisory Committee, who will serve three-year terms: Lloyd E. Douglas (University of North Carolina at Greensboro), Dagan Karp (Harvey Mudd College) and Monica Stephens (Spelman College). The Board also appointed Richard Taylor (Institute for Advanced Study) as the next co-chair of the Scientific Advisory Committee and appointed three distinguished new SAC members: Andrea Bertozzi (UCLA), Christopher Hacon (University of Utah), and Maryam Mirzakhani (Stanford).

The three-year term of our current Associate Director, David Auckly, is coming to a close this summer. Dave's enthusiasm for and tireless promotion of our outreach and education activities has made these programs remarkably successful and has stimulated an amazing growth in the number of students and mathematics educators that MSRI serves. We will miss Dave, but I'm pleased to report that we will have a new member of our Directorate to carry on this important work. Professor Alissa S. Crans, from Loyola Marymount College, will be joining us at the end of July as our new Director of Educational Activities. She has a wonderful record of service to the mathematics community in promoting diversity, outreach, and education activities, and we are excited about working with her to extend that record through MSRI's programs.

Finally, I am very pleased to announce our new Development Director, Alexander Gonzalez, Jr., who just joined our staff in March (see page 4 for his background). We are looking forward to working with Alex to help enhance MSRI's capabilities in outreach and service to the mathematics community.

MSRI invites **membership applications** for the 2013–2014 academic year in these positions:

Research Professors	by October 1, 2012
Research Members	by December 1, 2012
Postdoctoral Fellows	by December 1, 2012

In the academic year 2013–2014, the research programs are:

Mathematical General Relativity, Aug 19 to Dec 20, 2013

Organized by Yvonne Choquet-Bruhat (U. of Paris), Piotr T. Chruściel (U. of Vienna), Greg Galloway (U. of Miami), Gerhard Huisken (Albert Einstein Institute), James Isenberg (U. of Oregon; chairman), Sergiu Klainerman (Princeton U.), Igor Rodnianski (Princeton U.), Richard Schoen (Stanford U.).

Optimal Transport: Geometry and Dynamics, Aug 19 to Dec 20, 2013

Organized by Luigi Ambrosio (Scuola Normale Superiore di Pisa), Yann Brenier (CNRS, Université de Nice), Panagiota Daskalopoulos (Columbia U.), Lawrence C. Evans (U. of California at Berkeley), Wilfrid Gangbo (Georgia Inst. of Technology), Robert J. McCann (U. of Toronto, chair), Felix Otto (Universitat Bonn), and Neil S. Trudinger (Australian National U.).

Model Theory, Arithmetic Geometry and Number Theory, Jan 20 to May 23, 2014

Organized by: Ehud Hrushovski (Hebrew U., Israel) , François Loeser (Université Pierre et Marie Curie, France), David Marker (U. of Illinois, Chicago), Thomas Scanlon (U. of California, Berkeley), Sergei Starchenko (U. of Notre Dame), Carol Wood (Wesleyan U., chair).

Algebraic Topology, Jan 20 to May 23, 2014

Organized by Vigleik Angeltveit (Australian National U.), Andrew J. Blumberg (U. of Texas-Austin), Gunnar Carlsson (Stanford U.), Teena Gerhardt (Michigan State U.), Michael A. Hill (U. of Virginia; chair), and Jacob Lurie (Harvard U.)

MSRI uses **MathJobs** to process applications for its positions. Interested candidates must apply online at www.mathjobs.org after August 1, 2012.

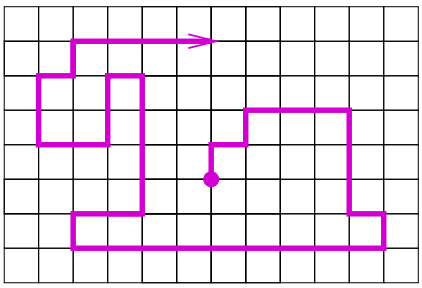
Self-avoiding walks

Mireille Bousquet-Mélou and Anthony J. Guttmann

We are now halfway through the Random Spatial Processes program at MSRI. This program follows recent progress in understanding a variety of two-dimensional models of combinatorics and statistical physics (some of the "buzzwords", or key objects of study, are listed on page 1). One particular model, the self-avoiding walk model, is conceptually simple, yet is still shrouded in mystery. There are however promising signs on the horizon, and a sense among the experts that a breakthrough is potentially imminent.

