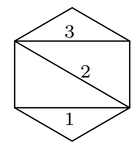
Lecture 3A: Exercises

Exercise 3Aa. *Given a fixed numbering $n+1,\ldots,2n+3$ of the edges of an $(n+3)$-gon Q, show that a triangulation of Q is determined uniquely by its edge-adjacency matrix.*

Exercise 3Ab. *Show that the top $n\times n$ submatrix of the edge-adjacency matrix is skew-symmetric. (Thus it is an exchange matrix.)*

Exercise 3Ac. *Fix a numbering $n+1,\ldots,2n+3$ of the edges of an $(n+3)$-gon Q. Let T and T' be triangulations of Q related by a diagonal flip. Number the diagonals of T and T' so that the flipped diagonal is k in both, and the non-flipped diagonals have the same labeling in both. Show that the edge-adjacency matrices of T and T' are related by the matrix mutation μ_k.*

Exercise 3Ad. *Consider the triangulation shown to the right, with diagonals labeled. Interpret the top square part of its edge-adjacency matrix an the exchange matrix. Take $\mathbf{x}=(x_1,x_2,x_3)$ and $\mathbb{P}=\{1\}$. Find the cluster variables and exchange matrices in the pattern.*



Recommendation: Use a drawing of the diagonal flips graph to organize your calculation of exchange matrices. Use a drawing of the dual simplicial complex to organize your calculation of cluster variables.

Exercise 3Ae. *Verify, for Φ of type A_3, that the action of τ_+ on $\Phi_{\geq -1}$ corresponds to a reflection of the hexagon, acting on diagonals. Same for τ_-. Verify that these reflections generate the symmetry group of the hexagon.*

Exercises, in order of priority

There are more exercises than you can be expected to complete in a *half* day. Please work on them in the order listed. Exercises on the first line constitute a minimum goal. It would be profitable to work all of the exercises eventually.

3Ab, 3Ac,

3Ae, 3Ad 3Aa.

Lecture 3B: Exercises

Exercise 3Ba. *Show that the action of each s_i is an isometry (in the sense of K). That is, $K(s_i(x),s_i(y))=K(x,y)$ for any $x,y\in V$.*

Exercise 3Bb. *Prove: If β is a root, then its associated co-root $\beta^\vee$ is $2\frac{\beta}{K(\beta,\beta)}$.*

Exercise 3Bc. *Show that the action of s_i^* on the basis of fundamental weights is*

$$s_i^*(\rho_j)=\begin{cases}\rho_j-\sum_{k=1}^n K(\alpha_k^\vee,\alpha_j)\rho_k & \text{if } i=j, \text{ or}\\ \rho_j & \text{if } i\neq j.\end{cases}$$

Exercise 3Bd. *Show that $t^*:V^*\to V^*$ is a reflection (in the sense that it fixes a hyperplane and has an eigenvector -1). Indeed, show that t^* fixes $\beta^\perp=\{x\in V^*:\langle x,\beta\rangle=0\}$.*

Exercises, in order of priority

There are more exercises than you can be expected to complete in a *half* day. Please work on them in the order listed. Exercises on the first line constitute a minimum goal. It would be profitable to work all of the exercises eventually.

3Ba, 3Bb,

3Bd, 3Bc.