Multi-Item Mechanisms: Revenue Maximization from Samples

Yannai A. Gonczarowski

Harvard

SLMath (MSRI) Summer School
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Based upon (but all typos are my own):

Learning Simple Auctions, Jamie Morgenstern and Tim Roughgarden, 2016

The Sample Complexity of Up-to-ε Multi-Dimensional Revenue Maximization, Y.A.G. and S. Matthew Weinberg, 2021

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- **Sample complexity**: the number of samples required for learning a "good enough" auction.
  - CS: "polynomially" = many *vs.* "exponentially" = too many

Setting

- Standard setup: one **seller**, n **items** for sale, m (potential) **buyers**.
- Buyer i's **valuation** for item j is **independently** drawn from some distribution V_{ij} supported on $[0, H]$.
- Each buyer's valuation is **additive** across items.
- Seller wishes to find an **auction mechanism** that would yield good **revenue in expectation** over $\bigtimes_{i,j} V_{ij}$.

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$$\forall i, v_i, v_i' : \quad \mathbb{E}_{v_{-i} \sim V_{-i}} \left[v_i(\text{outcome}) - \text{payment}_i \mid i \text{ bids } v_i \right] \geq \mathbb{E}_{v_{-i} \sim V_{-i}} \left[v_i(\text{outcome}) - \text{payment}_i \mid i \text{ bids } v_i' \right].$$

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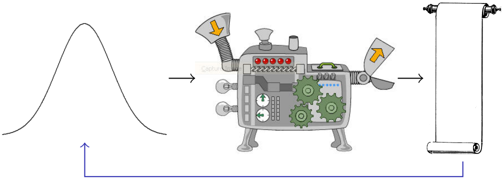
Two standard settings:

- **Bayesian** revenue maximization: seller has **complete knowledge** of $\bigtimes V_{ij}$.
- Revenue maximization **from samples**: seller has access to **polynomially many samples** from $\bigtimes V_{ij}$. ("PAC learning-like.")
  - Benchmark remains the optimal auction given $\bigtimes V_{ij}$.

Bayesian Auction (Mechanism) Design

- The seller is given
  a distribution from which the buyers' types (item valuations) are drawn, but does not know
  the realizations.

- The goal: find a **truthful** auction that **maximizes** the revenue of the seller, **in expectation** over this distribution.



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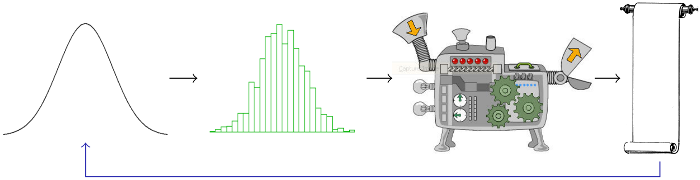

*Up to an additive ε, with high probability (**PAC learning**-like).

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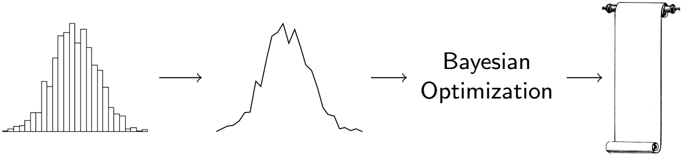

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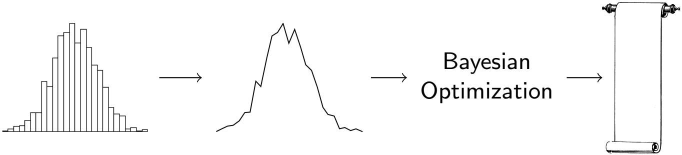

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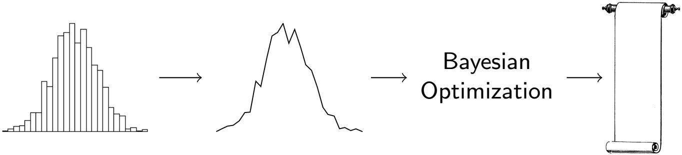

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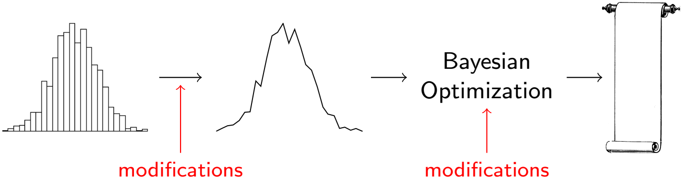

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- Fix samples $(v^{(1)}, \ldots, v^{(s)})$ and fix thresholds $t_1, \ldots, t_s$. How big can we make s while keeping $(v^{(1)}, \ldots, v^{(s)})$ shatterable?

Bounding the Pseudodimension

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Multi-Item (and more generally, multi-parameter)	Some percentage: CHK'07, CHMS'10, CMS'15, HN'12, BILW'14, RW'15, Yao'15, CDW'16, CM'16, CZ'17, HR'19	Some percentage: MR'16, BSV'16, CD'17, S'17, BSV'18
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From Bayesian Design to Empirical Design

- Revenue maximization from samples ubiquitously seen as the "next step" beyond Bayesian revenue maximization:
  1. Understand the **structure** of good-revenue auctions.
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Sample Complexity: A **Nonparametric** Approach

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Notation

m **buyers**, n **items**,
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Theorem (**G.** and Weinberg, 2021)

For every $\varepsilon, \delta > 0$, the sample complexity of learning, w.p. $1-\delta$, an IR and ε-BIC auction that maximizes revenue (among all such auctions) up to an additive ε is $\text{poly}(m, n, H, 1/\varepsilon, \log 1/\delta)$.

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- BIC: $\forall i, v_i, v_i'$:

$$\mathbb{E}_{v_{-i} \sim V_{-i}} [v_i(\text{outcome}) - \text{payment}_i \mid i \text{ bids } v_i] \geq$$
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- Proof **assumes nothing** regarding structure/dimensionality of optimal-, or approximately optimal-, revenue auctions.
- Holds even **far beyond** additive valuations.

Strengthened Results for Special Cases

Corollary (Single Buyer (Digital Goods), Many Items)

For $m = 1$ buyer (recall: also models selling digital goods), for every $\varepsilon, \delta > 0$, the sample complexity of learning, w.p. $1-\delta$, an IR and **IC** *mechanism that maximizes revenue (among all such mechanisms) up to an additive ε is* $\mathrm{poly}(n, H, 1/\varepsilon, \log 1/\delta)$.

Cf. DHN'14: fails for correlated distributions.

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Corollary (Single Item, Many Buyers)

For $n=1$ item, for every $\varepsilon, \delta > 0$, the sample complexity of **efficiently** *learning, w.p. $1-\delta$, an IR and* **DSIC** *auction that maximizes revenue (among all IR and BIC/DSIC auctions) up to an additive ε is* $\mathrm{poly}(m, H, 1/\varepsilon, \log 1/\delta)$.

