New developments in the geometry and physics of Gromov-Witten theory (May 22-27,2006)
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Mirror duality has demonstrated the striking e ectiveness of concepts of modern physics in enumerative geometry. It is of the same type as the simple radius inversion duality seen in string compactifications on S1. This type was early discovered, because it shows up in every term in the string genus expansion and can be studied in 2d conformal field theory.

Meanwhile the studies of topological gauge and string theories have revealed another type of duality, which relate non perturbative large N results in gauge theory to the full genus expansion of topological string theory with non-trivial target spaces. It is a topological version of 't Hoofts old picture of string theory as large N gauge theory, which was made concrete for the N = 4 supersymmetric case by Maldacena. We expect that this new duality has a similar impact on the study of open and closed Gromov-Witten invariants as mirror duality.

It was known since the work of Witten in 1992 that Chern-Simons theory on 3-manifolds is equivalent to open topological string theory on non-compact Calabi-Yau spaces. In 1999 Gopakumar and Vafa found identities between the large N expansion of certain Chern-Simons amplitudes on 3-manifolds and closed topological string amplitudes on dual non-compact Calabi-Yau geometries. While for simple Calabi-Yau geometries these identities between gauge theoretic knot and link invariants and closed/open Gromow-Witten invariants have been proven [Okounkov, Pandharipand.Liu,Liu,Zhou], physicists [Aganagic, Marino, Klemm, Vafa] have generalized the dualities to calculate open and closed Gromow-Witten invariants on general non-compact toric varieties and found as an important structure in this calculations the so called topological vertex.

Further progress in this area requires a deeper understanding of the integrable structure which underlies these relations. Integrable structures have also became a main focus in the Maldacena AdS/CFT correspondence in particular in its pp-wave limit. Since the work of Witten and Kontsevich it is known that the KdV hierarchy and the string equations govern 2d gravity or equivalently intersection theory on the moduli space of Riemann surfaces. On more general target space of P n, Virasoro hierarchies was proved by Givental. The Toda hierarchy governs the solution of the c = 1 string and also underlies the equivariant Gromow-Witten theory of P 1 as recently shown by Pandharipande and Okounkov. The hermitian matrix models are known to be powerful tools in obtaining the t -function, the logarithm of the all genus free energy, in these cases.

More recently Dijkgraaf and Vafa found a holomorphic matrix model, whose spectral density is directly related to the algebraic equation describing the local Calabi-Yau in question. There is evidence for the conjecture that the large N expansion of this matrix model gets identified in a double scaling limit with the genus expansion of topological string amplitudes. This raises the question about the relation between the two matrix descriptions of topological string theory. The local structure of simple non-compact Calabi-Yau space is spec-ified by canonical two complex dimensional singularities. A relation between these singularities and KdV was found by Giventhal using the vertex operator formulation of Kac and Wakimoto. Does this lead to a hierarchy decribing the topological string on every non-compact Calabi-Yau? What are the analogs of the Virasoro or the W8 constraints for these hierachies. What is it ferminionic description in terms of GL(8) transformation on infinite Grassmannians? By mirror symmetry these integrable structures relate to the complex structure deformations theory of Kodaira and Spencer and its applications to higher genus topological string calculations, i.e. to Ray-Singer torsion for g = 1 and Kodaira and Spencer gravity for higher genus. The relation between closed and open topological string should give us an open string version of Kodaira-Spencer theory.

On the mathematical side, great progress has been made on studying more general Gromov-Witten invariants such as relative (Ionel-Li-Li-Ruan), open (Fukayao-Oh), contact type (Eliashberg-Hofer-Givental) and hamiltonian (Salamon-other) Gromov-Witten invariants.

At the moment many of these topics only loom on the horizon, but we expect major progress by the time of the program. There is also reason [Donagi-Pantev] to expect that a proper understanding of these dualities should incorporate various gerbe-theoretic twistings as well.

Cycles on algebraic varieties can be represented by objects in a derived category of equivariant sheaves, and Lagrangian submanifolds of symplectic manifolds lead to a Fukaya-Floer homological mirror category; this duality between A-branes and B-branes is rapidly becoming mathematically rigorous, providing a foundation for new developments involving relative (eg Lagrangian or orbifold) Gromov-Witten theory and large N dualities. When pushed further, these ideas blur into recent work on derived categories of sheaves on algebraic varieties [As-pinwall, Douglas, Seidel, Thomas]: in principle these categories encode all the information contained in the underlying object, and one might hope to move from a purely algebraic description of such a category [eg in terms of some quiver] to a more geometric (quotient) description. These are all connected to the very rich question of understanding string theory on spaces with singularities.