Modular Forms and Arithmetic
Titles and Abstracts

Matthew Baker *Torsion points on abelian varieties*

I will discuss some of Ken Ribet's beautiful results concerning torsion points on abelian varieties, and I'll give a survey of how these results have been extended and generalized by various authors.

Joel Bellaiche *Non trivial extensions of p-adic Galois representations that are trivial at p*

The Bloch-Kato Selmer group of a p-adic Galois representation of a number field is a space of extensions that are unramifed at almost all places, and that satisfies some Fontaine-theoretic properties at every places of K dividing p. The Bloch-Kato conjecture predicts the dimensions of those spaces, and some progresses have been made toward this conjecture, even if the general case seems still very far.

If we replace the Fontaine-Mazur conditions at some places dividing p by the condition of being trivial (or dually, by no condition at all) at those places, we obtain spaces that are often even more mysterious than the Bloch-Kato Selmer groups. Indeed, even in the simplest case of the trivial representation, knowing the dimension of those spaces is equivalent to a generalization of the Leopoldt conjecture, in the spirit of the one recently proposed by Calegari-Mazur, and in general even a conjectural formula for the dimension of those spaces seems unknown (only some special cases being in the scope of Jannsen's conjecture)

In my talk, I will discuss those issues and explain how one can construct, using generalizations of ideas of Ribet, together with the work of Chenevier and myself on the geometry of eigenvarieties, non-trivial elements in those spaces.

Kevin Buzzard *Ken Ribet and Fermat's Last Theorem*

If there existed positive integers a, b, c and $n \geq 3$ with $a^n + b^n = c^n$ (that is, if Fermat's Last Theorem had turned out to be wrong), then from this data one can construct a cubic curve (the so-called "Frey curve") with very strange properties. In the 1980s Ken Ribet made a crucial breakthrough by proving much much more—he showed that this cubic curve would actually be a counterexample to a standard conjecture in arithmetic geometry. At that point Fermat's Last Theorem arguably "ceased to be inaccessible", and it is certainly no coincidence that eight years later Fermat's Last Theorem was finally a theorem. I will talk about what Ken did (and a little about how he did it), explain why it was so important both mathematically and historically, and how it fits into the grand scheme of things today.

Frank Calegari *Towards a Torsion Jacquet-Langlands Correspondence for* GL(2)

Let $F/\mathbf{Q}$ be a number field. We conjecture a relationship between the integral cohomology of arithmetic quotients arising from inner forms of $\mathrm{GL}(2)/F$. If one considers rational cohomology, the conjecture follows from the classical Jacquet-Langlands correspondence. We provide some partial results and applications in the case when F has one complex place. This is joint work with Akshay Venkatesh.

Robert Coleman *Wide Open Spaces*

While investigating the p-adic geometry of modular curves with Ken Mcmurdy, we were forced to justify results I've "known" for years. Now I know what a wide open space really is.

Samit Dasgupta *Ribet's converse to Herbrand and the weak Gross–Stark conjecture*

Let H/F denote a finite extension of number fields. In the 1970s, Stark stated a series of conjectures relating the leading terms of the partial zeta-functions of H/F to the absolute values of certain units in H. The most explicit of these conjectures, known as the "rank one abelian Stark conjecture," applies when H/F is an abelian extension and all its partial zeta-functions vanish at $s = 0$. In 1982, Gross stated certain p-adic analogues of Stark's conjectures, including an analogue of the rank one abelian conjecture. In this talk we present an attack on Gross's p-adic analogue of Stark's rank one abelian conjecture that yields very strong partial results. Our technique is to consider certain p-adic families of modular forms constructed from Eisenstein series, and to study their associated Galois representations. The methods draw strongly from those of Ribet in his proof of the converse to Herbrand's theorem, and those of Greenberg and Stevens in their proof of the Mazur–Tate–Teitelbaum conjecture. This is joint work with Henri Darmon and Rob Pollack.

Matthew Emerton *Level lowering for p-adic modular forms*

Let V denote the space of all generalized p-adic modular functions (in the sense of Katz) of arbitrary level, with coefficients in some finite extension E of $\mathbf{Q}_p$. The group $\mathrm{GL}_2(\mathbf{A}_f^p)$ (where $\mathbf{A}_f^p$ is the ring of prime-to-p finite adeles) acts naturally on V. Suppose that f is an element of V which is an eigenform for the Hecke operators T_ℓ for all but finitely many primes ℓ, and let $V[f]$ denote the subspace of V consisting of all forms nearly equivalent to f (i.e. which are also T_ℓ eigenforms for all but finitely many ℓ, with the same eigenvalues as f for all but finitely many ℓ). One sees that $V[f]$ is a $\mathrm{GL}_2(\mathbf{A}_f^p)$-subrepresentation of V; what is its structure?

Associated to f is a Galois representation $\rho_f : G_{\mathbf{Q}} \longrightarrow \mathrm{GL}_2(E)$. In analogy with the classical theorem of Carayol–Deligne–Langlands, we conjecture that $V[f]$ is isomorphic to a restricted tensor product of representations π_ℓ of the groups $\mathrm{GL}_2(\mathbf{Q}_\ell)$, where π_ℓ is associated to the restriction of ρ_f to the decomposition group D_ℓ via a suitably normalized version of the local Langlands correspondence.