Cf. parametric approaches: even in "Myersonian" settings, generalizes slightly beyond previous "top-right table cell" results.

What Drives the Results of the Paper

- Assuming access to an **oracle** that can solve the Bayesian revenue maximization problem for **explicitly given** discrete distributions (but assuming nothing about the structure of its output!), we explicitly construct our learning algorithm.

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- In fact, we prove a general **black-box reduction/transformation**, converting "middle-column results" (Bayesian Auction Design) to "right-column results" (Empirical Auction Design):

Meta Theorem (**G.** and Weinberg, 2021)

For any percentage C:

If $\exists$ *algorithm for C% revenue maximization given an explicitly specified finite distribution,*

Then $\forall\, \varepsilon, \delta > 0$, $\exists$ *"as computationally efficient" algorithm for an ε less than C% revenue maximization w.p. $1-\delta$, given* $\text{poly}(m, n, H, 1/\varepsilon, \log 1/\delta)$ *samples from the underlying (not necessarily finite) distribution.*

- Latter loses ε in IC compared to former.
- But, for a single buyer (digital goods) OR a single item: no loss in IC.

Learning-Algorithm Outline

Notation

m **buyers**, n **items**, S **samples**.

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$$(w_{11}^1, \ldots, w_{mn}^1) \qquad (w_{11}^2, \ldots, w_{mn}^2) \qquad \cdots \qquad (w_{11}^S, \ldots, w_{mn}^S)$$

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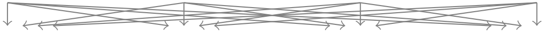

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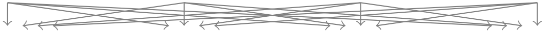

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$$\bigtimes W_{ij}$$

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modify auction to round its inputs to ε-grid

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Optimal auctions have very high dimensionality! Why is there **no overfitting**?

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Deliberately introduced controlled model misspecification for statistical analysis to rule out overfitting

$\bigtimes W_{ij}$

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μ

modify auction to round its inputs to ε-grid

$\hat{\mu}$

Algorithm Analysis

μ

$\times W_{ij}$

round inputs down

$\hat{\mu}$

Algorithm Analysis

μ $\times W_{ij}$

round inputs down

$\hat{\mu}$ $\times V_{ij}$

Algorithm Analysis

μ $\quad$ $\times W_{ij}$

round inputs down

round down to ε-grid

$\hat{\mu}$ $\quad$ $\times V_{ij}$

Algorithm Analysis

μ $\times W_{ij}$

round inputs down

$\times \lfloor V_{ij} \rfloor_\varepsilon$

round down to ε-grid

$\hat{\mu}$ $\times V_{ij}$

Algorithm Analysis

μ $\qquad$ $\times W_{ij}$

draw S samples

$\times \lfloor V_{ij} \rfloor_{\varepsilon}$

round inputs down $\qquad$ round down to ε-grid

$\hat{\mu}$ $\qquad$ $\times V_{ij}$

Algorithm Analysis

μ $\qquad$ $\times W_{ij}$

draw S samples

μ $\qquad$ $\times \lfloor V_{ij} \rfloor_{\varepsilon}$

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round down to ε-grid

$\hat{\mu}$ $\qquad$ $\times V_{ij}$

Algorithm Analysis

μ —is BIC and optimal for→ $\times W_{ij}$

μ $\| $ μ

draw S samples

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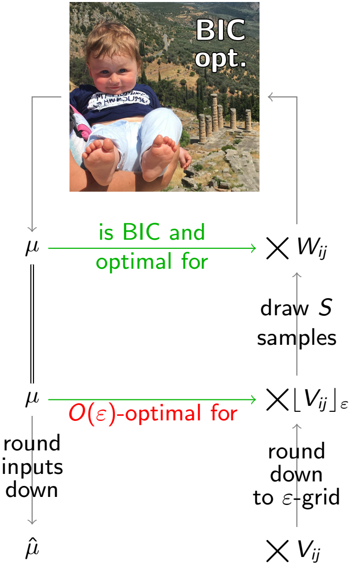
$\times V_{ij}$

Algorithm Analysis



Algorithm Analysis

μ
attains similar revenue

on $\bigtimes W_{ij}$ and $\bigtimes \lfloor V_{ij} \rfloor_\varepsilon$:

μ — is BIC and optimal for → $\bigtimes W_{ij}$

draw S samples

μ — $O(\varepsilon)$-optimal for → $\bigtimes \lfloor V_{ij} \rfloor_\varepsilon$

round inputs down

round down to ε-grid

$\hat{\mu}$ $\qquad$ $\bigtimes V_{ij}$

Algorithm Analysis

μ —is BIC and optimal for→ $\bigtimes W_{ij}$

μ —$O(\varepsilon)$-optimal for→ $\bigtimes \lfloor V_{ij} \rfloor_\varepsilon$

$\bigtimes \lfloor V_{ij} \rfloor_\varepsilon$ —draw S samples→ $\bigtimes W_{ij}$

$\bigtimes V_{ij}$ —round down to ε-grid→ $\bigtimes \lfloor V_{ij} \rfloor_\varepsilon$

μ —round inputs down→ $\hat{\mu}$

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- Enough to take a union bound over $(S+1)^{\frac{mnH}{\varepsilon}}$ random variables.

Algorithm Analysis

μ — is BIC and optimal for → $\bigtimes W_{ij}$

μ — $O(\varepsilon)$-optimal for → $\bigtimes \lfloor V_{ij} \rfloor_\varepsilon$

round inputs down: μ → $\hat{\mu}$

draw S samples: $\bigtimes \lfloor V_{ij} \rfloor_\varepsilon$ → $\bigtimes W_{ij}$

round down to ε-grid: $\bigtimes V_{ij}$ → $\bigtimes \lfloor V_{ij} \rfloor_\varepsilon$

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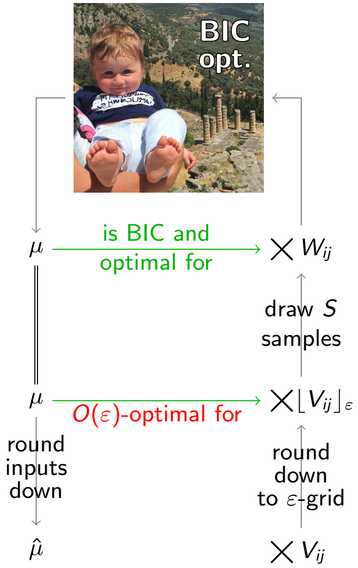
Algorithm Analysis



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Algorithm Analysis

μ — is BIC and optimal for → $\bigtimes W_{ij}$

μ — $O(\varepsilon)$-optimal for → $\bigtimes \lfloor V_{ij} \rfloor_\varepsilon$

μ round inputs down → $\hat{\mu}$

$\bigtimes V_{ij}$ round down to ε-grid → $\bigtimes \lfloor V_{ij} \rfloor_\varepsilon$ draw S samples → $\bigtimes W_{ij}$

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Algorithm Analysis

μ — is BIC and optimal for → $\bigtimes W_{ij}$

μ — $O(\varepsilon)$-optimal for → $\bigtimes \lfloor V_{ij} \rfloor_\varepsilon$

round inputs down: μ → $\hat{\mu}$

draw S samples

round down to ε-grid: $\bigtimes V_{ij}$

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- Grows with S… moderately.

