Fall 2020

EMISSARY

Mathematical Sciences Research Institute

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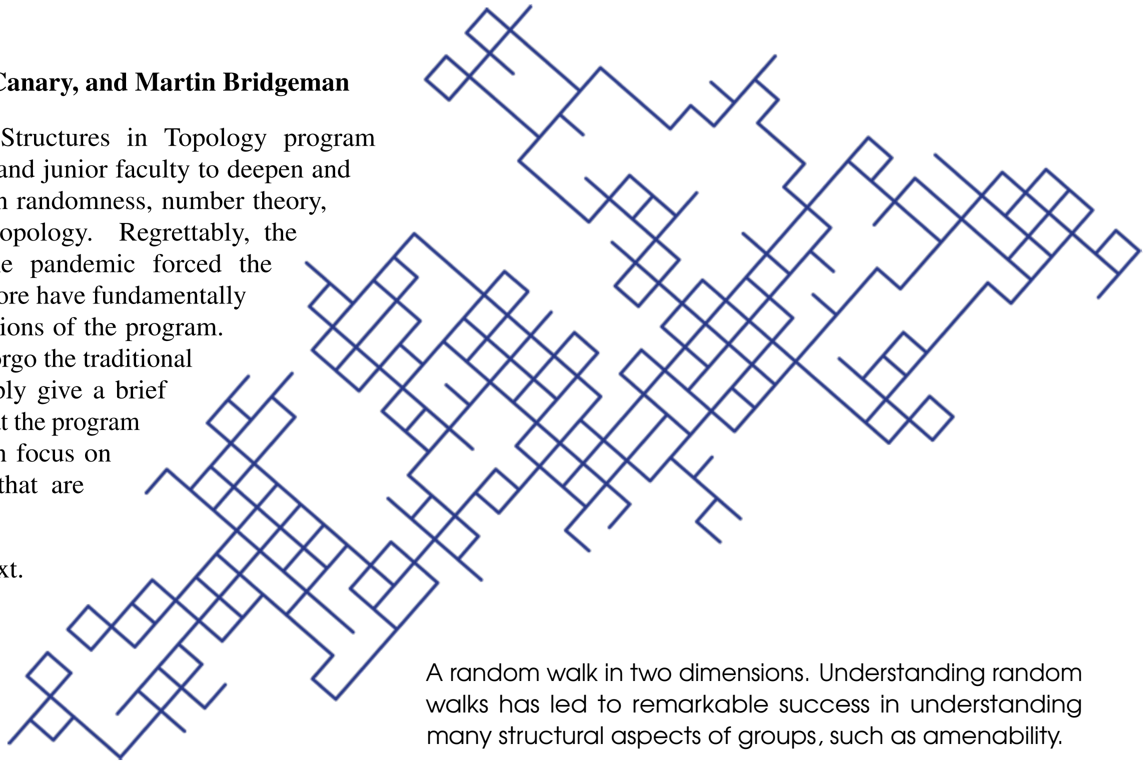
Random and Arithmetic Structures in Topology

Alan Reid, Richard Canary, and Martin Bridgeman

The Random and Arithmetic Structures in Topology program aimed to bring together experts and junior faculty to deepen and develop the connections between randomness, number theory, group actions, geometry, and topology. Regrettably, the precautions necessitated by the pandemic forced the program to be altered, and therefore have fundamentally changed the nature and expectations of the program. Given this, we have decided to forgo the traditional form of these articles, and simply give a brief overview of some of the topics that the program has aimed to highlight, and then focus on a description of the activities that are happening virtually at MSRI.

We begin with some context. Methods from ergodic theory have long been an important tool to study questions in number theory, geometry,

(continued on page 12)



A random walk in two dimensions. Understanding random walks has led to remarkable success in understanding many structural aspects of groups, such as amenability.

The Validated Numerical Computations of Mathematical Functions working group from ADJOINT 2020 (clockwise from top right): Bonita V. Saunders (lead), Ron Buckmire, Rachel Vincent-Farley, and Sean Brooks.

