INTRODUCTION TO DERIVED ALGEBRAIC GEOMETRY
MSRI, SUMMER 2023
MINICOURSE ON DG CATEGORIES

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Introduction

There are many good texts on DG categories available. My goal is to give an accessible introduction from the point of view of an 'end-user': someone who wants to learn the framework of DG categories as fast as possible in order to apply it in their research. Generally, I look for balance between examples and abstract foundations: my natural inclination would be to prioritize examples and applications, but I do not want to 'black-box' the foundations. I do not aim for a very detailed expositions, or for maximal generality: if you are interested, such more systematic text would be easy to find. Instead, I want to present a (subjective) view on how the theory is put together.

As far as the prerequisites, I expect some familiarity with commutative algebra, algebraic geometry, and homological algebra. If you saw derived functors between derived categories of quasi-coherent sheaves, you should be ready. In fact, you probably do not even need this much, but some parts may then appear unmotivated.

Tentative plan

(1) DG categories and DG functors. Definitions and examples.
(2) Derived category of modules. Semi-free resolutions.
(3) Modules over a DG category.
(4) The Yoneda embedding and (quasi)-functors.
(5) Pre-triangulated and ind- completions.
(6) 'Large' vs 'small' worlds: compactly generated DG categories.
(7) DG categories of quasicoherent sheaves. Thomason-Trobaugh Theorem.
(8) Working with DG categories: quotients, duals.
(9) Quasicoherent sheaves again. The Fourier-Mukai transforms.

References

[1] M. Bökstedt, A. Neeman. *Homotopy limits in triangulated categories.*
[2] V. Drinfeld, *DG quotients of DG categories.*
[3] B. Keller. *Deriving DG categories.*
[4] N. Spaltenstein, *Resolutions of unbounded complexes.*
[5] B. Toën. *Lectures on DG-categories.*