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round inputs down

round down to ε-grid

$\hat{\mu}$ $\bigtimes V_{ij}$

μ
attains similar revenue

on $\bigtimes W_{ij}$ and $\bigtimes \lfloor V_{ij} \rfloor_\varepsilon$:

- Enough to take a union bound over $(S+1)^{\frac{mnH}{\varepsilon}}$ random variables.
- Need convergence for a single variable w.p. $\geq 1 - \frac{\delta}{(S+1)^{\frac{mnH}{\varepsilon}}}$.
- Need #samples $S \approx \log \frac{(S+1)^{\frac{mnH}{\varepsilon}}}{\delta} \approx \text{poly}(m,n,{}^{H}/_{\varepsilon}) \cdot \log S + \log {}^{1}/_{\delta}$.
- Grows with S... moderately.
- $S = \text{poly}(m,n,H,{}^{1}/_{\varepsilon},\log {}^{1}/_{\delta})$ samples suffice!

Algorithm Analysis

μ is BIC and optimal for $\bigtimes W_{ij}$

draw S samples

$\lfloor \text{OPT} \rfloor_\varepsilon$ μ $O(\varepsilon)$-optimal for $\bigtimes \lfloor V_{ij} \rfloor_\varepsilon$

round inputs down

round down to ε-grid

$\hat{\mu}$ $\bigtimes V_{ij}$

μ
attains similar revenue

on $\bigtimes W_{ij}$ and $\bigtimes \lfloor V_{ij} \rfloor_\varepsilon$:

- Enough to take a union bound over $(S+1)^{\frac{mnH}{\varepsilon}}$ random variables.
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- Need #samples $S \approx \log \frac{(S+1)^{\frac{mnH}{\varepsilon}}}{\delta} \approx \text{poly}(m, n, {}^{H}\!/\!{}_{\varepsilon}) \cdot \log S + \log {}^{1}\!/\!{}_{\delta}$.
- Grows with S... moderately.
- $S = \text{poly}(m, n, H, {}^{1}\!/\!{}_{\varepsilon}, \log {}^{1}\!/\!{}_{\delta})$ samples suffice!

Algorithm Analysis

μ is BIC and optimal for $\bigtimes W_{ij}$

draw S samples

$\lfloor \text{OPT} \rfloor_\varepsilon$ $\quad$ μ $O(\varepsilon)$-optimal for $\bigtimes \lfloor V_{ij} \rfloor_\varepsilon$

round inputs down

round down to ε-grid

$\hat{\mu}$ $\quad$ $\bigtimes V_{ij}$

μ
attains similar revenue

on $\bigtimes W_{ij}$ and $\bigtimes \lfloor V_{ij} \rfloor_\varepsilon$:

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- Need convergence for a single variable w.p. $\geq 1 - \frac{\delta}{(S+1)^{\frac{mnH}{\varepsilon}}}$.
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- Grows with S... moderately.
- $S = \text{poly}(m,n,H,{}^1/_\varepsilon,\log {}^1/_\delta)$ samples suffice!

Algorithm Analysis

$\lfloor \mathrm{OPT} \rfloor_\varepsilon$ ‖ $\lfloor \mathrm{OPT} \rfloor_\varepsilon$

μ — is BIC and optimal for → $\bigtimes W_{ij}$

μ — $O(\varepsilon)$-optimal for → $\bigtimes \lfloor V_{ij} \rfloor_\varepsilon$

round inputs down → $\hat{\mu}$

$\bigtimes V_{ij}$ — round down to ε-grid → $\bigtimes \lfloor V_{ij} \rfloor_\varepsilon$ — draw S samples → $\bigtimes W_{ij}$

μ
attains similar revenue

on $\bigtimes W_{ij}$ and $\bigtimes \lfloor V_{ij} \rfloor_\varepsilon$:

- Enough to take a union bound over $(S+1)^{\frac{mnH}{\varepsilon}}$ random variables.
- Need convergence for a single variable w.p. $\geq 1 - \frac{\delta}{(S+1)^{\frac{mnH}{\varepsilon}}}$.
- Need #samples $S \approx \log \frac{(S+1)^{\frac{mnH}{\varepsilon}}}{\delta} \approx \mathrm{poly}(m,n,{}^{H}/_{\varepsilon}) \cdot \log S + \log {}^{1}/_{\delta}$.
- Grows with S... moderately.
- $S = \mathrm{poly}(m,n,H,{}^{1}/_{\varepsilon},\log {}^{1}/_{\delta})$ samples suffice!

Algorithm Analysis

$\lfloor \text{OPT} \rfloor_\varepsilon \overset{Rev}{\leq} \mu$ — is BIC and optimal for → $\bigtimes W_{ij}$

$\lfloor \text{OPT} \rfloor_\varepsilon$ μ — $O(\varepsilon)$-optimal for → $\bigtimes \lfloor V_{ij} \rfloor_\varepsilon$

round inputs down: $\hat{\mu}$

draw S samples

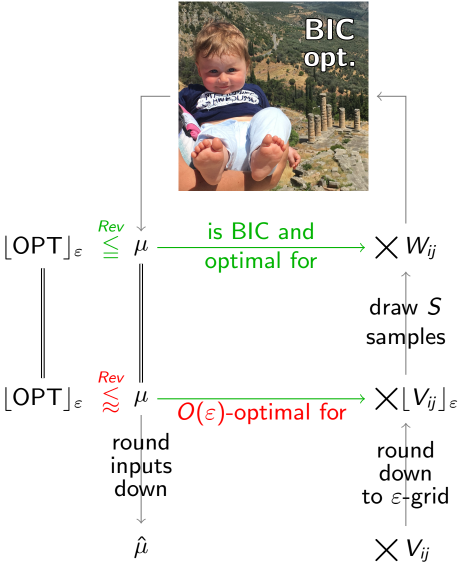
round down to ε-grid: $\bigtimes V_{ij}$

μ
attains similar revenue

on $\bigtimes W_{ij}$ and $\bigtimes \lfloor V_{ij} \rfloor_\varepsilon$:

- Enough to take a union bound over $(S+1)^{\frac{mnH}{\varepsilon}}$ random variables.
- Need convergence for a single variable w.p. $\geq 1 - \frac{\delta}{(S+1)^{\frac{mnH}{\varepsilon}}}$.
- Need #samples $S \approx \log \frac{(S+1)^{\frac{mnH}{\varepsilon}}}{\delta} \approx \text{poly}(m,n,{}^{H}/_{\varepsilon}) \cdot \log S + \log {}^{1}/_{\delta}$.
- Grows with S... moderately.
- $S = \text{poly}(m,n,H,{}^{1}/_{\varepsilon},\log {}^{1}/_{\delta})$ samples suffice!

