5 min talk at MSRI

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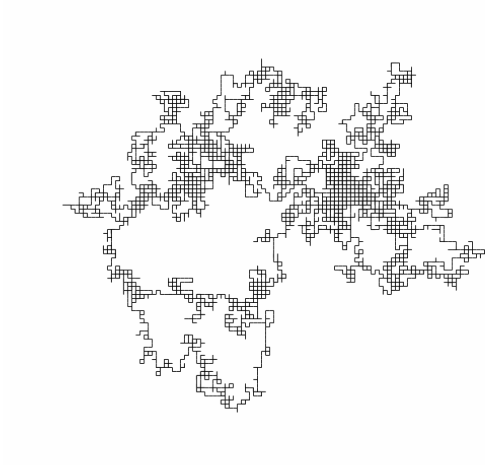
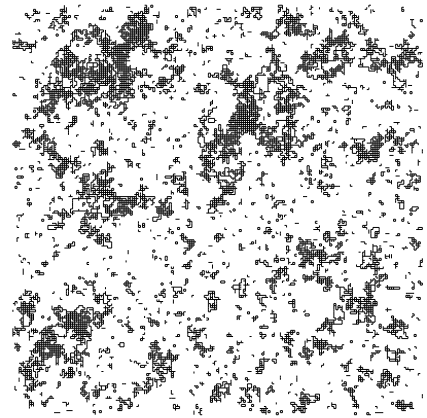
Two dimensional random geometry, conformal invariance

Brownian loop



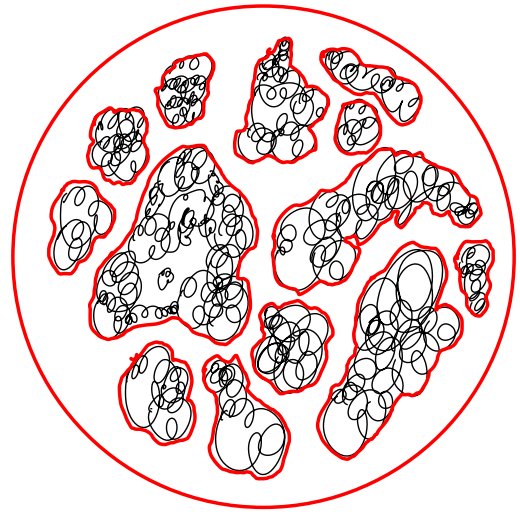
Brownian loop-soup



introduced by Lawler and Werner (2004)

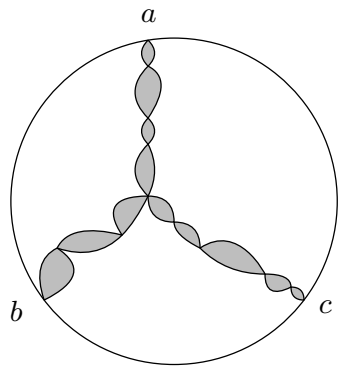
Structure of the clusters in the Brownian loop-soup

1. Outer boundary of the clusters: Conformal loop ensemble CLE (Sheffield and Werner), closely related to Schramm-Loewner evolutions SLE
2. Occupation time field: Gaussian free field GFF (Le Jan)
3. CLE are 0-level-lines of GFF (Miller and Sheffield)

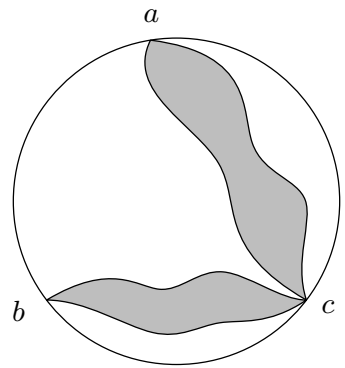


- Simultaneous coupling of the Brownian loop-soup, CLE and GFF (Q. and Werner) which leads to nemerous results on the structure of loops in the clusters.
- Existence (and non-existence) of multiple points on the boundaries of clusters. Generalized disconnection exponent and radial hypergeometric SLE (Q.). Computation of Hausdorff dimension (Gao, Li and Q. in progress)

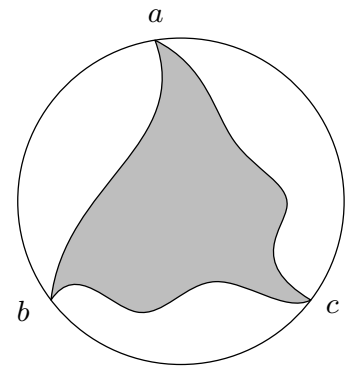
Conformal restriction measures. Conformal restriction: the trichordal case (Q., first work in PhD)



