MSRI 5-minute talks

Itay Glazer

Weizmann Institute of Science

Advisor: Rami Aizenbud

GRTA program

1. Representation theory of real reductive groups.
2. Algebraic geometry and analysis on algebraic varieties.

February 15, 2018

Definition

1) Let G be a finite group and $H \leq G$ a subgroup. We say that (G, H) is a ***Gelfand pair*** *if $\forall \pi \in \mathrm{Irr}(G)$, we have $\dim_{\mathbb{C}}(\pi^*)^H \leq 1$, or equivalently*

$$\dim_{\mathbb{C}} \mathrm{Hom}_G(\pi, \mathbb{C}[G/H]) \leq 1.$$

2) This notion can be generalized to a pair (G, H) where G is a reductive group and $H \leq G$ is a closed subgroup.

Gelfand pairs and a conjecture of van Dijk

Definition

1) Let G be a finite group and $H \leq G$ a subgroup. We say that (G, H) is a ***Gelfand pair*** *if $\forall \pi \in \mathrm{Irr}(G)$, we have $\dim_{\mathbb{C}}(\pi^*)^H \leq 1$, or equivalently*

$$\dim_{\mathbb{C}} \mathrm{Hom}_G(\pi, \mathbb{C}[G/H]) \leq 1.$$

2) This notion can be generalized to a pair (G, H) where G is a reductive group and $H \leq G$ is a closed subgroup.

The following are Gelfand pairs:

- (S_n, S_{n-1}), $(G \times G, \triangle G)$.
- $(\mathrm{GL}_n(\mathbb{C}), \mathrm{O}_n(\mathbb{C}))$, $(\mathrm{GL}_n(\mathbb{C}), \mathrm{GL}_m(\mathbb{C}) \times \mathrm{GL}_{n-m}(\mathbb{C}))$, (G, K), where G is a real reductive group and K a maximal compact subgroup.

Gelfand pairs and a conjecture of van Dijk

Definition

1) Let G be a finite group and $H \leq G$ a subgroup. We say that (G, H) is a ***Gelfand pair*** *if $\forall \pi \in \mathrm{Irr}(G)$, we have $\dim_{\mathbb{C}}(\pi^*)^H \leq 1$, or equivalently*

$$\dim_{\mathbb{C}} \mathrm{Hom}_G(\pi, \mathbb{C}[G/H]) \leq 1.$$

2) This notion can be generalized to a pair (G, H) where G is a reductive group and $H \leq G$ is a closed subgroup.

The following are Gelfand pairs:

- (S_n, S_{n-1}), $(G \times G, \triangle G)$.
- $(\mathrm{GL}_n(\mathbb{C}), \mathrm{O}_n(\mathbb{C}))$, $(\mathrm{GL}_n(\mathbb{C}), \mathrm{GL}_m(\mathbb{C}) \times \mathrm{GL}_{n-m}(\mathbb{C}))$, (G, K), where G is a real reductive group and K a maximal compact subgroup.

Conjecture (van Dijk)

Let G be a complex reductive group, $\theta : G \to G$ a complex involution, and H an open subgroup of G^θ. Then (G, H) is a Gelfand pair.