Lecture 5: Exercises

Exercise 5a. *If s is initial or final in c, then $\omega_c(\beta, \beta') = \omega_{scs}(s\beta, s\beta')$ for all roots β and β'.*

Exercise 5b. *Let s be initial in c and let t be a reflection in W. Then $\omega_c(\alpha_s, \beta_t) \geq 0$, with equality only if s and t commute.*

Exercise 5c. *Let $J \subseteq S$ and let c' be the Coxeter element of W_J obtained by deleting all the letters in $S \setminus J$ from a reduced word for c. Let V_J be the subspace of V spanned by simple roots corresponding to elements of J. Then ω_c restricted to V_J is $\omega_{c'}$.*

Exercise 5d. *Let s be initial in c, let v be c-sortable and let $r \in S$. Show that*

1. *If $v \not\geq s$ then*
$$\mathrm{cl}_c^r(v) = \begin{cases} -\alpha_s & \textit{if } r = s, \textit{ or} \\ \mathrm{cl}_{sc}^r(v) & \textit{if } r \neq s \end{cases}$$
2. *If $v \geq s$ then $\mathrm{cl}_c^r(v) = \sigma_s(\mathrm{cl}_{scs}^r(sv))$.*

The sets $\mathrm{cl}_{sc}^r(v)$ and $cl_{scs}^r(sv)$ are defined by induction on the rank of W or on the length of v.

Exercises, in order of priority

Although the lecture series is now over, and it's hard to say when these exercises could be "due," I've still put them in order of priority for you. The first line still constitutes a minimum immediate goal. It would be profitable to work all of the exercises eventually.

5a, 5b, 5d,

5c.