Introduction

A self-avoiding walk (SAW) on a lattice can be loosely described as a path joining adjacent vertices that never revisits a vertex. More precisely, a SAW of length n is a sequence of distinct vertices $w_0, w_1, \ldots, w_n$ such that each vertex is a nearest neighbor of its predecessor:



The problem of studying self-avoiding walks was initially proposed as a mathematical problem in a posthumously published paper by Orr in 1947, and shortly thereafter was proposed as a model of long chain polymers in dilute solution by Flory in 1949. Despite this 65 year history, and the deceptive simplicity of the problem, we know remarkably little about it rigorously. Making the problem more frustrating is that there exists a large number of results in the mathematical physics literature that are believed to be exact, but which have so far eluded mathematical proof, in large part.

Typical questions include the number c_n of SAW of length n, where these are considered equivalent up to translations. The number c_n grows exponentially. Indeed, this is one of the few properties we can easily prove, by virtue of the obvious sub-multiplicative inequality $c_{n+m} \le c_n c_m$. It follows that

$$\mu := \lim_{n\to\infty} c_n^{1/n} = \inf_n c_n^{1/n}$$

exists. Even the value of this "growth constant" μ is difficult to estimate. Only in 2010 was μ for one lattice (by lattice we mean a periodic graph), the honeycomb lattice, actually proved to take the value $\sqrt{2+\sqrt{2}}$ (see next section). For other lattices we have only numerical estimates. For example, for the square lattice the best current estimate is $\mu = 2.638158530323 \pm 2\times 10^{-12}$, based on extensive series of Jensen.

In fact it is believed that $c_n \sim \text{const.}\, \mu^n n^g$, so long as $d \neq 4$. The *critical exponent* g is believed to depend on the dimension, but not on the details, of the lattice. In particular, it is predicted to be a rational number, namely $11/32$, in two dimensions. Despite the simplicity of this value, we still cannot prove the existence of this exponent, let alone establish its value rigorously.

Given the exponential growth rate of c_n, an enormous amount of effort has been expended to develop efficient methods for counting SAWs. For the square lattice, Jensen has extended the known series to 79 step walks, for which he finds $c_{79} = 1019471029355746619378790007192 3676$. Methods for calculating these astonishing numbers are quite complicated, but the best current algorithm still has an exponential complexity, of about 1.3^n (while direct counting would have complexity 2.64^n).

Another important measure is the average size of a SAW of length n, taken uniformly at random. The most common measure is the mean-square end-to-end distance, which is believed to behave as

$$\mathbb{E}_n(|w_n|^2) \sim \text{const.}\, n^{2\nu},$$

(for lattices in dimensions other than 4), where ν is another critical exponent. Again, its existence hasn't been proved, but it is accepted that for two-dimensional lattices $\nu = 3/4$.

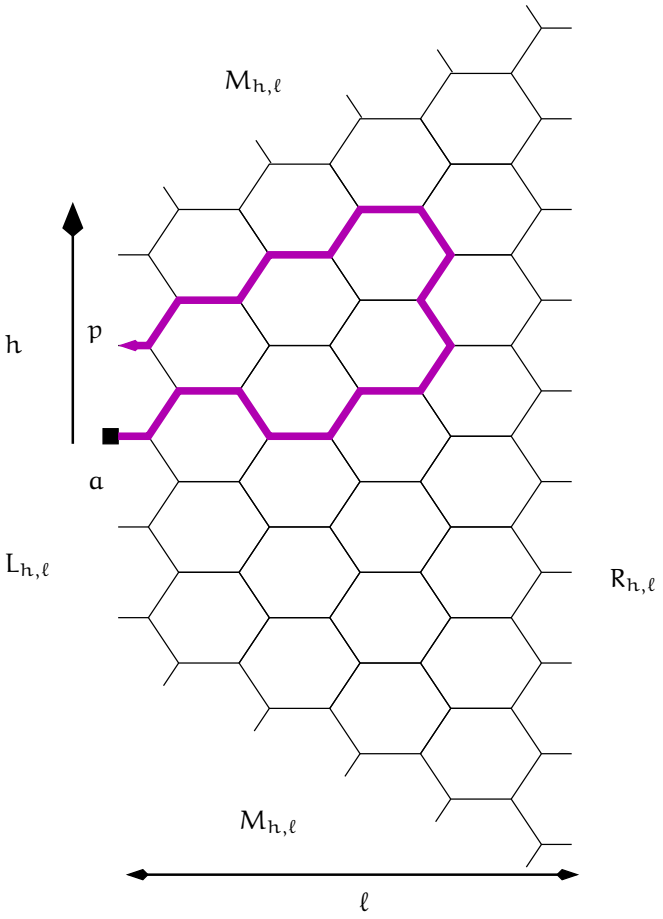
Finally, a veritable treasure trove of rigorous results would be unlocked if we could prove that, in the large size limit, two-dimensional random SAWs are described by one of the SLE processes (Schramm–Loewner Evolution), which, in the past 20 years, have been proved to describe the limit of several discrete models in combinatorics and statistical physics. Indeed, Lawler, Schramm and Werner proved that, if the scaling limit of SAWs exists and is conformally invariant, then this limit has to be $\text{SLE}_{8/3}$. This has been checked via simulation by Kennedy. This would in particular imply the conjectured values of the exponents g and ν.

