Another comb. char. of triang's:

Thm: [de Loera et al.]

$\emptyset \neq T \in \binom{\mathcal{A}}{d+1}$ is a triang of $\mathcal{A}$ iff

(CP) For every interior facet τ of a simplex $\sigma \in T$ there are two simplices
$\sigma, \sigma' \in T$, $\sigma \neq \sigma'$, $\tau < \sigma$, $\tau < \sigma'$
s.t.
$\chi(\tau, \sigma \setminus \tau) = -\chi(\tau, \sigma' \setminus \tau)$ "Cocircuit Property"

(EP) conv $\mathcal{A}$ has a simplicial facet or there is a point p in the interior of conv $\mathcal{A}$ in general position that is contained in conv σ for exactly one simplex $\sigma \in T$. "Extension Property"

Rem.: (EP) can also be checked by chirotope values.

Rem.: This one is implemented in TOPCOM.

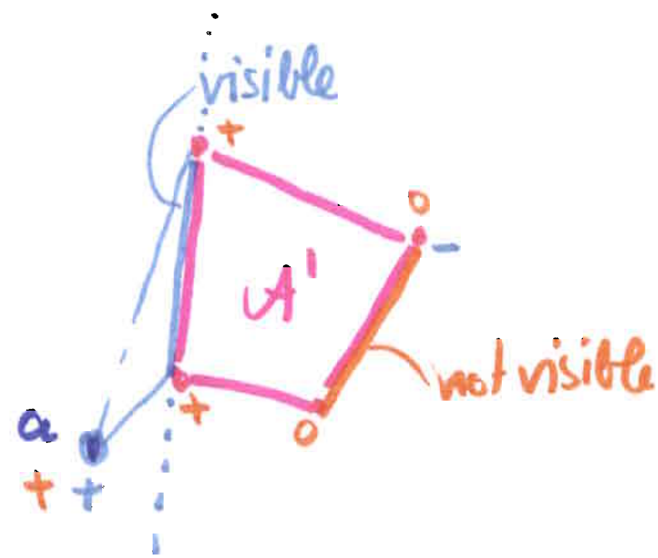
Obs.: ▷ Check triang. prop. ✓

Visible facets and cocircuits:

Prop.: Let $\mathcal{A}' \subsetneq \mathcal{A} \subset \mathbb{R}^d$ and $a \in \mathcal{A} \setminus \mathcal{A}'$.
A facet of $\mathcal{A}$ is visible by a iff the cocircuit defined by the facet has different signs on a and on at least one other point in $\mathcal{A}'$!
("+" "−" or "−" "+", not "+" "0" etc.)

Cor.: Placing triang. can be computed via cocircuit signatures

Example:



Obs.: ▷ Find one triang. ✓

BFS vs. Equivariant BFS (EBFS):

Alg. BFS

Input: $G=(V,E), v_0 \in V, \Gamma: V \to 2^V$ oracle

Output: $|V|$

1. $cnt \leftarrow 1, \mathcal{G} \leftarrow \{\underbrace{(v_0,\Gamma(v_0))}_{=:\, v_0 \text{ for brevity}}\}, \mathcal{W} \leftarrow \emptyset$
2. While $\mathcal{G} \neq \emptyset$
3.   For all $v \in \mathcal{G}$
4.   For all $w \in \Gamma(v)$ unmarked
5.   If $w \in \mathcal{G} \cup \mathcal{W}$
6.   mark $w \in \Gamma(v)$
7.   else
8.   mark $w \in \Gamma(v)$
9.   mark $v \in \Gamma(w)$
10. $\mathcal{W} \leftarrow \mathcal{W} \cup \{w\}, cnt \leftarrow cnt+1$
11.   endif
12.   endfor
13.   endfor
14. $\mathcal{G} \leftarrow \mathcal{W}, \mathcal{W} \leftarrow \emptyset$
15. endwhile
16. return cnt

Alg. EBFS

Input: $G=(V,E), v_0 \in V, \Gamma: V \to 2^V$ oracle, $\Pi(G)$ oracle

Output: $|V|$

1. $cnt \leftarrow 1, \mathcal{G} \leftarrow \{v_0\}, \mathcal{W} \leftarrow \emptyset$
2. While $\mathcal{G} \neq \emptyset$
3.   For all $v \in \mathcal{G}$
4.   For all $w \in \Gamma(v)$ unmarked
5.   If $\exists \pi \in \Pi(G): \pi(w) \in \mathcal{G} \cup \mathcal{W}$
6.   mark $\pi(v)$ in $\Gamma(\pi(w))$
7.   mark w in $\Gamma(v)$
8.   else
9.   mark w in $\Gamma(v)$
10.   mark v in $\Gamma(w)$
11. $\mathcal{W} \leftarrow \mathcal{W} \cup \{w\}, cnt \leftarrow cnt+1$
12.   endif
13.   endfor
14.   endfor
15. $\mathcal{G} \leftarrow \mathcal{W}, \mathcal{W} \leftarrow \emptyset$
16. endwhile
17. return cnt

$\triangleright$ Find comp. of flip gr. ✓

Obs.: Can be applied to flip graph: symm. in $\mathcal{A}$ $\rightsquigarrow$ symm. in $\mathcal{G}_{\mathcal{A}}$.

Triang's and independent sets

Def.: The conflict graph on the set of all d-simplices in $\mathcal{A}$ is the graph $G^{\wedge} = (V, E)$ with

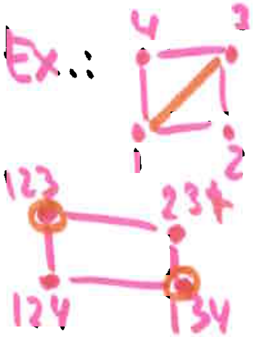
$$V := \left\{ \sigma \in \binom{\mathcal{A}}{d+1} : \dim \sigma = d \right\}$$

$$E := \left\{ \{\sigma, \sigma'\} : \sigma \text{ and } \sigma' \text{ do not intersect properly} \right\}$$

$$\Leftrightarrow \quad \operatorname{conv} \sigma \cap \operatorname{conv} \sigma' \neq \operatorname{conv}(\sigma \cap \sigma')$$

$$\Leftrightarrow \quad \exists \text{ circuit } (C_+, C_-) \text{ with } C_+ \subseteq \sigma,\ C_- \subseteq \sigma'$$

Ex.:



Prop.: Every triang. of $\mathcal{A}$ is a maximal independent set in $G^{\wedge}$.

Example: dim 1 and 2: triang's $\overset{\text{1-to-1}}{\Longleftrightarrow}$ max. indep. sets.

dim 3 : $\exists$ partial triangs that cannot be extended.

Obs.: Enumeration of all max. indep. sets yields all non-extendable partial triangs. Checking (UP) $\Longrightarrow$ all triang's.

▷ Find all triang's. ✓ (e.g., backtracking DFS)

Rem.: Do not know to use symmetries here.