Asymptotics of integrals arising in the representation of correlations functions in the Sinh-Gordon model

Research interests: Statistical mechanics, β-ensembles, Riemann-Hilbert problems, LDP techniques

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What my work is about

- **Main reference:** *Asymptotic expansion of a partition function related to the sinh-model*, G. Borot, A. Guionnet, K. Kozlowski (2016)
- **Framework:** Computing the correlation functions in the Sinh-Gordon model, a relativistic quantum field theory.
- **Current goal:** obtaining the N-asymptotic expansion of

$$\mathcal{Z}_N^\beta[V] = \int_{\mathbb{R}^N} \prod_{i<j}^{N} \left(\prod_{k=1}^{2} \sinh\left[\pi\omega_k(\lambda_i - \lambda_j)\right] \right)^\beta \prod_{i=1}^{N} e^{-V(\lambda_i)} d^N\lambda$$

- via LDP techniques: $\underset{N\infty}{\sim} e^{N^2 \log N \mathcal{E}(\mu_{\text{eq}})}$ and DS-equations to obtain the behaviour of fluctuations

$$\mathbb{E}_N^{V,\beta}\left[\int \phi d\left(\mu_{\text{eq}} - N^{-1} \sum_{i=1}^{N} \delta_{\lambda_i} \right) \right]$$

- $d\mu_{\text{eq}}^{(N)} = \rho_{\text{eq}}^{(N)} 1_{[a_N, b_N]} dx$ can be approximated by Riemann-Hilbert techniques, since

$$V' = \mathcal{S}[\rho_{\text{eq}}^{(N)}] \text{ where } \mathcal{S} \text{ is a Wiener-Hopf type operator}$$

- Behaviour near the endpoints a_N / b_N allows to push the asymptotic expansion of $\log \mathcal{Z}_N^\beta[V]$ further thanks to the DS-equations.