The honeycomb lattice

A breakthrough was achieved in 2010, when Duminil-Copin and Smirnov proved that the growth constant on the honeycomb lattice is $\mu = \sqrt{2+\sqrt{2}}$, as predicted by Nienhuis 30 years previously. The argument is actually so simple that we can sketch it here.

We consider SAWs that start from a point a located on the left side of a trapezoid $\mathcal{T}$ on the honeycomb lattice, having width ℓ and height h, as shown at the top of the next page. For p a mid-edge of $\mathcal{T}$, let $F(p)$ be the generating function of SAWs w that end at p, weighted by the number of vertices $v(w)$ and the number of turns $T(w)$ (a left turn counts $+1$, a right turn -1):

$$F(p) \equiv F(p; x, \alpha) := \sum_{w: a \rightsquigarrow p} x^{v(w)} e^{i\alpha T(w)}.$$



A trapezoid $\mathcal{T}$ on the honeycomb lattice.

For instance, the walk in the figure above visits 17 vertices, makes 10 left turns and 7 right turns, so that its contribution to $F(p)$ is $x^{17}e^{3i\alpha}$. Then, if v is any vertex of $\mathcal{T}$ and p_1, p_2, p_3 are the three mid-edges adjacent to it, the following local identity holds:

$$(p_1-v)F(p_1)+(p_2-v)F(p_2)+(p_3-v)F(p_3)=0, \qquad (1)$$

provided

$$x = x_c := 1/\sqrt{2+\sqrt{2}},$$

which is the reciprocal of the conjectured growth constant, and $\alpha = -5\pi/24$. (We regard the honeycomb lattice as embedded in the complex plane $\mathbb{C}$, so $p_i - v$ is a complex number). This identity is easily proved by grouping as pairs or triplets the SAWs that contribute to its left-hand side, as depicted in the figure below. One then checks that the contribution of each group is zero.

Let us now sum (1) over all vertices v of $\mathcal{T}$. Due to the terms $p_i - v$, all terms $F(p)$ such that p is not a mid-edge of the border disappear. After a few more reductions based on symmetries, one is left with

$$\left(\cos\frac{3\pi}{8}\right) L_{h,\ell}(x_c) + \frac{1}{\sqrt{2}} M_{h,\ell}(x_c) + R_{h,\ell}(x_c) = 1,$$

where $L_{h,\ell}(x)$ (resp. $R_{h,\ell}(x)$, $M_{h,\ell}(x)$) is the generating function of SAWs w that end on the left side (resp. right side, top or bottom side) of $\mathcal{T}$, weighted by the number of vertices $v(w)$.

By letting h and then ℓ tend to infinity, Duminil-Copin and Smirnov derived from this identity that the generating function of SAWs diverges at x_c, but converges when $x < x_c$. This means that its radius of convergence is x_c, so that the growth constant is $1/x_c = \sqrt{2+\sqrt{2}}$.

