MSRI 5-minute talks

Yotam Hendel

Weizmann Institute of Science (Rehovot)

Advisors: Rami Aizenbud and Joseph Bernstein

GRTA program

Research interests: representation theory of p-adic groups, Gelfand pairs, analysis on algebraic varieties

February 15, 2018

Convolution of algebraic morphisms improves their smoothness properties (joint with Itay Glazer)

Convolution of algebraic morphisms improves their smoothness properties (joint with Itay Glazer)

- Let G be an algebraic group, X and Y be smooth algebraic varieties and let $\varphi : X \to G$ and $\psi : Y \to G$ be algebraic morphisms.

Convolution of algebraic morphisms improves their smoothness properties (joint with Itay Glazer)

- Let G be an algebraic group, X and Y be smooth algebraic varieties and let $\varphi : X \to G$ and $\psi : Y \to G$ be algebraic morphisms.
- We can define the convolution $\varphi * \psi : X \times Y \to G$ of φ and ψ via $\varphi * \psi(x, y) = \varphi(x) \cdot \psi(y)$.

Convolution of algebraic morphisms improves their smoothness properties (joint with Itay Glazer)

- Let G be an algebraic group, X and Y be smooth algebraic varieties and let $\varphi : X \to G$ and $\psi : Y \to G$ be algebraic morphisms.
- We can define the convolution $\varphi * \psi : X \times Y \to G$ of φ and ψ via $\varphi * \psi(x, y) = \varphi(x) \cdot \psi(y)$.

Theorem (holds for $G = V$ vector space, in preparation for general G)

Let $\varphi : X \to G$ be an algebraic morphism with dense image, then there exists $N \in \mathbb{N}$ such that $\varphi^N := \varphi * \ldots * \varphi$ has fibers with "good" singularity properties (i.e. φ^N is a flat map with reduced fibers of rational singularities).

Convolution of algebraic morphisms improves their smoothness properties (joint with Itay Glazer)

- Let G be an algebraic group, X and Y be smooth algebraic varieties and let $\varphi : X \to G$ and $\psi : Y \to G$ be algebraic morphisms.
- We can define the convolution $\varphi * \psi : X \times Y \to G$ of φ and ψ via $\varphi * \psi(x, y) = \varphi(x) \cdot \psi(y)$.

Theorem (holds for $G = V$ vector space, in preparation for general G)

Let $\varphi : X \to G$ be an algebraic morphism with dense image, then there exists $N \in \mathbb{N}$ such that $\varphi^N := \varphi * \ldots * \varphi$ has fibers with "good" singularity properties (i.e. φ^N is a flat map with reduced fibers of rational singularities).

- Let φ be a word map (e.g. $\varphi(g_1, g_2) = g_1 g_2 g_1^{-1} g_2^{-1}$ is the commutator map), can we find an N as above which is independent of the rank of G?

Convolution of algebraic morphisms improves their smoothness properties (joint with Itay Glazer)

- Let G be an algebraic group, X and Y be smooth algebraic varieties and let $\varphi : X \to G$ and $\psi : Y \to G$ be algebraic morphisms.
- We can define the convolution $\varphi * \psi : X \times Y \to G$ of φ and ψ via $\varphi * \psi(x, y) = \varphi(x) \cdot \psi(y)$.

Theorem (holds for $G = V$ vector space, in preparation for general G)

Let $\varphi : X \to G$ be an algebraic morphism with dense image, then there exists $N \in \mathbb{N}$ such that $\varphi^N := \varphi * \ldots * \varphi$ has fibers with "good" singularity properties (i.e. φ^N is a flat map with reduced fibers of rational singularities).

- Let φ be a word map (e.g. $\varphi(g_1, g_2) = g_1 g_2 g_1^{-1} g_2^{-1}$ is the commutator map), can we find an N as above which is independent of the rank of G?
- This has applications to the study of algebraic families of random walks on $G(\mathbb{Z}/p^k\mathbb{Z})$.

Convolution of algebraic morphisms improves their smoothness properties (joint with Itay Glazer)

- Let G be an algebraic group, X and Y be smooth algebraic varieties and let $\varphi : X \to G$ and $\psi : Y \to G$ be algebraic morphisms.
- We can define the convolution $\varphi * \psi : X \times Y \to G$ of φ and ψ via $\varphi * \psi(x, y) = \varphi(x) \cdot \psi(y)$.

Theorem (holds for $G = V$ vector space, in preparation for general G)

Let $\varphi : X \to G$ be an algebraic morphism with dense image, then there exists $N \in \mathbb{N}$ such that $\varphi^N := \varphi * \ldots * \varphi$ has fibers with "good" singularity properties (i.e. φ^N is a flat map with reduced fibers of rational singularities).

- Let φ be a word map (e.g. $\varphi(g_1, g_2) = g_1 g_2 g_1^{-1} g_2^{-1}$ is the commutator map), can we find an N as above which is independent of the rank of G?
- This has applications to the study of algebraic families of random walks on $G(\mathbb{Z}/p^k\mathbb{Z})$.