8. Exercise List. Wednesday Week 2

Exercise 8.1. Show that the only positive steady state in a stoichiometric compatibility class of the network

$$2X_1 \xrightarrow{\kappa_1} X_2 \qquad 2X_2 \xrightarrow{\kappa_2} 4X_1 \quad X_1 + X_2 \xrightarrow{\kappa_3} 3X_1$$

from Exercise 7.2 is asymptotically stable.

Exercise 8.2. Consider the network from Exercise 4.3, which is not injective but monostationary:

$$E + S_1 \underset{\kappa_2}{\overset{\kappa_1}{\rightleftharpoons}} Y_1 \qquad E + S_2 \underset{\kappa_4}{\overset{\kappa_3}{\rightleftharpoons}} Y_2$$

$$S_2 + Y_1 \underset{\kappa_6}{\overset{\kappa_5}{\rightleftharpoons}} Y_3 \underset{\kappa_8}{\overset{\kappa_7}{\rightleftharpoons}} S_1 + Y_2 \qquad Y_3 \underset{\kappa_{10}}{\overset{\kappa_9}{\rightleftharpoons}} E + P.$$

Show that the only positive steady state in each stoichiometric compatibility class is asymptotically stable. To this end, show that the Hurwitz determinants derived from the characteristic polynomial and the parametrization in terms of λ and h are all positive.

Exercise 8.3. Consider the network following network

$$E + S_0 \xrightarrow{\kappa_1} Y_1 \xrightarrow{\kappa_2} E + S_1 \xrightarrow{\kappa_3} E + S_2$$

$$F + S_2 \xrightarrow{\kappa_4} Y_3 \xrightarrow{\kappa_5} F + S_0.$$

This network models the phosphorylation of substrate in two sites, such that phosphorylation occurs in a distributive and sequential way, but dephosphorylation occurs in a *processive* way, that is, one encounter with the enzyme leads to full dephosphorylation. The version given here is a simplification of the network studied in [Conradi, Mincheva, Shiu, "Emergence of oscillations in a mixed-mechanism phosphorylation system". Bulletin of Mathematical Biology, vol. 81, no. 6, 1829-1852 (2019).]

Show that the network admits a Hopf bifurcation for some choices of parameters.

Note that if you use Maple, it can well be the case that the Newton polytope of the relevant polynomial cannot be constructed. You can use SAGEMath, which computes it very fast, by first finding the list of exponents in any software you use.