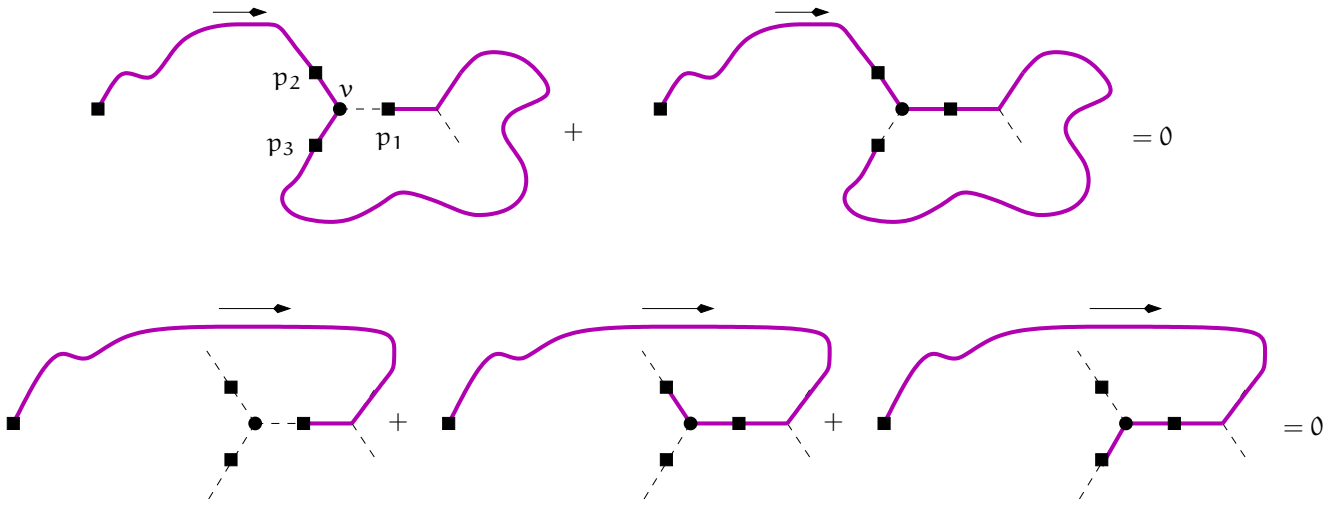
Higher dimensions

Little is known about dimension three, besides quite precise numerical estimates of the growth constant and the critical exponents. Dimension four is known as the upper critical dimension. This is the highest dimension that "feels" the self-avoiding walk constraint. One has the predictions

$$c_n \sim \text{const.}\ \mu^n (\log n)^{1/4}$$

and

$$\mathbb{E}_n(|w_n|^2) \sim \text{const.}\ n(\log n)^{1/4}.$$



A very local proof of the local identity (1).

In dimensions greater than 4, it has been proved that the freedom of walks to escape into another dimension makes them behave like random walks. More precisely, Hara and Slade proved in 1992 a number of important results using the so-called lace expansion. In particular,

$$c_n = \text{const.}\, \mu^n \left(1+O(n^{-\varepsilon})\right)$$

and

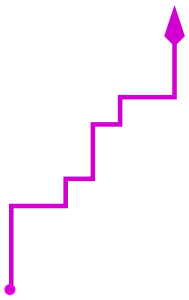
$$\mathbb{E}_n(|w_n|^2) \sim \text{const.}\, n \left(1+O(n^{-\varepsilon})\right),$$

where all constants, including ε, are positive. This establishes the value of the critical exponents, defined earlier, as $g = 0$ and $\nu = \frac{1}{2}$. In fact, Hara and Slade showed that the appropriately scaled SAWs converge weakly to Brownian motion in dimension $d > 4$, thus giving insight into these results.

Exactly solvable models

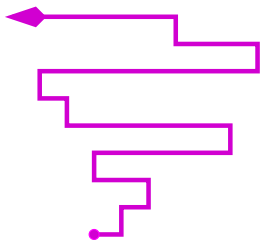
Given how difficult the study of general SAWs seems to be, it is natural to focus on more tractable subclasses of walks.

A first very simple subclass is provided by *directed* walks, that is, walks that take at most two kinds of steps, for instance N and E. Clearly, there are 2^n n-step walks of this type, and the end-to-end distance is linear in n. The associated generating function, $\sum_{n\geq 0} 2^n x^n$, is rational, equal to $1/(1-2x)$.



A directed walk.

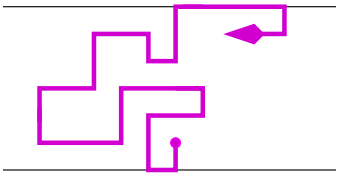
Similar results hold for *partially directed walks*, that is, walks that take at most three types of steps, like N, E, W. Their generating function is $(1+x)/(1-2x-x^2)$, and the associated growth constant is $1+\sqrt{2} \simeq 2.41$. These walks can be alternatively described by saying that, between successive visits to the same horizontal line, the walk consists of E or W steps only.