African Diaspora Joint Mathematics Workshop

Following Summer 2019's pilot program, MSRI hosted the African Diaspora Joint Mathematics Workshop (ADJOINT) this past summer. ADJOINT provides collaborative research opportunities to Black mathematicians and statisticians to work on exciting research projects in small groups of 4–5 participants and research leaders. The program was inspired in part by the success of MSRI's Summer Research in Mathematics program (SRM), first held in summer 2018.

As you can infer from the now-familiar Zoom-style group portrait (at left) of the Validated Numerical Computations of Mathematical Functions working group, ADJOINT 2020 took place virtually. Despite its not being on-site, MSRI was able to provide library, computing, and other resources to the participants to help support an atmosphere that proved to be infectious and conducive to collaborative research and learning. You can read a report on ADJOINT 2020 from another team, the Epidemiological Modeling working group, on page 4.

The View from MSRI

David Eisenbud, Director

Great Grant News

Last spring I wrote that we were in the final step of the years-long process of renewing the NSF grant that currently supplies about half of our overall funding, but two thirds of what we spend directly on core science. All went as planned, and we received the largest grant we have ever gotten: $25 million over five years. I've written about this process in every "View from MSRI" for a couple of years, and I'm heartily glad that it is over — and that it ended with success!

Adapting Programs

Needless to say, our two fall scientific programs are not what they would be if there were no COVID-19, and I was very concerned last spring that they might really collapse; I'm happy to report now that, despite the changes, both programs, Definability, Decidability, and Computability in Number Theory (DDC) and Random Arithmetic Structures in Topology (RAS), are bubbling, with many seminars each week. You can read a bit about them starting on the cover and on page 6.

Of course participation in our programs is much easier this year: lots of mathematicians who would not have been able to come physically to MSRI are taking part remotely. Some have told us that they much prefer remote workshops, too, though I think everyone who would have come for a longer stay would prefer to be here in person.

One of the options we offered the programs, as we became aware of the situation last spring, was to have an in-person, one-month "reunion" microprogram when that became possible. This semester's DDC program, and also the Higher Categories and Categorification program from last spring, will take advantage of this possibility. We will spend the money we've saved in per diems and travel assistance this year to bring people together for these events, now being planned.

Another initial worry we had was for the postdocs in our programs. Unlike the more senior members, who get only expenses, the postdocs generally depend on MSRI salaries to live. Fortunately, the NSF has allowed us great flexibility: We are able to pay them with NSF funds this year while they participate in their programs no matter where in the world they are — and they are indeed spread over many countries! Each of them has a senior mentor and they seem to be doing well. We are also able to provide small teaching buyouts for a few, more senior members so that they would have time to participate fully.

David Eisenbud

Autumn arrives at MSRI.

National Math Festival, in Your Own Home

The next National Math Festival, organized by MSRI in partnership with the Institute for Advanced Study and MoMath, the National Museum of Mathematics, was planned for April 2021; it was to bring many thousands of people together for two days at the Washington Convention Center. Meanwhile, the convention center was set up as a field hospital for COVID, and announced that until the virus was gone they would host only very small gatherings. As a result, the National Math Festival will be online, with events starting this December and climaxing in intensity next April. Take a look at the current schedule on pages 8–9. Now you can have a front-row seat no matter where you live!

Opening Our Online Door to You

Our web site is a major portal to MSRI — indeed, almost the only portal in this pandemic era! As of early October we are in the midst of a major refresh. From the early stages I've seen so far, I'm very optimistic that the new version will be far more consistent, easier to navigate, and attractive than the current one. I think that the rollout should occur sometime before the end of 2020. Stay tuned!

