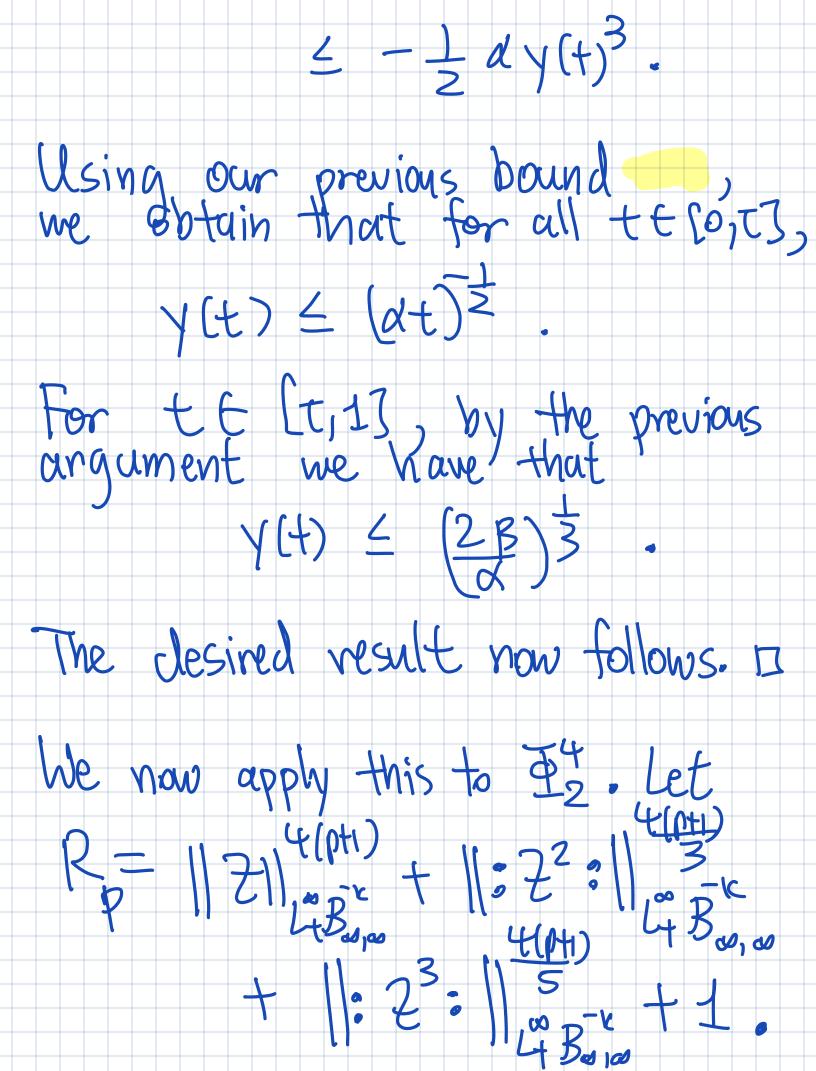
Comina claim from intinity.

Ref: Lee Moinut-Weber Space time localization. For an alternative proof For Pz in unit volume, recall the L'estimate from Session 3: 2p 0 th 112p + m 112p + 112p + 112p + 112p + 1 1 2 (pt1)
4 1 1 1 2 (pt1) For simplicity, we take  $\lambda = 1$ . Using this, we will show that