Algorithm Analysis



μ
attains similar revenue

on $\times W_{ij}$ and $\times \lfloor V_{ij} \rfloor_\varepsilon$:

- Enough to take a union bound over $(S+1)^{\frac{mnH}{\varepsilon}}$ random variables.
- Need convergence for a single variable w.p. $\geq 1 - \frac{\delta}{(S+1)^{\frac{mnH}{\varepsilon}}}$.
- Need #samples $S \approx \log \frac{(S+1)^{\frac{mnH}{\varepsilon}}}{\delta} \approx \mathrm{poly}(m,n,{}^{H}\!/\!_{\varepsilon}) \cdot \log S + \log {}^{1}\!/\!_{\delta}$.
- Grows with S... moderately.
- $S = \mathrm{poly}(m,n,H,{}^{1}\!/\!_{\varepsilon},\log {}^{1}\!/\!_{\delta})$ samples suffice!

Algorithm Analysis

$\lfloor \text{OPT} \rfloor_\varepsilon \overset{Rev}{\leq} \mu$ — is BIC and optimal for → $\bigtimes W_{ij}$

$\lfloor \text{OPT} \rfloor_\varepsilon \overset{Rev}{\lessapprox} \mu$ — $O(\varepsilon)$-optimal for → $\bigtimes \lfloor V_{ij} \rfloor_\varepsilon$

draw S samples

round inputs down

round down to ε-grid

$\hat{\mu}$

$\bigtimes V_{ij}$

μ, as well as $\lfloor \text{OPT} \rfloor_\varepsilon$, attains similar revenue

on $\bigtimes W_{ij}$ and $\bigtimes \lfloor V_{ij} \rfloor_\varepsilon$:

- Enough to take a union bound over $(S+1)^{\frac{mnH}{\varepsilon}}$ random variables.
- Need convergence for a single variable w.p. $\geq 1 - \frac{\delta}{(S+1)^{\frac{mnH}{\varepsilon}}}$.
- Need #samples $S \approx \log \frac{(S+1)^{\frac{mnH}{\varepsilon}}}{\delta} \approx \text{poly}(m,n,{}^{H}/_{\varepsilon}) \cdot \log S + \log {}^{1}/_{\delta}$.
- Grows with S... moderately.
- $S = \text{poly}(m,n,H,{}^{1}/_{\varepsilon},\log {}^{1}/_{\delta})$ samples suffice!

Algorithm Analysis

Unknown, yet well defined; independent of the samples

$\lfloor \mathrm{OPT} \rfloor_\varepsilon \overset{Rev}{\leq} \mu$ — is BIC and optimal for → $\times W_{ij}$

$\lfloor \mathrm{OPT} \rfloor_\varepsilon \overset{Rev}{\lessapprox} \mu$ — $O(\varepsilon)$-optimal for → $\times \lfloor V_{ij} \rfloor_\varepsilon$

draw S samples

round inputs down → $\hat{\mu}$

round down to ε-grid ← $\times V_{ij}$

μ, as well as $\lfloor \mathrm{OPT} \rfloor_\varepsilon$, attains similar revenue

on $\times W_{ij}$ and $\times \lfloor V_{ij} \rfloor_\varepsilon$:

- Enough to take a union bound over $(S+1)^{\frac{mnH}{\varepsilon}}$ random variables.
- Need convergence for a single variable w.p. $\geq 1 - \frac{\delta}{(S+1)^{\frac{mnH}{\varepsilon}}}$.
- Need #samples $S \approx \log \frac{(S+1)^{\frac{mnH}{\varepsilon}}}{\delta} \approx \mathrm{poly}(m,n,{}^{H}\!/\!_{\varepsilon}) \cdot \log S + \log {}^{1}\!/\!_{\delta}$.
- Grows with S... moderately.
- $S = \mathrm{poly}(m,n,H,{}^{1}\!/\!_{\varepsilon},\log {}^{1}\!/\!_{\delta})$ samples suffice!

Algorithm Analysis

Unknown, yet well defined; independent of the samples

$\lfloor \mathrm{OPT} \rfloor_\varepsilon \overset{Rev}{\leq} \mu$ — is BIC and optimal for → $\bigtimes W_{ij}$

$\lfloor \mathrm{OPT} \rfloor_\varepsilon \overset{Rev}{\lessapprox} \mu$ — $O(\varepsilon)$-optimal for → $\bigtimes \lfloor V_{ij} \rfloor_\varepsilon$

draw S samples

round inputs down → $\hat{\mu}$

round down to ε-grid ← $\bigtimes V_{ij}$

μ, as well as $\lfloor \mathrm{OPT} \rfloor_\varepsilon$, attains similar revenue

on $\bigtimes W_{ij}$ and $\bigtimes \lfloor V_{ij} \rfloor_\varepsilon$:

- Enough to take a union bound over $(S+1)^{\frac{mnH}{\varepsilon}}+1$ random variables.
- Need convergence for a single variable w.p. $\geq 1 - \frac{\delta}{(S+1)^{\frac{mnH}{\varepsilon}}}$.
- Need #samples $S \approx \log \frac{(S+1)^{\frac{mnH}{\varepsilon}}}{\delta} \approx \mathrm{poly}(m,n,{}^{H}\!/\!_{\varepsilon}) \cdot \log S + \log {}^{1}\!/\!_{\delta}$.
- Grows with S... moderately.
- $S = \mathrm{poly}(m,n,H,{}^{1}\!/\!_{\varepsilon},\log {}^{1}\!/\!_{\delta})$ samples suffice!

