Theorem 5.2: $(\mathbb{M}, +, \cdot, \dots)$ field (dp-finite)
One of the following holds:
1) $\mathbb{M}$ has finite Morley rank
2) There are two disjoint heavy

proof: $(X_1 \dots X_n, P)$ crit conf.
3.10 $Q_i \subseteq X_i$ infinit $H = Q_1 \times \dots \times Q_n \setminus P$ narrow
$P' = \underbrace{Q_1 \times \dots \times Q_n} \cap P$ broad

$rk(P) = rk(P') \Rightarrow$ critical conf its image Y
is a critical set. Y heavy!

$Q_i \subseteq \mathbb{M}$
$P \subseteq Q_1 \times \dots \times Q_n$ $\quad \Sigma: P \to \mathbb{M}$ finite fibers $\quad Y$ image.

- Q_i have Morley rank 1:
  $W = \Sigma Q_i$ heavy $\quad W - W = \mathbb{M}$
  surjection $(Q_1 \times \dots \times Q_n)^2 \to \mathbb{M}$
  $\mathbb{M}$ has finite Morley rk.
  otherwise $\delta \notin W - W$
  $W, W + \delta$ disjoint and heavy.

- Q_1 has Morley > 1:
  Claim (5.1): infinitely many broad types in $Q_1 \times \dots \times Q_n$

  $(D_i)_{i \in \omega}$ $\quad D_i \subseteq Q_1$ $\quad \underbrace{D_i \times Q_2 \times \dots \times Q_n}_{\text{broad}} \ni p_i$

  $\Sigma: P' \to \mathbb{M}$ finite fibers

  $\Sigma_*: S_{P'} \to S_{\mathbb{M}}$
  p_1, p_2 broad in P' st $\Sigma_* p_1 \neq \Sigma_* p_2$
  $Y = Y_1 \dot\cup Y_2$ $\quad p_i \in Y_i$

$rk(Y_i) = rk(p_i) = rk(P') = rk(Y) \Rightarrow$ heavy
$\{\varepsilon : X + \varepsilon \cap X \text{ heavy}\}$ $\quad \exists \delta \neq 0$ $\quad Y_i + \delta \cap Y$ broad

$X - \infty X$ nghb of U

6.5.5: $RM(M)$ infinite $\Rightarrow$ $\forall a \neq 0$ in M $\exists U$ nghb of 0
$a \notin U$ ($a \in M \preccurlyeq \mathbb{M} \Rightarrow U$ M-def)

6.2 6/ Infinitesimals

Def: M small model

$$I_M := \bigcap_{\substack{X\ M\text{-def} \\ \text{heavy}}} X - \infty X$$

- $M \preccurlyeq M'$: $I_{M'} \subseteq I_M$

6.9:
- $I_M \subseteq X$ def $\Rightarrow$ heavy
- $I_M \neq \{0\}$ $\quad 0 \in I_M$
- $I_M \cdot M \subseteq I_M$
- $RM(M)$ infinite $\quad I_M(M) = \{0\}$

Def: $X \subseteq \mathbb{M}$ M-def $\delta \in \mathbb{M}$ M-displace X if

$$X(M) + \delta \cap X = \emptyset$$
$$[x \in X(M),\ x + \delta \notin X]$$

6.12: M defines crit. conf, $\varepsilon \in I_M$ and ε M-displ. some $X \subseteq \mathbb{M}$ M-def then X is light.
[X heavy, not displaced by infinitesimal]

Lemma: 6.11 $\quad M \preccurlyeq M' \quad \varepsilon, \varepsilon' \in \mathbb{M}$
$tp(\varepsilon'/M')$ heir of $tp(\varepsilon/M)$

then
1) ε M-inf $\Rightarrow$ ε' M'-inf
2) X M-def ε M-displ. X $\Rightarrow$ ε' M'-displ. X

proof wma $\varepsilon' = \varepsilon$

1) ε not M'-inf: $b' \in M'$ $\phi(\mathbb{M}, b')$ heavy
$\varepsilon \notin \phi(\mathbb{M}, b') - \infty\, \phi(\mathbb{M}, b') \rightarrow \underset{\text{over } M}{\Theta}(b', \varepsilon)$
$\exists b \in M$ st $\models \Theta(b, \varepsilon)$ holds $\quad \varepsilon$ is not M-inf.