A partially directed walk.

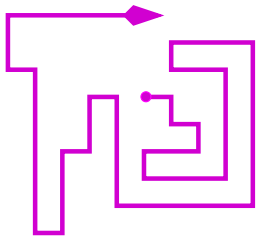
This suggests the following natural extension: in a *weakly directed walk*, the portion of the walk between two visits to the same horizontal line contains only N, S and E steps, or, symmetrically, N, S and W steps. The generating function of these walks is also known explicitly, but is much more complex than the two series above. The growth constant is also larger, being about 2.54, but the end-to-end distance remains linear.

Another class of SAWs having a rational generating function consists of SAW confined to a strip of width k, for any (fixed) k. From a combinatorial viewpoint, these walks have a rather uninteresting structure (and their end-to-end distance remains disappointingly linear), but they provide good lower bounds, improving with k, on the growth constant.



A walk confined to a strip.

Let us finally mention a more attractive class: prudent walks. A walk is *prudent* if, at any stage of its growth, it takes a step that does not point in the direction of a previously visited vertex.



A prudent walk.

The generating function of prudent walks is not known explicitly, but is described by an explicit functional equation. The growth constant is about 2.48. Random prudent walks of large size are believed to "stick" to a diagonal (and have linear end-to-end distance). However, for the *kinetic* distribution on prudent walks, where at each stage one of the prudent steps is chosen uniformly at random, the limiting behavior is very different, as walks proceed in large horizontal and vertical segments (Beffara, Friedli, Velenik).

Mireille Bousquet-Mélou (mireille.bousquet@labri.fr) *is a CNRS Researcher at LaBRI, Université de Bordeaux, France.*

Anthony J. Guttmann (guttmann@unimelb.edu.au) *is a Professor at the University of Melbourne, Victoria, Australia.*

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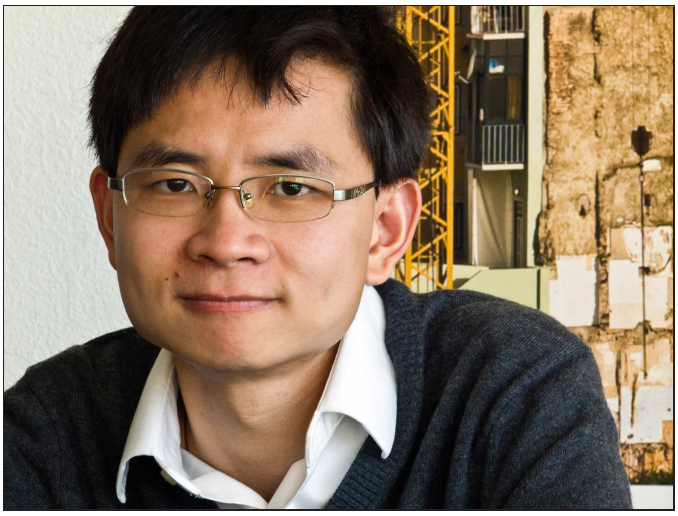
Focus on the Scientists: Jian Ding and Vadim Gorin

Elchanan Mossel

In this issue we focus on two MSRI postdoctoral fellows in the Spring 2012 Random Spatial Processes program.

Jian Ding is originally from Hunan in China and attended Peking University for his undergraduate studies. He then joined the Statistics department at Berkeley where he completed his Ph.D. under the guidance of Yuval Peres in 2011. Jian is on leave from Stanford, where he is a Szegö Assistant Professor.



Sheila Newbery

Jian's research area is centered around discrete probability, in particular problems with a combinatorial flavor. He studies Gibbs measures, Markov chain Monte Carlo methods, random walks and random graphs.

A breakthrough result of Jian with James Lee and Yuval Peres relates the cover time of a random walk on a graph to the "Gaussian free field". The cover time of a random walk on a graph is the expected time it takes the random walk to visit all the vertices of the graph. While it is relatively easy to estimate the cover time by running the random walk over and over again, a major open problem stated by Aldous (who is a member of the program) and Fill in 1996 was whether it is possible to obtain such an estimate without randomization. Previous progress towards the solution of this problem involved a work by Kahn, Kim, Lovász and Vu (2000). The results of Jian and his collaborators give a deterministic estimate of the cover time stated in terms of the expected maximum of a Gaussian process. Such maxima were extensively studied by many mathematicians and statisticians including Talagrand, as they play a crucial role in the theory of empirical processes.

