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Supervisor: Prof. Uri Onn

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Representation Growth

Pick a group G.

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Key tool: *The Representation Zeta Function*

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- Finite groups of Lie-type
- p-adic integration and geometry
- Analysis on algebraic varieties
- Representations of groups over finite rings
- Model theory

A Nice Question

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For any prime p, there exist $W_1, \ldots, W_r \in \mathbb{Q}(x)$ and $n_1, \ldots, n_r \in \mathbb{N}$, such that

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- ✓ Principal congruence subgroups [AKOV]
- ✓ Regular characters [Krakowski, Onn& Singla, S.- Classical groups]