Algorithm Analysis

Unknown, yet well defined; independent of the samples

$\lfloor \mathrm{OPT} \rfloor_\varepsilon \overset{Rev}{\leq} \mu$ — is BIC and optimal for → $\bigtimes W_{ij}$

$\lfloor \mathrm{OPT} \rfloor_\varepsilon \overset{Rev}{\lessapprox} \mu$ — $O(\varepsilon)$-optimal for → $\bigtimes \lfloor V_{ij} \rfloor_\varepsilon$

draw S samples

round inputs down → $\hat{\mu}$

round down to ε-grid ← $\bigtimes V_{ij}$

μ, as well as $\lfloor \mathrm{OPT} \rfloor_\varepsilon$, attains similar revenue

on $\bigtimes W_{ij}$ and $\bigtimes \lfloor V_{ij} \rfloor_\varepsilon$:

- Enough to take a union bound over $(S+1)^{\frac{mnH}{\varepsilon}}+1$ random variables.
- Need convergence for a single variable w.p. $\geq 1 - \frac{\delta}{(S+1)^{\frac{mnH}{\varepsilon}}+1}$.
- Need #samples $S \approx \log \frac{(S+1)^{\frac{mnH}{\varepsilon}}}{\delta} \approx \mathrm{poly}(m,n,{}^{H}\!/\!_{\varepsilon}) \cdot \log S + \log {}^{1}\!/\!_{\delta}$.
- Grows with S... moderately.
- $S = \mathrm{poly}(m,n,H,{}^{1}\!/\!_{\varepsilon},\log {}^{1}\!/\!_{\delta})$ samples suffice!

Algorithm Analysis

Unknown, yet well defined; independent of the samples

$\lfloor \mathrm{OPT} \rfloor_\varepsilon \overset{Rev}{\leqq} \mu$ — is BIC and optimal for → $\bigtimes W_{ij}$

$\lfloor \mathrm{OPT} \rfloor_\varepsilon \overset{Rev}{\lessapprox} \mu$ — $O(\varepsilon)$-optimal for → $\bigtimes \lfloor V_{ij} \rfloor_\varepsilon$

draw S samples

round inputs down: $\hat{\mu}$

round down to ε-grid: $\bigtimes V_{ij}$

μ, as well as $\lfloor \mathrm{OPT} \rfloor_\varepsilon$, attains similar revenue

on $\bigtimes W_{ij}$ and $\bigtimes \lfloor V_{ij} \rfloor_\varepsilon$:

- Enough to take a union bound over $(S+1)^{\frac{mnH}{\varepsilon}+1}$ random variables.
- Need convergence for a single variable w.p. $\geq 1 - \frac{\delta}{(S+1)^{\frac{mnH}{\varepsilon}+1}}$.
- Need #samples $S \approx \log \frac{(S+1)^{\frac{mnH}{\varepsilon}+1}}{\delta} \approx \mathrm{poly}(m,n,H/\varepsilon) \cdot \log S + \log 1/\delta$.
- Grows with S... moderately.
- $S = \mathrm{poly}(m,n,H,1/\varepsilon,\log 1/\delta)$ samples suffice!

Algorithm Analysis

Unknown, yet well defined; independent of the samples

$\lfloor \mathrm{OPT} \rfloor_\varepsilon \overset{Rev}{\leqq} \mu$ —is BIC and optimal for→ $\bigtimes W_{ij}$

$\lfloor \mathrm{OPT} \rfloor_\varepsilon \overset{Rev}{\lessapprox} \mu$ —$O(\varepsilon)$-optimal for→ $\bigtimes \lfloor V_{ij} \rfloor_\varepsilon$

draw S samples

round inputs down → $\hat{\mu}$

$\bigtimes V_{ij}$ — round down to ε-grid

μ, as well as $\lfloor \mathrm{OPT} \rfloor_\varepsilon$, attains similar revenue

on $\bigtimes W_{ij}$ and $\bigtimes \lfloor V_{ij} \rfloor_\varepsilon$:

- Enough to take a union bound over $(S+1)^{\frac{mnH}{\varepsilon}+1}$ random variables.
- Need convergence for a single variable w.p. $\geq 1 - \frac{\delta}{(S+1)^{\frac{mnH}{\varepsilon}+1}}$.
- Need #samples $S \approx \log \frac{(S+1)^{\frac{mnH}{\varepsilon}+1}}{\delta} \approx \mathrm{poly}(m,n,{}^{H}\!/\!_{\varepsilon}) \cdot \log S + \log {}^{1}\!/\!_{\delta}$.
- Grows with S... moderately.
- $S = \mathrm{poly}(m,n,H,{}^{1}\!/\!_{\varepsilon},\log {}^{1}\!/\!_{\delta})$ samples suffice!

Algorithm Analysis

Unknown, yet well defined; independent of the samples

$\lfloor \text{OPT} \rfloor_\varepsilon \overset{Rev}{\leq} \mu$ — is BIC and optimal for → $\times W_{ij}$

$\lfloor \text{OPT} \rfloor_\varepsilon \overset{Rev}{\lessapprox} \mu$ — $O(\varepsilon)$-optimal for → $\times \lfloor V_{ij} \rfloor_\varepsilon$

draw S samples

round inputs down → $\hat{\mu}$

round down to ε-grid

$\times V_{ij}$

μ, as well as $\lfloor \text{OPT} \rfloor_\varepsilon$, attains similar revenue

on $\times W_{ij}$ and $\times \lfloor V_{ij} \rfloor_\varepsilon$:

- Enough to take a union bound over $(S+1)^{\frac{mnH}{\varepsilon}+1}$ random variables.
- Need convergence for a single variable w.p. $\geq 1 - \frac{\delta}{(S+1)^{\frac{mnH}{\varepsilon}+1}}$.
- Need #samples $S \approx \log \frac{(S+1)^{\frac{mnH}{\varepsilon}+1}}{\delta} \approx \text{poly}(m,n,{}^{H}/_{\varepsilon}) \cdot \log S + \log {}^{1}/_{\delta}$.
- Grows with S... moderately.
- $S = \text{poly}(m,n,H,{}^{1}/_{\varepsilon},\log {}^{1}/_{\delta})$ samples suffice!

Algorithm Analysis

Unknown, yet well defined; independent of the samples

$\lfloor \mathrm{OPT} \rfloor_\varepsilon \overset{Rev}{\leq} \mu$ — is BIC and optimal for → $\bigtimes W_{ij}$

$\lfloor \mathrm{OPT} \rfloor_\varepsilon \overset{Rev}{\lessapprox} \mu$ — is $O(\varepsilon)$-BIC and $O(\varepsilon)$-optimal for → $\bigtimes \lfloor V_{ij} \rfloor_\varepsilon$

μ — round inputs down → $\hat{\mu}$

$\bigtimes V_{ij}$ — round down to ε-grid → $\bigtimes \lfloor V_{ij} \rfloor_\varepsilon$ — draw S samples → $\bigtimes W_{ij}$

μ, as well as $\lfloor \mathrm{OPT} \rfloor_\varepsilon$, attains similar revenue

on $\bigtimes W_{ij}$ and $\bigtimes \lfloor V_{ij} \rfloor_\varepsilon$:

- Enough to take a union bound over $(S+1)^{\frac{mnH}{\varepsilon}+1}$ random variables.
- Need convergence for a single variable w.p. $\geq 1 - \frac{\delta}{(S+1)^{\frac{mnH}{\varepsilon}+1}}$.
- Need #samples $S \approx \log \frac{(S+1)^{\frac{mnH}{\varepsilon}+1}}{\delta} \approx \mathrm{poly}(m,n,{}^{H}/_{\varepsilon}) \cdot \log S + \log {}^{1}/_{\delta}$.
- Grows with S... moderately.
- $S = \mathrm{poly}(m,n,H,{}^{1}/_{\varepsilon},\log {}^{1}/_{\delta})$ samples suffice!

