Relative representation theory – Harmonic analysis over spherical varieties

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Observation

Representation theory of G

$$\Updownarrow$$

Harmonic analysis on G w.r.t. the two sided action of $G \times G$

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Conclusion

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Conjecture

Let G be a reductive algebraic group scheme and X be a spherical G space (i.e. over any algebraically closed field, the Borel acts with finitely may orbits on X). Then

$$\sup_{F \text{ is a finite or local field}} \left(\sup_{\rho \in \mathrm{irr}(G(F))} \langle F(X), \rho \rangle \right) < \infty.$$