Lecture Series A: Problem Set 5

1. Let G be a $\mathrm{Gr}_{k,n}$-graph. Show that the face labels of G have size k.

2. Consider the plabic graph G constructed as follows:

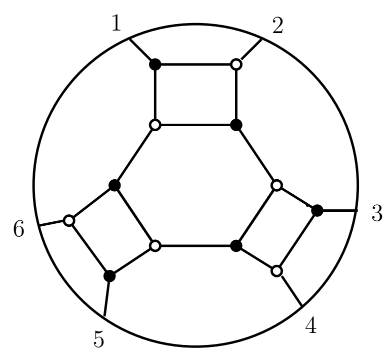
- Draw a grid with $k-1$ rows of squares and $n-k-1$ columns of squares.
- Color the lower left corner of the grid black, then color the corners of all squares so they are bipartite.
- Attach a boundary leg to each white vertex along the top and bottom boundaries, to each black vertex along the left and right boundaries, and to any corner of the grid that doesn't yet have a leg attached. Label the legs $1, \ldots, n$ clockwise, starting at the lower left corner.

Draw a picture of G and its dual quiver Q_G. Prove that G is a $\mathrm{Gr}_{k,n}$-graph.

3. Suppose G, G' are $\mathrm{Gr}_{k,n}$-graphs.

a.) If G, G' are related by a (B) or (C) move, prove that $\Sigma_G = \Sigma_{G'}$.
b.) If G, G' are related by a square move at f, prove that $Q_{G'} = \mu_f(Q_G)$.

4. Let G be the following $\mathrm{Gr}_{3,6}$-graph.



a) Compute Σ_G and $\mu_f(\Sigma_G)$, where f is the hexagonal face.
b) The seed $\mu_f(\Sigma_G)$ has one non-Plücker cluster variable x. Find an expression for x as a polynomial in Plücker coordinates.
c) The cluster variable x is a tensor invariant $[W]$ for some web W. Find W. Hint: take your expression from b) and look at a single term. Express that term as a tensor diagram, and see what happens when you apply the uncrossing relation.

5. On Wednesday, we produced seeds Σ_T for $\mathrm{Gr}_{2,n}$ from triangulations T of the n-gon. Today, we produced seeds Σ_G for $\mathrm{Gr}_{k,n}$ from $\mathrm{Gr}_{k,n}$-graphs. For each triangulation T, give a construction of a $\mathrm{Gr}_{2,n}$-graph $G(T)$ so that $\Sigma_T = \Sigma_{G(T)}$.