DAG Exercises - 4 - Derived stacks

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IV.1. Prove that $\mathrm{C}_\bullet(\mathbb{CP}^\infty, \mathbf{Q})$ is equivalent to the free cdga over $\mathbf{Q}$ on a homologically degree 2 generator. Show that there is in fact only one cdga (up to equivalence) with this homology ring.

IV.2. Show that quasi-affine schemes are separated.

IV.3. Write down an example of a non-separated scheme.

IV.4. Say that a scheme is quasi-separated if the diagonal map $X \to X \times_{\mathbf{Z}} X$ is quasi-compact (the inverse images of affine opens are quasi-compact). Is your example from IV.3 quasi-separated? If it is, find a non-quasi-separated scheme.

IV.5. Make precise the notion of a quotient by an étale equivalence relation. Show that away from characteristic 2, the relation $x \sim -x$ generates an étale equivalence relation on $\mathbf{A}^1 \setminus \{0\}$. Show that the quotient is an algebraic space which is not a scheme.

References