A very recent related work by Jian estimates the maximum of the two-dimensional Gaussian free field, by quantifying the fluctuation of the peak of this "random surface", thus strengthening a recent result of Bramson and Zeitouni.

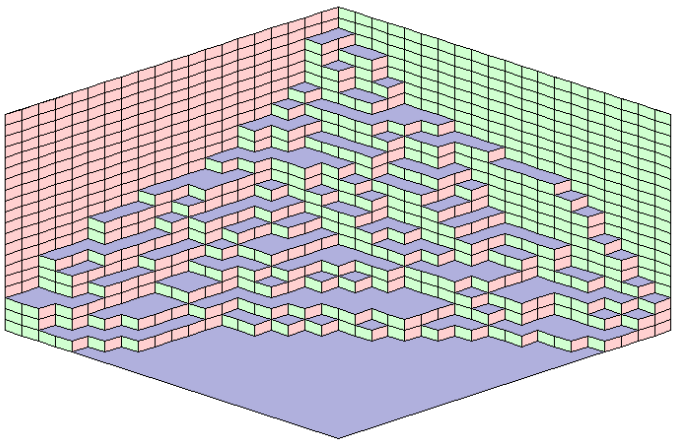
Vadim Gorin is originally from Moscow, where he attended one of the top mathematical schools (where he enjoyed advanced math in the company of 20 boys and 4 girls). He then attended simultaneously the Moscow State University and the Independent University of Moscow for his undergraduate studies. He has two PhDs, one from Moscow State University and one from Utrecht University.

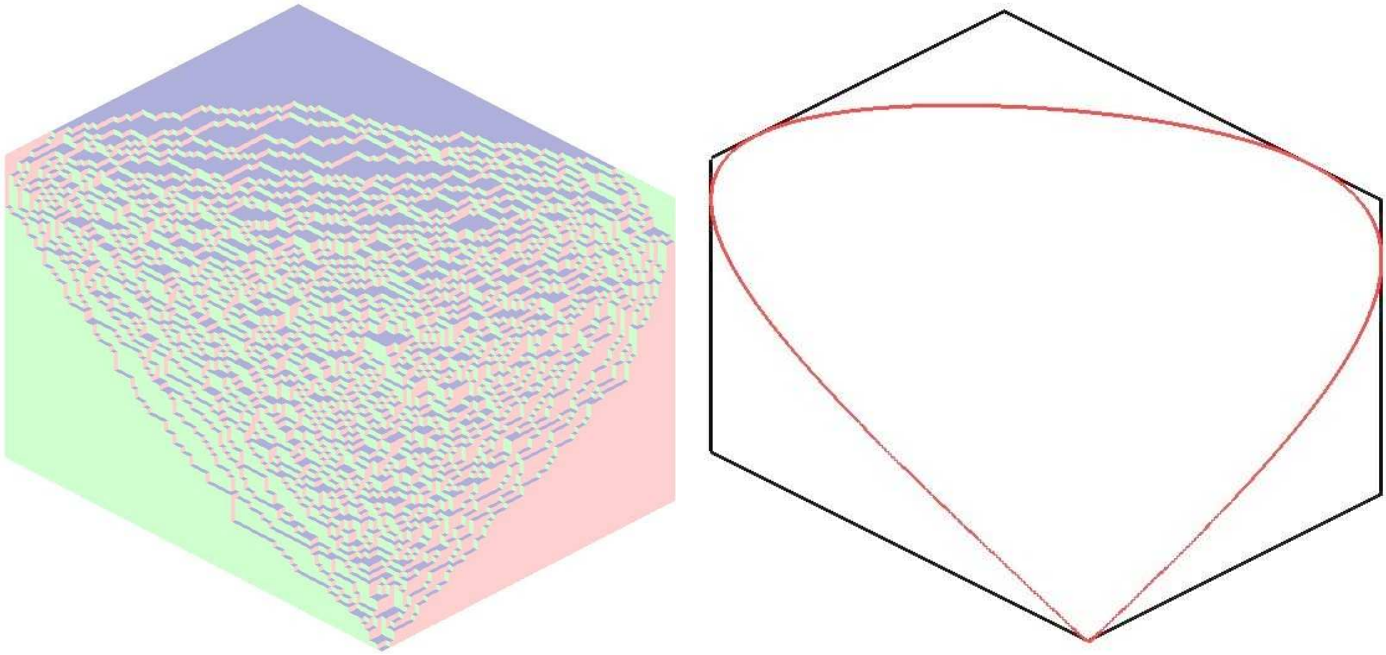
Sheila Newbery

In 2004 he took a course on algebraic combinatorics with Grigori Olshanski and was fascinated by this beautiful research area which became central in his work.