Contents

Focus on the Scientist: Martin Bridgeman

Martin Bridgeman is a Simons Professor in this semester's program on Random and Arithmetic Structures in Topology. His research interests are in hyperbolic geometry, Kleinian groups, and the geometry of representation varieties, and he has an outstanding track record of significant contributions to these areas. Most recently he has had a major impact on the rapidly developing field of higher Teichmüller theory. Martin is known for his energy and collaborative spirit, which is seen in joint work with 15 co-authors from across the globe.

Martin Bridgeman

Martin grew up in Dublin and attended Trinity College, Dublin, graduating in 1987. He received his Ph.D. in 1994 from Princeton University under the direction of Bill Thurston. He is currently a professor at Boston College, where he has been a faculty member since 1999. He is in demand both as a collaborator and speaker and has held many visiting positions at universities and institutes.

Whilst one cannot do justice to Martin's creativity and geometric intuition in this short space, we can record some highlights of his work: his celebrated proof of a uniform bound on the average bending of convex pleated planes in hyperbolic 3-space, and his discovery of a beautiful identity relating the orthospectrum of a hyperbolic surface with geodesic boundary with its area. Remarkably, applying this identity to specific surfaces he recovers classical number theoretic identities of Euler, Abel, Ramanujan, and Lewin.

He has had a longstanding, ongoing collaboration with Dick Canary, which most recently has led to a very fruitful collaboration on the thermodynamic formalism for character varieties (together with F. Labourie and A. Sambarino). Finally, in work with Brock and Bromberg, Martin has made major advances in the study of the renormalized volume of a hyperbolic 3-manifold.

Martin has been a visible presence in the hyperbolic geometry community for several decades. Perhaps his most important contribution has been (since 2006 up until recently) as the founder and principal organizer for the William Rowan Hamilton Geometry and Topology Workshop, an annual conference put on by the Hamilton Institute in Dublin (this conference series was sadly discontinued recently). It is fair to say that this conference simply would not have run and had the success it enjoyed, without Martin's tireless efforts.

Martin was also a driving force in the creation of a Ph.D. program at Boston College. This program is less than ten years old and is already producing outstanding students, with Martin being one of the leading mentors in the program, graduating two Ph.D. students from the inaugural class.

— Alan Reid, Richard Canary, Jeff Brock, and Francois Labourie

MSRI Director Search

Ed Baker and Hélène Barcelo

The summer of 2022 will mark the beginning of a new era in the history of MSRI. In July 2022, MSRI's longstanding leader David Eisenbud will retire from his position as Director, a position he has held twice — first from 1997–2007 and again from 2013 to the present. MSRI has thrived during David's tenure, enjoying tremendous success under his thoughtful, dedicated, and loving guidance.

MSRI faces an important challenge in finding a replacement for David. In recognition of the importance, MSRI's Board of Trustees decided to immediately begin the process of finding David's replacement in its last meeting in March 2020.

The first step in the process was to form a Director Search Committee to operate under the guidance of Board Chair Edward D. Baker and MSRI's Deputy Director Hélène Barcelo. This Committee was formed by the end of March and consists of the following members of the Board of Trustees: Edward D. Baker (Chair of the Committee), Ian Agol (University of California, Berkeley), Deborah Loewenberg Ball (University of Michigan), Hélène Barcelo (MSRI), Dan S. Freed (University of Texas, Austin), Maria M. Klawe (Harvey Mudd College), Mark Kleiman (Factorial Partners), Henry K. Schenck (Auburn University), Paul Seidel (Massachusetts Institute of Technology), Roger Strauch (The Roda Group), Terence Tao (University of California, Los Angeles), and Rodolfo H. Torres (University of California, Riverside).

The Director Search Committee has been hard at work since its formation, benefiting enormously from recommendations offered by MSRI's academic sponsors, and is pleased to have attracted an exceptional and diverse pool of candidates for the position. A first round of interviews took place in early October 2020, with a second round planned for early November 2020. The committee is hopeful that it will be in position to identify a final choice for MSRI's next director by the spring of 2021.