Algorithm Analysis

Unknown, yet well defined; independent of the samples

$\lfloor \mathrm{OPT} \rfloor_\varepsilon \overset{Rev}{\leq} \mu$ is BIC and optimal for $\times W_{ij}$

$\lfloor \mathrm{OPT} \rfloor_\varepsilon \overset{Rev}{\lessapprox} \mu$ is $O(\varepsilon)$-BIC and $O(\varepsilon)$-optimal for $\times \lfloor V_{ij} \rfloor_\varepsilon$

draw S samples

round inputs down: $\hat{\mu}$

round down to ε-grid: $\times V_{ij}$

μ, as well as $\lfloor \mathrm{OPT} \rfloor_\varepsilon$, attains similar revenue and has similar **reduced form**

on $\times W_{ij}$ and $\times \lfloor V_{ij} \rfloor_\varepsilon$:

- Enough to take a union bound over $(S+1)^{\frac{mnH}{\varepsilon}+1}$ random variables.
- Need convergence for a single variable w.p. $\geq 1 - \frac{\delta}{(S+1)^{\frac{mnH}{\varepsilon}+1}}$.
- Need #samples $S \approx \log \frac{(S+1)^{\frac{mnH}{\varepsilon}+1}}{\delta} \approx \mathrm{poly}(m,n,{}^{H}/_{\varepsilon}) \cdot \log S + \log {}^{1}/_{\delta}$.
- Grows with S... moderately.
- $S = \mathrm{poly}(m,n,H,{}^{1}/_{\varepsilon},\log {}^{1}/_{\delta})$ samples suffice!

Algorithm Analysis

Unknown, yet well defined; independent of the samples

$\lfloor \mathrm{OPT} \rfloor_\varepsilon \overset{Rev}{\leqq} \mu$ is BIC and optimal for $\bigtimes W_{ij}$

draw S samples

$\lfloor \mathrm{OPT} \rfloor_\varepsilon \overset{Rev}{\lessapprox} \mu$ is $O(\varepsilon)$-BIC and $O(\varepsilon)$-optimal for $\bigtimes \lfloor V_{ij} \rfloor_\varepsilon$

round inputs down

round down to ε-grid

$\hat{\mu}$

$\bigtimes V_{ij}$

μ, as well as $\lfloor \mathrm{OPT} \rfloor_\varepsilon$, attains similar revenue and has similar **reduced form** $\left\{\mathbb{E}_{v_{-i}}\left[v_i(o) - p_i \mid \text{bid } v_i'\right]\right\}_{i,v_i,v_i'}$ on $\bigtimes W_{ij}$ and $\bigtimes \lfloor V_{ij} \rfloor_\varepsilon$:

- Enough to take a union bound over $(S+1)^{\frac{mnH}{\varepsilon}} + 1$ random variables.
- Need convergence for a single variable w.p. $\geq 1 - \frac{\delta}{(S+1)^{\frac{mnH}{\varepsilon}} + 1}$.
- Need #samples $S \approx \log \frac{(S+1)^{\frac{mnH}{\varepsilon}} + 1}{\delta} \approx \mathrm{poly}(m,n,{}^{H}\!/\!_{\varepsilon}) \cdot \log S + \log {}^{1}\!/\!_{\delta}$.
- Grows with S... moderately.
- $S = \mathrm{poly}(m,n,H,{}^{1}\!/\!_{\varepsilon},\log {}^{1}\!/\!_{\delta})$ samples suffice!

Algorithm Analysis

Unknown, yet well defined; independent of the samples

$\lfloor \mathrm{OPT} \rfloor_\varepsilon \overset{Rev}{\leqq} \mu$ is BIC and optimal for $\bigtimes W_{ij}$

$\lfloor \mathrm{OPT} \rfloor_\varepsilon \overset{Rev}{\approx} \mu$ is $O(\varepsilon)$-BIC and $O(\varepsilon)$-optimal for $\bigtimes \lfloor V_{ij} \rfloor_\varepsilon$

draw S samples

round inputs down

$\hat{\mu}$

round down to ε-grid

$\bigtimes V_{ij}$

μ, as well as $\lfloor \mathrm{OPT} \rfloor_\varepsilon$, attains similar revenue and has similar **reduced form** $\{\mathbb{E}_{v_{-i}}[v_i(o) - p_i \mid \text{bid } v_i']\}_{i,v_i,v_i'}$ on $\bigtimes W_{ij}$ and $\bigtimes \lfloor V_{ij} \rfloor_\varepsilon$:

- Enough to take a union bound over $\left((S+1)^{\frac{mnH}{\varepsilon}}+1\right) \cdot \left(1+m\lceil \frac{H}{\varepsilon} \rceil^{2n}\right)$ random variables.
- Need convergence for a single variable w.p. $\geq 1 - \frac{\delta}{(S+1)^{\frac{mnH}{\varepsilon}}+1}$.
- Need #samples $S \approx \log \frac{(S+1)^{\frac{mnH}{\varepsilon}}+1}{\delta} \approx \mathrm{poly}(m,n,{}^{H}\!/\!_{\varepsilon}) \cdot \log S + \log {}^{1}\!/\!_{\delta}$.
- Grows with S... moderately.
- $S = \mathrm{poly}(m,n,H,{}^{1}\!/\!_{\varepsilon},\log {}^{1}\!/\!_{\delta})$ samples suffice!

