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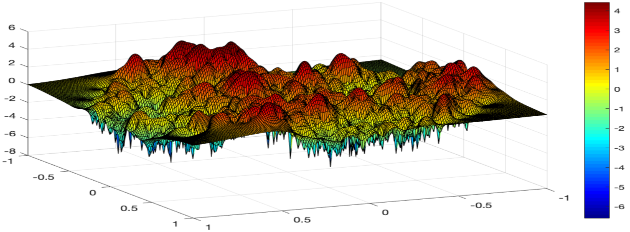
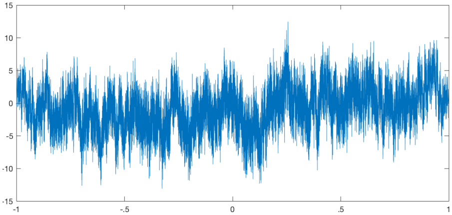
1. (Mesoscopic) Fluctuations of eigenvalues
   - Unitary-invariant ensembles & β-ensembles
   - High-temperature or mean-field regime $\beta = \mathscr{O}(1/N)$.
   - Stein's method and combinatorial methods (cumulants)

2. Characteristic polynomials
   - Extreme values

$$\frac{\max_{|z|\leq 1}\left\{\log\det|z-\mathbf{G}_N| - N\varphi(z)\right\}}{\log N} \to \frac{1}{\sqrt{2}} \quad \text{in probability.}$$



2. Characteristic polynomials
   - Gaussian multiplicative chaos



For GUE,

$$h_N(x) = \sum_{j=1}^{N} \mathbb{1}_{\lambda_j \leq x} - N \int_{-1}^{x} \frac{\sqrt{1-t^2}}{\pi/2} dt$$

$$\frac{e^{\gamma h_N(x)}}{\mathbb{E} e^{\gamma h_N(x)}} dx \to GMC_\gamma, \quad \text{in distribution as } N \to \infty \text{ for } \gamma \in [0, \gamma_c].$$

   - Scaling limits of characteristic polynomial.

Let $\varphi_N(z) = \det(z - \mathrm{G}\beta\mathrm{E}_N) e^{-Nz^2}$. With overwhelming probability

$$\varphi_N(1 + \lambda/2N^{2/3}) \simeq c_N \mathrm{SAi}_\beta(\lambda) \frac{e^{\gamma \mathfrak{X}_N}}{\mathbb{E} e^{\gamma \mathfrak{X}_N}}, \qquad \gamma = \sqrt{2/\beta}.$$

3. Free fermions & determinantal processes (in dimension ≥ 2)
   - Universality of local fluctuations
   - Central limit theorems