"We express immense gratitude to Professor Eisenbud for his long-term commitment to MSRI. His leadership has been instrumental to MSRI's extraordinary success, particularly during these unprecedentedly difficult times," commented Edward D. Baker. "The incoming director will have a unique opportunity to advance MSRI's multifaceted mission at a pivotal moment in its history. The Search Committee and I are pleased with the quality of the candidates that we have identified and are confident that our efforts will result in an outstanding successor to Professor Eisenbud."

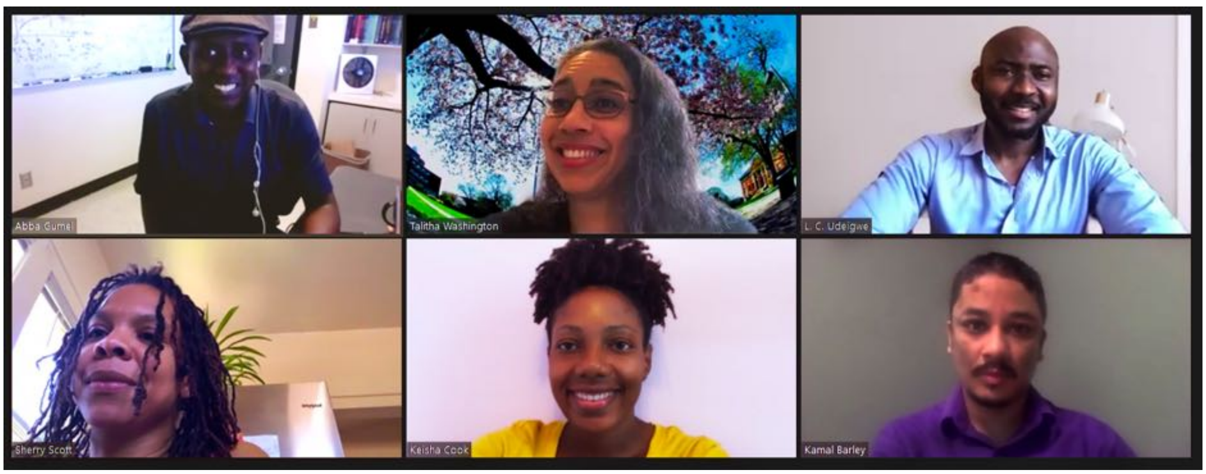
MSRI Creates Summer Research Workshop for American Mathematicians of African Diaspora

Abba Gumel

MSRI formed the African Diaspora Joint Mathematics Workshop (ADJOINT) to provide collaborative research opportunities to Black mathematicians and statisticians to work on exciting research projects in small groups of 4–5 participants and research leaders. The main aim of the workshop is to establish and promote research communities that will foster and strengthen research productivity and career development among their participants.

The workshop is designed to catalyze research collaborations, provide support for conferences to increase the visibility of the the researchers, and to develop a sense of community among the researchers who attend. ADJOINT will enhance the mathematical and statistical sciences and its community by positively affecting the research and careers of Black mathematicians, thereby increasing the impact of their work.

Participation in ADJOINT begins with a two-week (normally in-person) summer workshop at MSRI. Support continues throughout the academic year (and beyond) by providing the research groups with the resources and support they need to advance their projects after leaving MSRI.



The epidemiological modeling working group brainstorming on model formation. Clockwise from top left: Abba Gumel (lead), Talitha Washington, Lawrence Udeigwe, Kamal Barley, Keisha Cook, and Sherry Scott.

The 2020 ADJOINT program took place virtually (due to the novel Coronavirus pandemic) June 15–26, 2020. A total of five working groups, each consisting of established and up-and-coming talented junior faculty, were selected for ADJOINT 2020. Each group worked on exciting and challenging research topics and/or areas in mathematics and statistics.