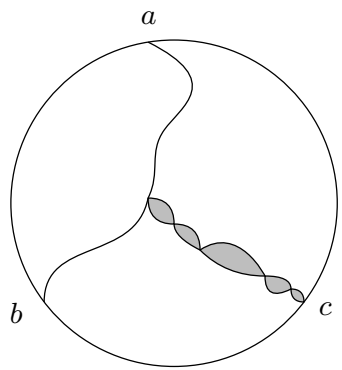
$\alpha = \beta = \gamma = 20/27$



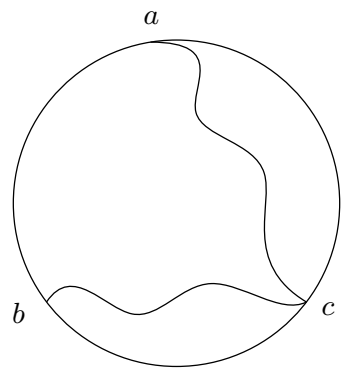
$\gamma = \tilde{\xi}(\alpha, \beta)$



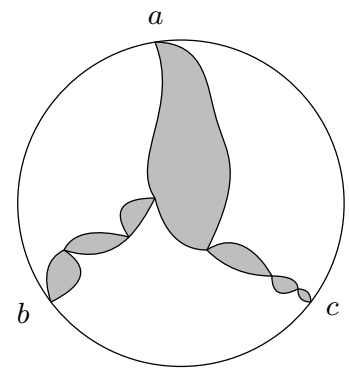
$\alpha, \beta, \gamma \geq 35/24$



$\alpha = \beta = 5/8, \gamma = 1$



$\alpha = \beta = 5/8, \gamma = 2$

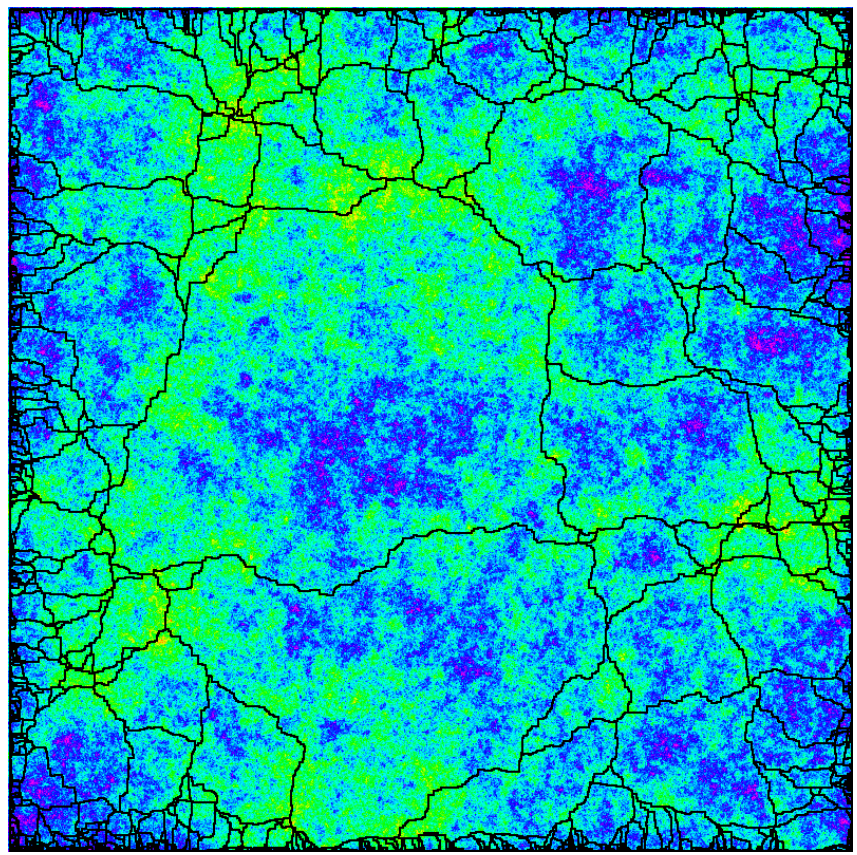


$\alpha > 24/35, \beta, \gamma < 35/24$

Constructed by introducing chordal hypergeometric SLEs.

