Random matrices and random landscapes

(see also Gérard's UCB course!)

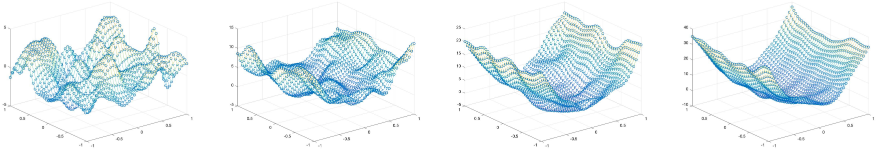
Ben(jamin) McKenna

MSRI Postdoc

(just finished Ph.D. at NYU Courant; after, IST Austria, then Harvard)

August 30, 2021

$$\mathcal{H}_2(x) = \text{random noise}(x) + \lambda \cdot \text{deterministic signal}(x), \quad x \in [-1,1]^2$$



complex ($\lambda = 0$) $\longrightarrow$ phase transition $\longrightarrow$ simple ($\lambda \gg 1$)

- I'm interested in the geometry of random landscapes, meaning smooth (Gaussian) random functions $\mathcal{H}_N : \mathbb{R}^N \to \mathbb{R}$; for example, Hamiltonians in statistical physics, or loss functions in data science.
- Specifically I count critical points, meaning study the random variable $\mathrm{Crt}(\mathcal{H}_N) = \#\{x : \nabla \mathcal{H}_N(x) = 0\}$, when $N \to +\infty$. Useful for ...
    - predicting dynamics of optimization
    - predicting physics phase transitions
    - finding ground states
- The Kac-Rice formula writes $\mathbb{E}[\mathrm{Crt}(\mathcal{H}_N)]$ in terms of

$$\mathbb{E}[\left|\det(\nabla^2 \mathcal{H}_N(x))\right| \mid \nabla \mathcal{H}_N(x) = 0]$$

  so I care about determinants of symmetric Gaussian random matrices.
- Often non-invariant: I study random landscapes that are far from isotropic, by studying random matrices that are far from GOE.