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KG-modules have support varieties via $H^*(G)$.
A fdim algebra: support varieties via $HH^*(A)$?

DEF A satisfies (Fg):
$HH^*(A)$ is Noetherian and $\mathrm{Ext}^*_A(S,S)$ f.g. over $HH^*(A)$.

Problem: Criteria for (Fg)?

DEF M is a criminal if M is bounded but not periodic.

$\exists$ a criminal $\Rightarrow$ No (Fg).

EX $A = K\langle X,Y\rangle/(X^2, Y^2, (XY)^k - q(YX)^k)$
$q \neq$ root of unity. Then A has criminals.
Every element in $HH^{>0}(A)$ is nilpotent.

Algebras beyond blocks

Fix defect group D.

- D cyclic: $B =$ Brauer tree algebra ($\Rightarrow$ Brauer graph algebra)

- $D =$ dihedral, semidihedral or quaternion, $p = 2$ ('tame')

[E, Skowronski]
Algebras via weighted surface algebras.
They (almost) generalize 'quaternion' blocks.
(Partial) degenerations generalize '(semi)dihedral' blocks.

Open Problem What are the indecomposable representations (char$(K) = 2$) of the quaternion group?