The Epidemiological Modeling Working Group

One of the five working groups, consisting of Kamal Barley (Stony Brook University), Keisha Cook (Tulane University), Abba Gumel (Arizona State University, group leader), Sherry Scott (Milwaukee School of Engineering), Lawrence Udeigwe (Manhattan College), and Talitha Washington (Clark Atlanta University), chose to address the most important public health and socioeconomic challenge facing mankind over the last 102 years, namely, the problem of the spread of COVID-19, the disease caused by the novel SARS-CoV-2 virus. Our group members have diverse backgrounds, training, and expertise in the mathematical sciences, ranging from analysis, dynamical systems, stochastic processes, and data analysis to fluid dynamics and mathematical biology.

We started with a general discussion on what each group member expected to gain from ADJOINT 2020. The responses included trying to submit a paper to a reputable mathematical biology journal, exploring collaboration opportunities within our group and the rest of the 2020 ADJOINT cohort, learning more about mathematical epidemiology, learning about funding opportunities in mathematical biology, and learning more about how to write winning research proposals.

Preliminary Work

The group leader gave a series of lectures introducing the basic principles of mathematical biology. In particular, the lectures covered the history of mathematical modeling of infectious diseases (dating back to the pioneering works of Daniel Bernoulli, Sir Ronald Ross, Kermack and McKendrick, and others).

We started with discussion on the fundamental compartmental modeling framework of the Kermack–McKendrick formulation of SIR (susceptible-infected-recovered) epidemic models, including detailed discussion on the fundamental assumptions (such as homogeneous mixing, exponentially-distributed waiting times in epidemiological compartments, no vital/demographic processes, to account for the assumption that the time scale of the epidemic is less than the human demographic timescale, large population size, . . .). We discussed the derivation of basic qualitative features of the SIR model, including proving the non-negativity and boundedness of solutions, invariance of solutions, computation of reproduction numbers, conditions for the existence and asymptotic stability of solutions, and bifurcation analyses.

The introductory lectures also covered extended versions of the classical Kermack–McKendrick SIR model, such as models with an exposed class (SEIR), demographic dynamics, multiple disease stages, and vaccination (and the computation of vaccine-derived herd immunity threshold). Various techniques for computing the reproduction number of disease transmission models, notably the inspection method, the standard linearization method, and the next generation operator method, were discussed. We covered ways to design effective control strategies based on the expression of the reproduction number of the model. We studied key related statistical concepts, such as parameter estimation, data fitting, and uncertainty quantification/analysis.

We also studied some of the fundamental research papers in mathematical biology, including the 2000 SIAM Review by Herbert Hethcote (titled "Mathematics of Infectious Diseases"). Lastly, the group members read some of the recommended texts on dynamical systems to refresh their background on the theory and methodologies of nonlinear dynamical systems (notably, the books by Stephen Strogatz and Lawrence Perko).

Model Formulation

After acquiring the basic background knowledge, the group was then ready to start addressing questions pertaining to the design of our model. We started with the formulation of a basic model for the spread of COVID-19 in the United States. The model was designed by splitting the total population into mutually exclusive compartments based on disease status. Some of the notable features of the basic model included the incorporation of a compartment for asymptomatically-infectious and pre-symptomatically-infectious individuals, as well as allowing for the assessment of numerous non-pharmaceutical interventions, such as the use of face masks, testing and contact-tracing of confirmed cases, and self-isolation.

We carried out an extensive literature search to find data and parameter values needed to fit (and parameterize) the model. We also carried out a detailed qualitative analysis of the model, including the computation of its reproduction number (an epidemiological quantity, computed as the spectral radius of some next generation matrix, which governs the severity or decline/control of disease outbreaks) and proving the global asymptotic stability property of the associated continuum of disease-free equilibria, using Lyapunov function theory and LaSalle's invariance principle.

Our code for fitting the data did not initially work well, and we decided to explore that after the two-week period of the workshop. We presented our work to the entire ADJOINT 2020 cohort during the first and second weeks of the workshop. After revising our code and fully calibrating/parametrizing the model, we carried out extensive numerical simulations of the parametrized model to answer some of the specific questions we were interested in.

