February 4, 2022 9:14 PM

ÖZGÜR ESENTEPE

UNIVERSITY OF CONNECTICUT

COMPLEMENTARY PROGRAM.

sntp.ca

February 4, 2022 9:19 PM

- For numbers: $ab=0 \Leftrightarrow a=0$ or $b=0$
- For matrices: $AB=0 \Leftrightarrow \text{col}(B) \subseteq \text{null}(A)$.
- For linear maps: $\alpha\beta=0 \Leftrightarrow \text{Im}(\beta) \subseteq \ker(\alpha)$. Homology: $\frac{\ker(\alpha)}{\text{Im}(\beta)}$.

- R ring. M an R-module. Usually not free.
  - We can approximate M by free modules.
    - $\cdots \to F_2 \xrightarrow{\partial_2} F_1 \xrightarrow{\partial_1} F_0 \to 0 \;:C$ s.t. $H^i(C)=0$ $i>0$, $H^0(C)=M$.
- N another R-module. Consider $\text{Hom}_R(M,N)$.
  - Replace by $\text{Hom}_R(C,N)$.
    - $0 \to \text{Hom}_R(F_0,N) \to \text{Hom}_R(F_1,N) \to \text{Hom}_R(F_2,N) \to \cdots$

i^{th} Homology of this is called $\text{Ext}^i_R(M,N)$.

- If R is the coordinate ring of a smooth thing: $\text{Ext}^i_R(*,-)=0$ for $i \gg 0$.
- Otherwise, there are M,N with $\text{Ext}^i_R(M,N) \neq 0$ for every i.

I study ring elements which uniformly annihilate all Ext.