

# Algebraic Combinatorics

$$s_\lambda(x_1, \dots, x_N) := \frac{\det \begin{bmatrix} x_i^{\lambda_j + N - j} \end{bmatrix}_{i,j=1}^N}{\prod_{1 \leq i < j \leq N} (x_i - x_j)}$$

$$= x_1^2 x_2^2 + x_1^2 x_3^2 + x_2^2 x_3^2 + x_1^2 x_2 x_3 + x_1 x_2^2 x_3 + x_1 x_2 x_3^2$$

1	1	1	1	2	1	1	2	1	2
2	2	3	3	3	3	2	3	2	3

$S_n$ ,  $GL_N$  representation theory

Kronecker coefficients etc

## Computational Complexity Theory

P vs NP? VP vs VNP? det vs per?  
Geometric Complexity Theory (GCT)

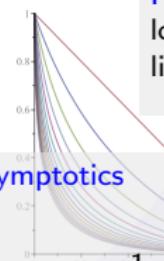
problem  $\in \#P$   
 $\iff$   
“Combinatorial interpretations”  
positive formulas

## Enumeration and asymptotics

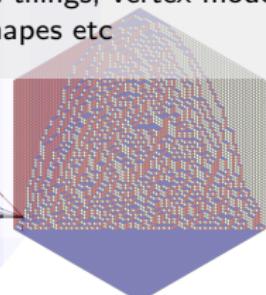
$$f^{\lambda/\mu} = |\lambda/\mu|! \sum_{D \in \mathcal{E}(\lambda/\mu)} \prod_{u \in [\lambda] \setminus D} \frac{1}{h(u)}$$

$$\sqrt{\binom{n}{k}} e^{-d\sqrt{n}} \leq \max_{\lambda \vdash n, \mu \vdash k, \nu \vdash n-k} c_{\mu, \nu}^{\lambda} \leq \sqrt{\binom{n}{k}}$$

Tilings  $\leftrightarrow$  plane partitions  
 Schur generating functions  
 NILP, RSK



## Probability, stat mech lozenge tilings, vertex models, limit shapes etc



## Some relevant papers: combinatorics + asymptotics / probability

- V. Gorin, G. Panova, Asymptotics of symmetric polynomials with applications to statistical mechanics and representation theory, *Ann. Probab.* (2015), 43(6), pp. 3052 – 3132.
- G. Panova, Lozenge tilings with free boundaries, *Lett. Math. Phys.*, (2015), 105(11), pp. 1551–1586.
- A.H. Morales, I. Pak, G. Panova, Hook formulas for skew shapes I, II, III, IV (2015–2021+)
- A.H. Morales, I. Pak, G. Panova, Asymptotics of the number of Standard Young Tableaux of skew shape, *Europ. J. Comb.* 70(2018), pp. 26–49.
- A.H. Morales, I. Pak, G. Panova, Asymptotics of principal evaluations of Schubert polynomials for layered permutations (with A. Morales, I. Pak), *Proc. Amer. Math. Soc.*, <https://doi.org/10.1090/proc/14369> (2018).
- I. Pak, G. Panova, D. Yeliussizov, On the largest Kronecker and Littlewood–Richardson coefficients, *J. Comb. Theory Ser. A*, 165 (2019), pp. 44–77.
- S. Melczer, G. Panova, R. Pemantle, Counting partitions inside a rectangle, *SIAM J. Discrete Math.*, 34(4), 2388–2410.
- S. H. Chan, I. Pak, G. Panova, Sorting probabilities for Young diagrams, [arXiv:2005.08390](https://arxiv.org/abs/2005.08390).
- S. H. Chan, I. Pak, G. Panova, Log-concavity in planar random walks, [arXiv:2106.10640](https://arxiv.org/abs/2106.10640)