One of our main findings is that asymptomatic and pre-symptomatic infectious individuals are the main drivers of the pandemic in the U.S., and that targeting this cohort, via widespread random testing, is crucial to combatting the pandemic. We also obtained results on the effectiveness of community lockdown measures.

Continuing and Future Work

We identified some related sub-projects to explore in the future, including studying the impact of COVID-19 on healthcare workers, exploring the impact of a potential population-level impact of a COVID-19 vaccine in curtailing the pandemic, and estimating, using a stochastic version of the model, the impact of superspreading events on the burden and dynamics of COVID-19. We are continuing to meet periodically to work on the project, and we will be presenting our work at major conferences, such as the Joint Meetings of the AMS. We are currently finalizing a manuscript for submission for peer review, and we expect to write a few more in the future.

ADJOINT 2020 participants in the final-day group meeting.

MSRI Support

Finally, we really must thank the management, technical, and support staff of the MSRI for the excellent resources, support, and atmosphere they provided to us during the two-week virtual workshop. We had full access to the library and computing resources at the MSRI, and this, I must say, was invaluable.

The atmosphere they created for us was infectious and conducive for collaborative research and learning. The weekly presentations by the various ADJOINT working groups were also very beneficial to us. In particular, they allowed us to receive constructive feedback from members of the other groups on our own presentations. Further, the presentations from members of the other groups gave us the opportunity to learn about the important mathematical problems and challenges they were addressing (in areas or branches of mathematics different from ours) and explore other collaboration opportunities on projects we never knew we would be interested on, let alone actually working on.

Overall, the ADJOINT 2020 workshop was an immensely beneficial experience, allowing us to work on, and be exposed to, exciting mathematical problems within an excellent, supportive research environment and atmosphere.

Hilbert's Tenth Problem

Valentina Harizanov and Alexandra Shlapentokh

Hilbert's Mathematical Problems

In 1900, David Hilbert presented a list of twenty-three problems to mathematicians gathered in Paris for the Second International Congress. Many problems on that list would greatly influence the development of mathematics in the 20th century, and many still exert a powerful pull on modern mathematics.

The tenth problem on the list was to devise a process that determines whether an arbitrary polynomial equation in several variables and integer coefficients had integer solutions. A Diophantine equation, named after Diophantus of Alexandria, is an equation of the form $q(X_1,\dots,X_n)=0$ where q is a polynomial in the variables $X_1,\dots,X_n$ with integer coefficients. In modern interpretation of Hilbert's question, he asked for an algorithm or a computer program that takes coefficients of the polynomial q as its input, and outputs a "yes" or "no" answer to the question whether q has roots in $\mathbb{Z}$. This question became known as Hilbert's tenth problem.

(Hilbert, most certainly, did not ask for a computer program since the first computers would appear more than forty years later; he also did not ask for a formal algorithm, since a rigorous mathematical theory of algorithms — computability theory — would not be developed until more than thirty years later.)

The answer to Hilbert's question had to wait almost seventy years. In 1970, Yuri Matiyasevich completed the proof started by Martin Davis, Hilary Putnam, and Julia Robinson by showing that an algorithm such as the one that Hilbert sought did not exist. To answer Hilbert's question, the four coauthors proved a much stronger result that connected number theory with computability theory.

"10. Determination of the Solvability of a Diophantine Equation.
Given a Diophantine equation with any number of unknown quantities and with rational integral numerical coefficients: *To devise a process according to which it can be determined by a finite number of operations whether the equation is solvable in rational integers.*"

David Hilbert, pictured next to the tenth problem from his presentation to the Second International Congress in Paris, as reported in translation in Bull. Amer. Math. Soc. **8** (1902), 437–479.