Algorithm Analysis

Unknown, yet well defined; independent of the samples

$\lfloor \mathrm{OPT} \rfloor_\varepsilon \overset{Rev}{\leq} \mu$ — is BIC and optimal for → $\times W_{ij}$

draw S samples

$\lfloor \mathrm{OPT} \rfloor_\varepsilon \overset{Rev}{\lessapprox} \mu$ — is $O(\varepsilon)$-BIC and $O(\varepsilon)$-optimal for → $\times \lfloor V_{ij} \rfloor_\varepsilon$

round inputs down: $\hat{\mu}$

round down to ε-grid: $\times V_{ij}$

μ, as well as $\lfloor \mathrm{OPT} \rfloor_\varepsilon$, attains similar revenue and has similar **reduced form** $\left\{\mathbb{E}_{v_{-i}}\left[v_i(o) - p_i \mid \text{bid } v_i'\right]\right\}_{i,v_i,v_i'}$ on $\times W_{ij}$ and $\times \lfloor V_{ij} \rfloor_\varepsilon$:

- Enough to take a union bound over $\left((S+1)^{\frac{mnH}{\varepsilon}}+1\right)\cdot\left(1+m\left\lceil\frac{H}{\varepsilon}\right\rceil^{2n}\right)$ random variables.
- Need convergence for a single variable w.p. $\geq 1 - \frac{\delta}{\left((S+1)^{\frac{mnH}{\varepsilon}}+1\right)\cdot\left(1+m\left\lceil\frac{H}{\varepsilon}\right\rceil^{2n}\right)}$.
- Need #samples $S \approx \log \frac{(S+1)^{\frac{mnH}{\varepsilon}}+1}{\delta} \approx \mathrm{poly}(m,n,{}^{H}/_{\varepsilon})\cdot\log S + \log {}^{1}/_{\delta}$.
- Grows with S... moderately.
- $S = \mathrm{poly}(m,n,H,{}^{1}/_{\varepsilon},\log {}^{1}/_{\delta})$ samples suffice!

Algorithm Analysis

Unknown, yet well defined; independent of the samples

$\lfloor \mathrm{OPT} \rfloor_\varepsilon \overset{Rev}{\leq} \mu$ — is BIC and optimal for → $\times W_{ij}$

draw S samples

$\lfloor \mathrm{OPT} \rfloor_\varepsilon \overset{Rev}{\lessapprox} \mu$ — is $O(\varepsilon)$-BIC and $O(\varepsilon)$-optimal for → $\times \lfloor V_{ij} \rfloor_\varepsilon$

round inputs down

$\hat{\mu}$

round down to ε-grid

$\times V_{ij}$

μ, as well as $\lfloor \mathrm{OPT} \rfloor_\varepsilon$, attains similar revenue and has similar **reduced form** $\left\{\mathbb{E}_{v_{-i}}\left[v_i(o) - p_i \mid \text{bid } v_i'\right]\right\}_{i,v_i,v_i'}$ on $\times W_{ij}$ and $\times \lfloor V_{ij} \rfloor_\varepsilon$:

- Enough to take a union bound over $\left((S+1)^{\frac{mnH}{\varepsilon}}+1\right)\cdot\left(1+m\left\lceil \frac{H}{\varepsilon}\right\rceil^{2n}\right)$ random variables.
- Need convergence for a single variable w.p. $\geq 1 - \frac{\delta}{\left((S+1)^{\frac{mnH}{\varepsilon}}+1\right)\cdot\left(1+m\left\lceil \frac{H}{\varepsilon}\right\rceil^{2n}\right)}$.
- Need #samples $S \approx \log\frac{\left((S+1)^{\frac{mnH}{\varepsilon}}+1\right)\cdot\left(1+m\left\lceil \frac{H}{\varepsilon}\right\rceil^{2n}\right)}{\delta} \approx \mathrm{poly}(m,n,{}^{H}/_{\varepsilon})\cdot\log S + \log {}^{1}/_{\delta}$.
- Grows with S... moderately.
- $S = \mathrm{poly}(m,n,H,{}^{1}/_{\varepsilon},\log {}^{1}/_{\delta})$ samples suffice!

Algorithm Analysis

Unknown, yet well defined; independent of the samples

$\lfloor \mathrm{OPT} \rfloor_\varepsilon \overset{Rev}{\leq} \mu$ — is BIC and optimal for → $\bigtimes W_{ij}$

draw S samples

$\lfloor \mathrm{OPT} \rfloor_\varepsilon \overset{Rev}{\lessapprox} \mu$ — is $O(\varepsilon)$-BIC and $O(\varepsilon)$-optimal for → $\bigtimes \lfloor V_{ij} \rfloor_\varepsilon$

round inputs down

round down to ε-grid

$\hat{\mu}$ $\qquad$ $\bigtimes V_{ij}$

μ, as well as $\lfloor \mathrm{OPT} \rfloor_\varepsilon$, attains similar revenue and has similar **reduced form** $\{\mathbb{E}_{v_{-i}}[v_i(o) - p_i \mid \text{bid } v_i']\}_{i,v_i,v_i'}$ on $\bigtimes W_{ij}$ and $\bigtimes \lfloor V_{ij} \rfloor_\varepsilon$:

- Enough to take a union bound over $\left((S+1)^{\frac{mnH}{\varepsilon}}+1\right) \cdot \left(1+m\left\lceil \frac{H}{\varepsilon} \right\rceil^{2n}\right)$ random variables.
- Need convergence for a single variable w.p. $\geq 1 - \frac{\delta}{\left((S+1)^{\frac{mnH}{\varepsilon}}+1\right) \cdot \left(1+m\left\lceil \frac{H}{\varepsilon} \right\rceil^{2n}\right)}$.
- Need #samples $S \approx \log \frac{\left((S+1)^{\frac{mnH}{\varepsilon}}+1\right) \cdot \left(1+m\left\lceil \frac{H}{\varepsilon} \right\rceil^{2n}\right)}{\delta} \approx \mathrm{poly}(m,n,{}^{H}/_{\varepsilon}) \cdot \log S + \log {}^{1}/_{\delta}$.
- Grows with S... moderately.
- $S = \mathrm{poly}(m,n,H,{}^{1}/_{\varepsilon},\log {}^{1}/_{\delta})$ samples suffice!