Much of Vadim's research deals with plane partitions and their relations to Gibbs measures, dynamics, and representation theory. He showed that locally a uniformly random boxed plane partition converges to a translation invariant Gibbs measure with position dependent slope. With Borodin he constructed extremely simple Markov chains that transition between the uniform measures on boxes of different shapes, which resulted in an efficient sampling algorithm.

Using the connection with random plane partitions, Vadim solved a number of problems regarding the representation theory of unitary groups. The techniques Vadim developed with his collaborators are general enough to allow analysis and sampling of more general and non-uniform measures. Here is an illustration of one of these measures (see also illustrations on the next page):





A random plane partition in a $70 \times 90 \times 70$ box. The parameters of the measure are tuned so that the theoretical boundary of the "frozen" region (shown in the second picture) has a node at the bottom corner. Courtesy Vadim Gorin.

Forthcoming Workshops

June 16, 2012 to July 29, 2012: *MSRI-UP 2012*

July 9, 2012 to July 27, 2012: *Mathematics Professional Development Institute 2012*

August 22, 2012 to August 24, 2012: *Connections For Women: Joint Workshop on Commutative Algebra and Cluster Algebras*, organized by Claudia Polini, Idun Reiten, Karen Smith, and Lauren Williams (chair)

August 27, 2012 to September 7, 2012: *Joint Introductory Workshop: Cluster Algebras and Commutative Algebra*, organized by David Eisenbud (co-chair), Bernhard Keller, Karen Smith, and Alexander Vainshtein (co-chair)

October 29, 2012 to November 2, 2012: *Cluster Algebras in Combinatorics, Algebra, and Geometry*, organized by Claire Amiot, Sergey Fomin, Bernard Leclerc, and Andrei Zelevinsky (chair)

December 3, 2012 to December 7, 2012: *Combinatorial Commutative Algebra and Applications*, organized by Winfried Bruns, Alicia Dickenstein, Takayuki Hibi, Allen Knutson (chair), and Bernd Sturmfels

January 24, 2013 to January 25, 2013: *Connections for Women: Noncommutative Algebraic Geometry and Representation Theory*, organized by Georgia Benkart, Ellen Kirkman (chair), and Susan Sierra

January 28, 2013 to February 1, 2013: *Introductory Workshop: Noncommutative Algebraic Geometry and Representation Theory*, organized by Michael Artin, Michel Van den Bergh (chair), and Toby Stafford

Making a Difference

MSRI has recently received from the American Mathematical Society an award for its "high level of commitment and successful efforts to improve diversity in the profession of mathematics in the United States."

Titled "Mathematics Programs that Make a Difference", the award commends MSRI for its leadership in promoting diversity in numerous ways, from a workshop series that encourages the participation of **women in mathematical research** to the mentoring of **undergraduates from underrepresented groups** through the MSRI-UP summer research program. MSRI has originated several **diversity-oriented mathematics workshops**, such as "Promoting Diversity at the Graduate Level in Mathematics" and the "Modern Math Workshop," which introduces fields of research to faculty and students, inspiring them to attend activities at the NSF-funded math institutes. MSRI also played a role in establishing the annual **Conference for African-American Researchers** (CAARMS) and the **Blackwell–Tapia Prize**, which honors a mathematician who has contributed significantly to their field as well as working to address underrepresentation of minorities in mathematics. MSRI is also part of the **Collaborative Diversity Initiative** (www.mathinstitutes.org/diversity.php).

According to Director Robert Bryant, "increasing diversity is an important part of the Institute's mission to develop the human potential within the mathematics community at all levels—from math education in schools to postdoctoral fellows and senior scientists."

MSRI
Mathematical Sciences Research Institute
17 Gauss Way, Berkeley CA 94720-5070
510.642.0143 • FAX 510.642.8609 • www.msri.org

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