Liouville quantum gravity (LQG) informally $e^{\gamma h}(dx^2 + dy^2)$ where h is a GFF

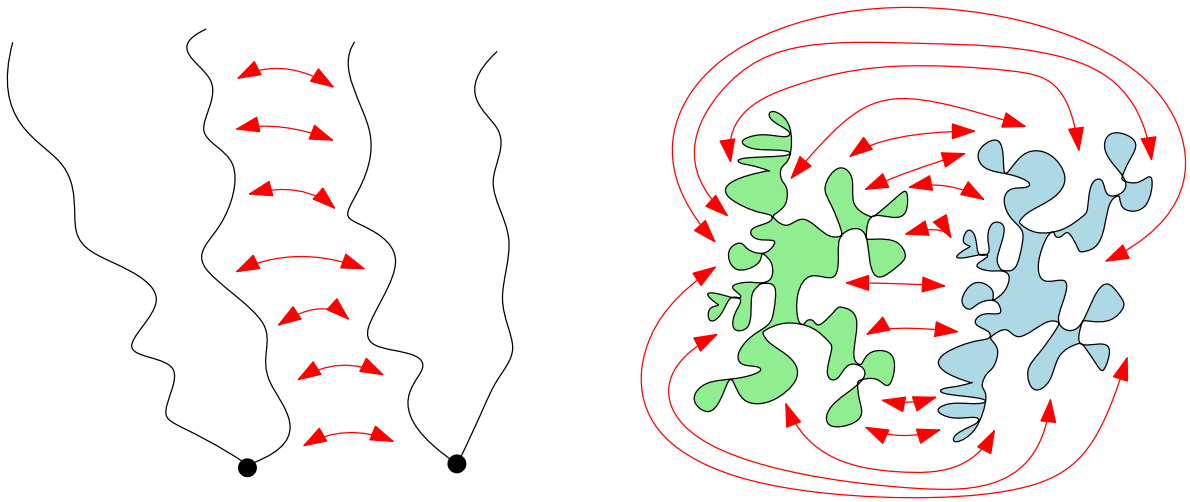
- The geodesics of the LQG are not SLE (Miller and Q.)
- Classification of the geodesics of the Brownian map (Miller and Q.)



Conformal welding

Condition 1 on the initial curve γ + Condition 2 on the image curve γ' $\Longrightarrow$ uniqueness of the welding

A **deterministic** result which applies to the welding of LQG surfaces with SLE interfaces (McEnteggart, Miller and Q.)



I am glad to talk to complex analysis people about it!

Scaling limit and conformal invariance of discrete lattice models

- Free-boundary dimer model (Berestycki, Lis and Q.)
- Conformal invariance of double random currents (Duminil-Copin, Lis and Q.)

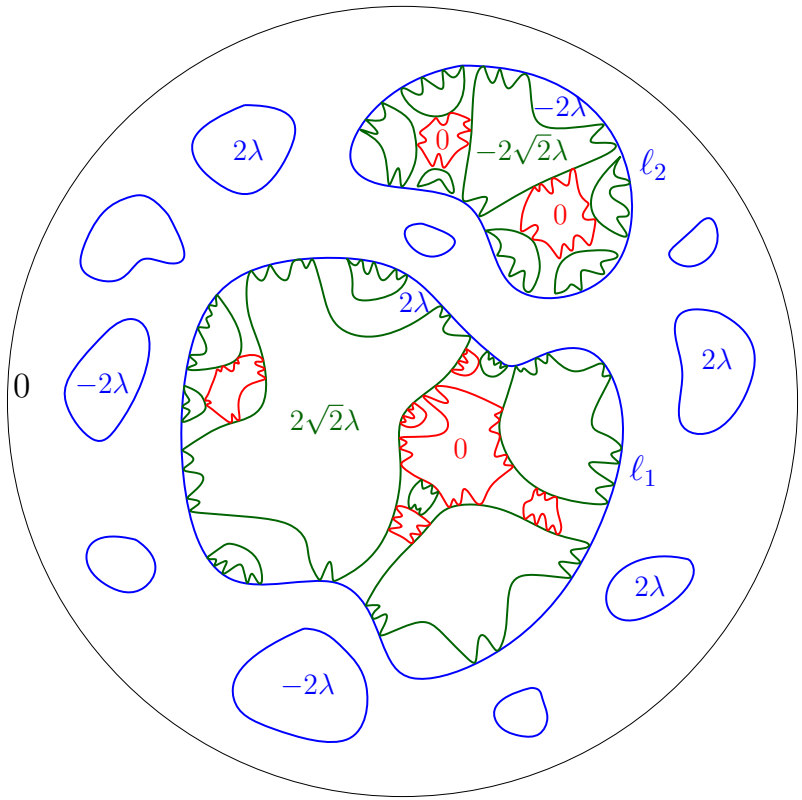


Figure: Scaling limit of the outer and inner boundaries of double random current clusters are level-loops of the GFF

Thank you for your attention !

Brownian loop-soup, Schram-Loewner evolution, conformal loop ensemble, restriction measures, Gaussian free field, Liouville quantum gravity, Brownian map, discrete lattice models...

Conformal welding, fractal metric spaces, geodesics...

(I am in room 225)