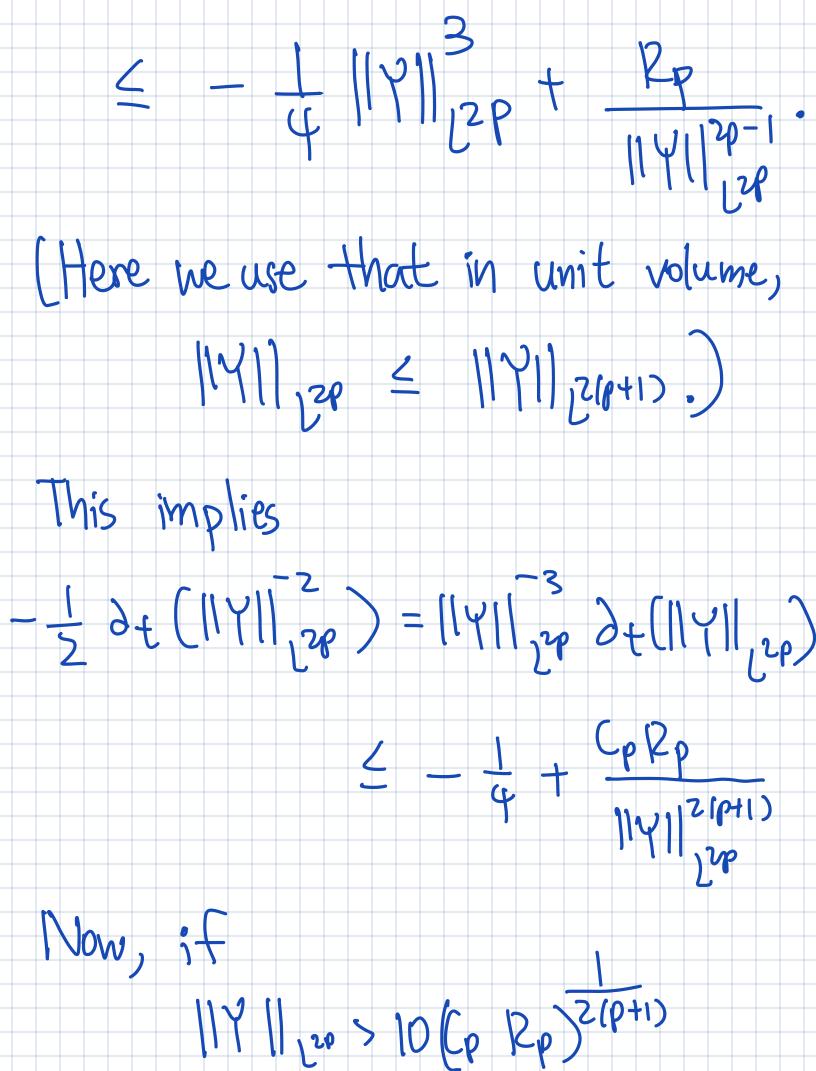
114 (t) 11 p satisfies a bound which is completely independent of 11710/11, p. Estimates of this type cure known as coming down from infinity. We will see that the essence of the penament reduces to understanding the Denamer of the ODE:  $\dot{y}(t) \leq -\alpha \dot{y}(t)^3$ . Here d70 is some garameter. We have that  $-\frac{1}{2}\frac{d}{dt}\frac{d}{dt} = \frac{1}{1}\frac{(t)}{(t)^3} = -\frac{1}{2}$ which implies

 $\frac{d}{dt} \gamma(t) \geq 2 \alpha$ Y(4) > Y(0) + 2at Y(t) \( \langle \langl  $\frac{2}{2\alpha t}$ Note that the bound is independent of y(b). More generally, we have the following Lemma. Let y: (0,13 -> (0,00) be

In cleed, if the sup is attained at some to E (0) 1], then the left derivative of y at to is  $\geq 0$ , and thus 0 \( \frac{1}{y(t\_0)} \( \frac{2}{-} \alpha y(t\_0)^3 + \beta \) 1 (to) 4 (B) 3. 1000, suppose that y(0) > (2B)3. T = inf3te(0)13: (28)3It no such t exists, set T = 1.
Then for all te lo, TJ, we have that  $\dot{y}(t) \leq -\alpha y(t)^3 + \beta$ 



Here, Lt is taken over the unit time interval. By the LP estimate, we have that t CP RP Thus, 9+ (11/11/56) = 9+ (11/11/56) 36 = 1 (|| \( \) | \( \) | \( \)  $\leq \frac{11}{11} \frac{1}{12} \frac{1}{12} \left( \frac{1}{12} \frac{1}$ 



for all t E (v, 1), then obtain 1 2 ( | Y | 7p ) 4 - 10 for all t E (0,1]. Otherwise, let t be the first time st 117(4)1120 E 10 (CP (SP) 2(0+1) for all t & Coit], have 117(4)11 & £ 5 for all te [t, 1], have 114 (4) 11,2p = 10 (Cp2p)2(p+1) Thus, obtain  $||Y(t)||_{2p} \leq \max(t^{\frac{1}{2}}(CpPp)^{\frac{2(p+1)}{2}})$ for all + E [0,1]. Using a similar argument, can Lemma. (Cf Lemma 2.7 Mourrat – Weber, Lemma 6.1 Hairer – Steele). Let u: [0,1] x II d > R satisfy  $(2+-1)u \leq -u^3 + C_0$ Men ||u(t)|| == = max[+=, (3). Pf. Have

$$\frac{1}{2p} \Rightarrow_{\xi} (||u||_{L^{2p}}^{2p}) \leq -||u||_{L^{2(p+1)}}^{2(p+1)}$$

$$+ C_{\delta} ||u||_{L^{2p}-1}^{2(p+1)}$$

$$+ C_{\delta} ||u||_{L^{2p}-1}^{2(p+1)}$$

$$+ C_{\delta} ||u||_{L^{2p}-1}^{2(p+1)}$$

$$+ C_{\delta} ||u||_{L^{2p}}^{2p}$$

$$-\frac{1}{2} \partial_{t} \left( ||u||_{2p}^{-2} \right) = ||u||_{2p}^{\frac{3}{2}} \partial_{t} \left( ||u||_{2p} \right)$$

$$\leq -\frac{1}{2} + \frac{C}{2} \left( \frac{1}{2} e^{\frac{1}{2}} e^{\frac{1}{2}} e^{\frac{1}{2}} \right)$$

$$||u||_{2p}^{2p} > ||0CC_{0}^{\frac{1}{2}}||$$
for all,  $t \in (0,1]$ , then obtain bound like
$$||u(t)||_{2p} \leq t^{\frac{1}{2}}$$
as before. Can handle other case similar to before.