Alan Sola (Stockholm University)
Spring 2022: Research Member
Analysis and Geometry of Random Spaces

- *Complex analysis and probability:* $\mathrm{ALE}(\alpha, \eta)$, $\mathrm{HL}(\alpha)$.
- *Harmonic analysis:* Littlewood-Paley theory, Orlicz spaces.
- *Operator theory:* shifts on Bergman/Dirichlet spaces.
- *Several complex variables:* stable polynomials, boundary behavior of rational functions.

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General case for $d = 2$: Corresponding statement not true but Kelly Bickel-Greg Knese-James Pascoe-AS. (2021) make *conjecture* (and prove true in many situations).
General case for $d \geq 3$: Probably hard.