Tensor categories without groups

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Algebraic group G, $Rep(G)$ the category of f.d. rep-s of G /$\mathbb{C}$.

- Symmetric monoidal category: operation $\otimes$, unit ($G \curvearrowright \mathbb{C}$), associativity and commutativity.
- Duals of objects, categorical dimension $\dim(V) \in \mathrm{End}_G(\mathbb{C})$:

$$\mathbb{C} \xrightarrow{1 \mapsto \sum_i e_i \otimes e_i^*} V \otimes V^* \xrightarrow{v \otimes f \mapsto f \otimes v} V^* \otimes V \xrightarrow{e_i^* \otimes e_j \mapsto \delta_{i,j}} \mathbb{C}$$

- Recover G from $Rep(G)$ using Tannakian reconstruction!

Similar categories which are not of the form $Rep(G)$ for any G?

- Vector superspaces ($V = V_{\bar{0}} \oplus V_{\bar{1}}$); $Rep(G)$ for supergroup G.
  - ★ Dimensions are integers: $\dim(V) = \dim V_{\bar{0}} - \dim V_{\bar{1}}$.
  - ★ Category not necessarily semisimple, rich representation theory.

- Deligne interpolation categories: $Rep(\underline{GL}_t)$, $Rep(\underline{S}_t)$ for $t \in \mathbb{C}$.
  - ★ Complex dimensions.
  - ★ Category not necessarily semisimple.
  - ★ Representation stability: behaviour of rep-s of $GL_n(\mathbb{C})$, S_n as $n \to \infty$.