Least gradient problems in metric measure spaces

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  - Lipschitz restriction of solvable data may not be solvable