

**INTRODUCTION TO DERIVED ALGEBRAIC GEOMETRY**  
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**MINICOURSE ON DG CATEGORIES**

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INTRODUCTION

There are many good texts on DG categories available. My goal is to give an accessible introduction from the point of view of an ‘end-user’: someone who wants to learn the framework of DG categories as fast as possible in order to apply it in their research. Generally, I look for balance between examples and abstract foundations: my natural inclination would be to prioritize examples and applications, but I do not want to ‘black-box’ the foundations. I do not aim for a very detailed expositions, or for maximal generality: if you are interested, such more systematic text would be easy to find. Instead, I want to present a (subjective) view on how the theory is put together.

As far as the prerequisites, I expect some familiarity with commutative algebra, algebraic geometry, and homological algebra. If you saw derived functors between derived categories of quasi-coherent sheaves, you should be ready. In fact, you probably do not even need this much, but some parts may then appear unmotivated.

TENTATIVE PLAN

- (1) DG categories and DG functors. Definitions and examples.
- (2) Derived category of modules. Semi-free resolutions.
- (3) Modules over a DG category.
- (4) The Yoneda embedding and (quasi)-functors.
- (5) ‘Large’ vs ‘small’ worlds: ind-completion and compact objects.
- (6) DG categories of quasicohherent sheaves. Thomason-Trobaugh Theorem.
- (7) Working with DG categories: quotients, duals.
- (8) Quasicohherent sheaves again. The Fourier-Mukai transforms.
- (9) What next? Tensor DG categories, categories over a scheme,...

REFERENCES

- [1] M. Bökstedt, A. Neeman. *Homotopy limits in triangulated categories*.
- [2] V. Drinfeld, *DG quotients of DG categories*.
- [3] B. Keller. *Deriving DG categories*.
- [4] N. Spaltenstein, *Resolutions of unbounded complexes*.
- [5] B. Toën. *Lectures on DG-categories*.