2) ε does not M'-displ X $\quad \underbrace{b' \in X(M) \text{ st } b' + \varepsilon \in X}_{M\text{-def}}$

realized by some $b \in M \Rightarrow \varepsilon$ does not M-displ X.

proof 6.12: X M-displ ε, ε M-inf.

Baisalov $(\varepsilon_i, M_i)_{i\in\omega}$:

- $\varepsilon_i \in M_{i+1}$
- ε_i is M_i-inf
- X is M_i-displ by ε_i

$\varepsilon \in M' \succcurlyeq M$ $\quad$ q global coheir of $tp(M'/M)$

$\varepsilon_0 = \varepsilon \quad M_0 = M \quad M_1 = M'$

$\Longrightarrow tp(\varepsilon_i/M_i)$ heir $tp(\varepsilon/M)$

$\alpha \in 2^{<\omega}$ $\quad X_\alpha = \{x \in X : x+\varepsilon_i \in X \text{ if } \alpha_i = 1\}$

NIP: $\quad \exists \alpha\ X_\alpha = \emptyset \quad X_\alpha$ light!

Claim: X_α heavy $\quad X_{\alpha 1}$ heavy

ε_n M_n-inf, X_α M_n-def $\quad X_\alpha \cap X_\alpha - \varepsilon$ heavy

$X_{\alpha 1} = X_\alpha \cap (X - \varepsilon_n)$ " $\Rightarrow$ heavy.

Claim. X_α heavy $\quad X_{\alpha 0}$ heavy:

$X(M_n) + \varepsilon_n \cap X = \emptyset \quad M_n$-displ

$\Rightarrow X(M_n) \subseteq X_{\alpha 0} = \{x \in X_\alpha : x+\varepsilon_n \notin X\}$

4.22: $X_{\alpha 0}$ heavy.

$\Rightarrow X$ is light.

6.15: M defines a crit. conf.

$\varepsilon_1, \varepsilon_2 \in M \Rightarrow \varepsilon_1 - \varepsilon_2$ inf.

X M-def and heavy, want $X \cap (X+\varepsilon_1-\varepsilon_2)$ heavy

$D_0 = \{x \in X : x+\varepsilon_i \in X\}$

$D_i = \{x \in X, x+\varepsilon_i \notin X\} \qquad i=1,2$

$X \subseteq D_0 \cup D_1 \cup D_2$

4.21: $\exists i$ M-def $X' \subseteq X$ heavy $X'(M) \subseteq D_i$

$i>0 \qquad x \in X'(M) \subseteq D_i$

$x+\varepsilon_i \notin X \Rightarrow \notin X'$

X' is M-displaced by ε_i

$\Longrightarrow X'$ light.

$i=0 \qquad \Rightarrow D_0$ heavy

$D_0 + \varepsilon_1 \subseteq X$

$D_0 + \varepsilon_2 \subseteq X \Rightarrow D_0 + \varepsilon_1 \subseteq X - \varepsilon_2 + \varepsilon_1$

$\underbrace{D_0 + \varepsilon_1}_{\text{heavy}} \subseteq X \cap (X + \varepsilon_1 - \varepsilon_2)$ heavy.

6.15: U M-def ngbh $\exists M$ def ngbh V st $V - V \subseteq U$ ($-$ continuous at 0)

proof: $M' \succ M$ define a crit. conf.

$$I_{M'} - I_{M'} \subseteq I_{M'} \subseteq I_M \subseteq U$$

compactness (+ ngbh are directed):

V M'-def ngbh st $V - V \subseteq U$

we can find such a V M-def $M' \succ M$ (+ situation is M-def).

Consequence: There is a (unique) topology on M, which is a group topology and whose ngbh of 0 are $U - U$, U heavy M-def.

6.19: $J \leq (M, +)$ type def / M

Suppose that every M-def $D_{\supseteq J}$ is heavy $\Rightarrow I_M \subseteq J$

6.20: $I_M = I_M^{\infty} = I_M^{00}$