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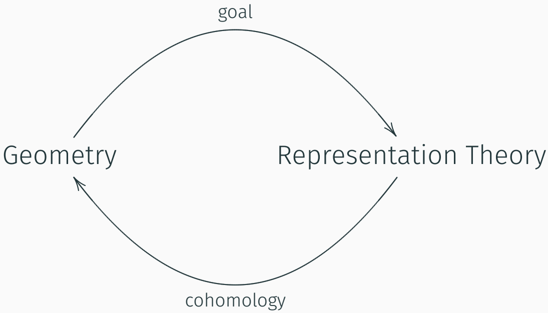
## Research Member, GRTA

**Research interests:** Representation theory and cohomology of algebraic groups and related structures, e.g., Lie algebras, finite group schemes, Lie superalgebras, Hopf algebras...


goal
Geometry
Representation Theory
cohomology

# Favorite example

$k = \overline{k}$ field of characteristic $p > n \geq 2$.

Let $M$ be a finite-dimensional rational $GL_n$-module.

When is $M$ projective for the $p$-Lie algebra $\mathfrak{g} = \mathfrak{gl}_n$, i.e., when is it projective as a module for the restricted enveloping algebra $u(\mathfrak{g})$?

## Invariants defined using cohomology (support varieties)

- $\mathrm{MaxSpec}(\mathrm{H}^\bullet(u(\mathfrak{g}), k)) \cong \mathcal{N}(\mathfrak{g}) := \{X \in \mathfrak{gl}_n : X \text{ is nilpotent}\}$
- $M \mapsto \{x \in \mathcal{N}(\mathfrak{g}) : M|_{\langle x \rangle} \text{ is not projective}\} \cup \{0\}$
  Moreover, $M$ maps to a set that is stable under conjugation.
- $M \mapsto \{0\}$ if and only if $M$ is projective for $u(\mathfrak{g})$
- Using the geometry of nilpotent orbits, suffices to check whether $M|_{\langle x \rangle}$ is free when $x \in \mathfrak{gl}_n$ is any single root vector.