Algorithm Analysis

Unknown, yet well defined; independent of the samples

$\lfloor \mathrm{OPT} \rfloor_\varepsilon \overset{Rev}{\leq} \mu$ — is BIC and optimal for → $\bigtimes W_{ij}$

$\lfloor \mathrm{OPT} \rfloor_\varepsilon \overset{Rev}{\lessapprox} \mu$ — is $O(\varepsilon)$-BIC and $O(\varepsilon)$-optimal for → $\bigtimes \lfloor V_{ij} \rfloor_\varepsilon$

draw S samples

round inputs down

round down to ε-grid

$\hat{\mu}$ — is $O(\varepsilon)$-BIC and $O(\varepsilon)$-optimal for → $\bigtimes V_{ij}$

μ, as well as $\lfloor \mathrm{OPT} \rfloor_\varepsilon$, attains similar revenue and has similar **reduced form** $\left\{\mathbb{E}_{v_{-i}}\left[v_i(o) - p_i \mid \text{bid } v_i'\right]\right\}_{i,v_i,v_i'}$ on $\bigtimes W_{ij}$ and $\bigtimes \lfloor V_{ij} \rfloor_\varepsilon$:

- Enough to take a union bound over $\left((S+1)^{\frac{mnH}{\varepsilon}}+1\right)\cdot\left(1+m\left\lceil \frac{H}{\varepsilon}\right\rceil^{2n}\right)$ random variables.
- Need convergence for a single variable w.p. $\geq 1 - \frac{\delta}{\left((S+1)^{\frac{mnH}{\varepsilon}}+1\right)\cdot\left(1+m\left\lceil \frac{H}{\varepsilon}\right\rceil^{2n}\right)}$.
- Need #samples $S \approx \log\frac{\left((S+1)^{\frac{mnH}{\varepsilon}}+1\right)\cdot\left(1+m\left\lceil \frac{H}{\varepsilon}\right\rceil^{2n}\right)}{\delta} \approx \mathrm{poly}(m,n,{}^H\!/\!_\varepsilon)\cdot\log S + \log {}^1\!/\!_\delta$.
- Grows with S... moderately.
- $S = \mathrm{poly}(m,n,H,{}^1\!/\!_\varepsilon,\log {}^1\!/\!_\delta)$ samples suffice!

(A Peak) Behind the Scenes

- ε-BIC vs. BIC revenue maximization:

Theorem (Rubinstein and Weinberg, 2015; see also DW12)

Let $\mathcal{W}$ be any joint distribution over arbitrary valuations, where the valuations of different buyers are independent.
The maximum revenue attainable by any IR and ε-BIC auction for $\mathcal{W}$ is at most $2m\sqrt{nLH\varepsilon}$ greater than the maximum revenue attainable by any IR and BIC auction for $\mathcal{W}$.

Algorithm Analysis

Unknown, yet well defined; independent of the samples

$\lfloor \mathrm{OPT} \rfloor_\varepsilon \overset{Rev}{\leq} \mu$ is BIC and optimal for $\bigtimes W_{ij}$

$\|$ $\|$ draw S samples

$\lfloor \mathrm{OPT} \rfloor_\varepsilon \overset{Rev}{\lessapprox} \mu$ is $O(\varepsilon)$-BIC and $O(\varepsilon)$-optimal for $\bigtimes \lfloor V_{ij} \rfloor_\varepsilon$

round inputs down; round down to ε-grid

$\hat{\mu}$ is $O(\varepsilon)$-BIC and $O(\varepsilon)$-optimal for $\bigtimes V_{ij}$

μ, as well as $\lfloor \mathrm{OPT} \rfloor_\varepsilon$, attains similar revenue and has similar **reduced form** $\left\{ \mathbb{E}_{v_{-i}} \left[v_i(o) - p_i \mid \text{bid } v_i' \right] \right\}_{i, v_i, v_i'}$ on $\bigtimes W_{ij}$ and $\bigtimes \lfloor V_{ij} \rfloor_\varepsilon$:

- Enough to take a union bound over $\left((S+1)^{\frac{mnH}{\varepsilon}} + 1 \right) \cdot \left(1 + m \left\lceil \frac{H}{\varepsilon} \right\rceil^{2n} \right)$ random variables.
- Need convergence for a single variable w.p. $\geq 1 - \frac{\delta}{\left((S+1)^{\frac{mnH}{\varepsilon}} + 1 \right) \cdot \left(1 + m \left\lceil \frac{H}{\varepsilon} \right\rceil^{2n} \right)}$.
- Need #samples $S \approx \log \frac{\left((S+1)^{\frac{mnH}{\varepsilon}} + 1 \right) \cdot \left(1 + m \left\lceil \frac{H}{\varepsilon} \right\rceil^{2n} \right)}{\delta} \approx \mathrm{poly}(m, n, H/\varepsilon) \cdot \log S + \log 1/\delta$.
- Grows with S... moderately.
- $S = \mathrm{poly}(m, n, H, 1/\varepsilon, \log 1/\delta)$ samples suffice!

(A Peak) Behind the Scenes

- ε-BIC vs. BIC revenue maximization:

Theorem (Rubinstein and Weinberg, 2015; see also DW12)

Let $\mathcal{W}$ be any joint distribution over arbitrary valuations, where the valuations of different buyers are independent.
The maximum revenue attainable by any IR and ε-BIC auction for $\mathcal{W}$ is at most $2m\sqrt{nLH\varepsilon}$ greater than the maximum revenue attainable by any IR and BIC auction for $\mathcal{W}$.

- Chernoff-style concentration bound for product distributions:

Theorem (Babichenko et al., 2017; see also DHP16)

Let $V_1, \ldots, V_\ell$ be discrete distributions. Let $S \in \mathbb{N}$.
For every i, draw S independent samples from V_i, and let W_i be the uniform distribution over these samples.
For every $\varepsilon > 0$ and $f : \bigtimes_{i=1}^{\ell} \operatorname{supp} V_i \to [0, H]$, we have that

$$\Pr\Big(\big|\mathbb{E}_{\times_{i=1}^{\ell} W_i}[f] - \mathbb{E}_{\times_{i=1}^{\ell} V_i}[f]\big| > \varepsilon\Big) \leq \frac{4H}{\varepsilon} \exp\Big(-\frac{\varepsilon^2 S}{8H^2}\Big).$$

Algorithm Analysis

Unknown, yet well defined; independent of the samples

$\lfloor \mathrm{OPT} \rfloor_\varepsilon \overset{Rev}{\leq} \mu$ is BIC and optimal for $\bigtimes W_{ij}$

draw S samples

$\lfloor \mathrm{OPT} \rfloor_\varepsilon \overset{Rev}{\lessapprox} \mu$ is $O(\varepsilon)$-BIC and $O(\varepsilon)$-optimal for $\bigtimes \lfloor V_{ij} \rfloor_\varepsilon$

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- Deliberate model misspecification as a **tool against overfitting.**

Conclusion & Further Research

- Main takeaway: **empirical revenue maximization not harder than Bayesian revenue maximization** in many settings: any result that holds given full information immediately implies a robust result from samples.
  - For many buyers AND many items, a fairly moderate price to pay: ε-IC.
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Open Question

Given an IR and ε-BIC auction for some product distribution, even in an additive multi-item setting, is it possible to transform it into an IR and (precisely) BIC auction with negligible ($\text{poly}(\varepsilon) \cdot \text{poly}(m, n, H)$) revenue loss using polynomially many samples from this product distribution?

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- Known ε-BIC-to-BIC reduction from samples (DHKN'17) requires a number of samples that is polynomial in the size of the type space = exponential in the number of items, but does **not** assume independence.

- Can independence come to the rescue?

Questions?