In our talk we will explain the proof of this conjecture, under the assumption that the reduced representation $\overline{\rho}_f$ is absolutely irreducible, modulo a certain level-lowering result for p-adic modular forms, namely the p-adic analogue of the famous level lowering result of Ribet for mod p modular forms. We will then describe two approaches to proving this level lowering conjecture under (slightly differing) additional mild hypotheses: in the first we apply a certain $R = \mathbf{T}$ theorem (due to Mark Kisin and the speaker, extending a result of Böckle), and in the second we develop a p-adic analogue of "Mazur's principle".

Benedict Gross *Ramification theory and finite subgroups of Lie groups*

Let k be a non-Archimedean local field. In this talk we study the interplay inherent in the theory of Langlands parameters, which are homomorphisms from the profinite Galois group of a separable closure of k to a complex Lie group G. The image D is both the Galois group of a finite extension field K/k and a finite subgroup of G. Hence D has a ramification filtration as well as a complex representation on $V = \mathrm{Lie}(G)$. I will compute the Swan conductor of $\mathrm{Gal}(K/k)$ on V, for the parameters of some simple supercuspidal representations. This is a report on joint work with Mark Reeder.

David Helm *On ℓ-adic families of admissible representations of* $\mathrm{GL}_2(\mathbf{Q}_p)$

The "mod ℓ" local Langlands correspondence of Marie-France Vigneras establishes a bijection between admissible representations of $\mathrm{GL}_n(\mathbf{Q}_p)$ and n-dimensional Frobenius-semisimple representations of $G_{\mathbf{Q}_p}$ over an algebraic closure of $\mathbf{F}_\ell$. For $n = 2$, we compute compare the deformation theory of a given admissible representation with that of the corresponding galois representation; in almost all cases there is a canonical isomorphism between the deformation rings. The failure of this to occur in all cases is closely connected to the existence of congruences between modular forms; when such congruences exist the deformation theory of the galois representation is "richer" than that of the corresponding admissible representation. Following a suggestion of Richard Taylor, we give an ad-hoc way of "fixing" this by considering lifts of admissible representations to $\mathrm{GL}_2(\mathbf{Q}_p)$-modules that are not free over their base rings, and show that $\mathrm{GL}_2(\mathbf{Q}_p)$-modules that arise from the theory of modular forms fit naturally into this framework.

Nicholas Katz

Mark Kisin *Shimura varieties mod p*

In his "Jugendtraum" paper Langlands outlined a program to determine the zeta function of a Shimura variety by describing its mod p points. The conjecture on the structure of these points was made precise by Langlands-Rapoport. Its proof should have applications to the construction of Galois representations associated to automorphic forms.

I will explain some recent progress towards the Langlands-Rapoport conjecture.

Elena Mantovan *Integral models for toroidal compactifications of Shimura varieties*

In the case of good reduction, smooth integral models for Shimura varieties of PEL type have been constructed by Faltings and Chai. In my talk I'll describe how their construction can be extended to the cases of bad reduction at unramified primes, and discuss the geometry of the resulting spaces. A useful tool in this context is provided by the language of 1-motives. This is joint work with Ben Moonen.

Barry Mazur *Construction of abelian extensions following Ken Ribet*

I'll start my talk with a specific numerical example (involving the prime $p = 691$) that illustrates some broad structural relationships in number theory. I will show how this structure was vastly generalized and clarified by Ken Ribet's celebrated 1976 result (that established a converse of a theorem of Herbrand).

The combined theorem, now known simply as *Herbrand–Ribet*, deals with H, the ideal class group modulo p of the cyclotomic number field K obtained by adjoining a p-th root of unity to the field of rational numbers.

Let p be any odd prime number. Ernst Kummer, beginning one of the great chapters of modern algebraic number theory, already understood deep arithmetic consequences that follows from knowledge of H. (For example, if H vanishes, then "enough" of the fundamental theorem of arithmetic holds in K to allow one to prove Fermat's Last Theorem for the equation $x^p + y^p = z^p$.) The Herbrand–Ribet theorem establishes a connection between **(a)** the structure of the action of $\mathrm{Gal}(K/\mathbf{Q})$ on H and **(b)** the divisibility (or non-divisibility) by p of the numerator of certain Bernoulli numbers.

This was a great advance for many reasons: first it gives a simple numerical way of determining important aspects of the structure of these ideal class groups; second, Ken proved what he did by the impressive route of actually constructing the abelian Galois extensions of K that correspond—via Class Field Theory—to the desired ideal classes; thirdly, Ribet's ingenious method is the gateway to much later progress (and is fun to describe). I will discuss—or rather hint at—some of the later developments.

Loic Merel *Modular symbols for global fields*

Bjorn Poonen *Cohomological obstructions to rational points*

In almost every instance where the non-existence of rational points on a variety over a number field has been proved, the non-existence can be explained by either the Brauer-Manin obstruction or a descent obstruction based on a vast generalization of Fermat's method of infinite descent. I will give an introduction to these obstructions, and discuss a 3-fold constructed in 2008 that proves for the first time that these obstructions are not always sufficient.

William Stein *Kolyvagin's Approach to the Birch and Swinnerton-Dyer Conjecture*

We revisit Kolyvagin's 1980s approach to proving the Birch and Swinnerton-Dyer conjecture, and suggest some related questions for elliptic curves with analytic rank at least 2.

Marie-France Vigneras