Small deviations principle and Chung's law of the iterated logarithm

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Small deviations principle:

X_t satisfies a SDP with rates α and β if there exists a constant $c > 0$ such that

$$\lim_{\varepsilon \to 0} -\varepsilon^{\alpha} |\log \varepsilon|^{\beta} \log \mathbb{P}\left(\max_{0 \leqslant t \leqslant 1} |X_t| < \varepsilon \right) = c.$$

Chung's LIL

X_t satisfies Chung's LIL at *infinity* with rate $a \in \mathbb{R}_+$ if there exists a constant $C > 0$ such that

$$\liminf_{t \to \infty} \left(\frac{\log \log t}{t} \right)^{a} \max_{0 \leqslant s \leqslant t} |X_s| = C \text{ a.s.}$$

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Example

$\lim_{\varepsilon \to 0} -\varepsilon^2 \log \mathbb{P}\left(\max_{0 \leqslant t \leqslant T} |b_t| < \varepsilon \right) = \lambda_1 T$,

$\liminf_{t \to \infty} \sqrt{\frac{\log \log t}{t}} \max_{0 \leqslant s \leqslant t} |b_s| = \lambda_1^2$ a.s..

References

1. *Small deviations and Chung's law of iterated logarithm for a hypoelliptic Brownian motion on the Heisenberg group*, C.-Gordina, to appear in the **Transactions of the AMS**.

2. *An application of the Gaussian correlation inequality to the small deviations for a Kolmogorov diffusion*, C., submitted.