Exercises for the Wednesday Afternoon

The solution of each subsection in this sheet is rather short, and you might try to solve them in your head. At the beginning of the tutorial on Thursday, some of you will have the opportunity to present their solution on board.

Problem 5.1 This problem derives an interesting interpretation of a virtual valuation $\varphi(v) = v - \frac{1-F(v)}{f(v)}$ and the regularity condition. Consider a strictly increasing distribution function F with a strictly positive density function f on the interval $[0, v_{\max}]$, with $v_{\max} < +\infty$.

For a single bidder with valuation drawn from F, for $q \in [0, 1]$, define $V(q) = F^{-1}(1-q)$ as the (unique) posted price that yields a probability q of a sale. Define $R(q) = q \cdot V(q)$ as the expected revenue obtained from a single bidder when the probability of a sale is q. The function $R(q)$, for $q \in [0, 1]$, is the *revenue curve* of F. Note that $R(0) = R(1) = 0$.

(a) What is the revenue curve for the uniform distribution on $[0, 1]$?

(b) Prove that the slope of the revenue curve at q (i.e., $R'(q)$) is precisely $\varphi(V(q))$, where φ is the virtual valuation function for F.

(c) Prove that a distribution is regular if and only if its revenue curve is concave.

Problem 5.2 *(H)* Consider a single bidder with valuation drawn from a regular distribution F that satisfies the assumptions of Problem 5.1. Let p be the *median* of F, meaning the value for which $F(p) = \frac{1}{2}$. Prove that the price p earns at least 50% of the expected revenue of the optimal posted price for F.