Computable and Computably Enumerable Sets

In the 1930s, Church, Gödel, Kleene, Post, Turing, and others developed computability theory as a precise mathematical theory of algorithms. One of the first results of computability theory was Turing's theorem that not all problems can be solved algorithmically; those that can be are called decidable or computable. Some undecidable problems can be described by computably enumerable sets. A nonempty set is *computably enumerable* if there is an algorithm that generates it by enumerating (listing) its elements. A set is *computable* or *decidable* if both the set and its complement are computably enumerable. From two algorithmic enumerations, for the set and its complement, we can devise a decision procedure for recognizing the elements of the set and, equivalently, the elements of its complement.

We can simultaneously effectively enumerate all computably enumerable sets. Namely, let $P_0,P_1,\dots,P_e,\dots$ be an algorithmic enumeration of all Turing machine programs, given by listing them systematically. On an input x, a Turing machine may halt and output the value of the partial function it computes or may compute forever. It can be shown that computably enumerable sets are exactly the domains of these partial functions. Hence we have an algorithmic enumeration of all computably enumerable sets as domains of partial computable functions: $W_0,W_1,\dots,W_e,\dots$

There are many sets that are computably enumerable but not computable. Turing's (diagonal) halting set is one of the first and most important such sets. The *halting set* H is defined to be the set of all inputs e on which the Turing machine program P_e halts. That is, $H=\{e : e\in W_e\}$. The set H is computably enumerable because it is enumerated by a procedure that simultaneously runs P_0 on 0, P_1 on 1, … , P_k on k, for bigger and bigger k's and more and more computational steps, and checks which computations halt. The complement of the halting set, $\overline{H}$, is not computably enumerable since otherwise, $\overline{H}=W_i$ for some i. If that were the case, then

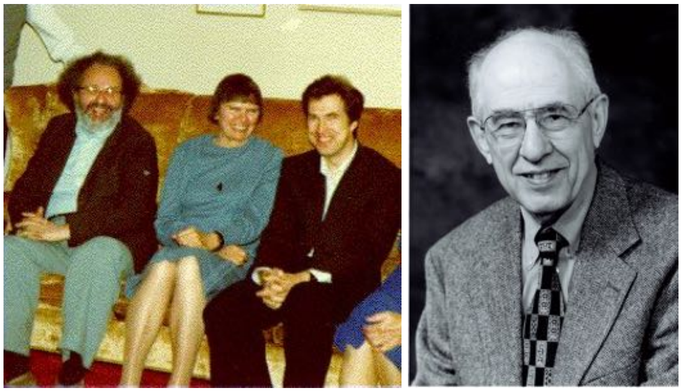
$$i\in\overline{H}\Leftrightarrow i\in W_i\Leftrightarrow i\in H,$$

which is a contradiction.

Diophantine Sets and Solution to Hilbert's Tenth Problem

One of the main notions related to Hilbert's tenth problem is that of a Diophantine set. We call a subset $A\subseteq\mathbb{Z}$ *Diophantine* if there exists a polynomial $p(T,X_1,\dots,X_r)\in\mathbb{Z}[T,X_1,\dots,X_r]$ such that for all $t\in\mathbb{Z}$ there exist $x_1,\dots,x_r\in\mathbb{Z}$ with $p(t,x_1,\dots,x_r)=0$ if and only if $t\in A$. We can think of A as the set of all parameter values for which the corresponding Diophantine equation has a solution. The polynomial $p(T,X_1,\dots,X_r)$ is called a Diophantine definition of A. It is not hard to see that every Diophantine set is computably enumerable. We can produce an effective listing of all $(r+1)$-tuples of integers, evaluate the polynomial at each $(r+1)$-tuple, and every time the polynomial is equal to 0, output the value t.

In order to solve Hilbert's problem, Davis, Putnam, Robinson, and Matiyasevich established that every computably enumerable set of integers is Diophantine. Since not all computably enumerable sets are computable, it follows that there exist Diophantine sets that are not computable. The last statement immediately implies a negative answer to Hilbert's question. Indeed, let $U\subset\mathbb{Z}$ be an undecidable Diophantine set, and let $f(T,Y_1,\dots,Y_m)$ be its Diophantine definition. If we had an algorithm to decide when polynomial equations have solutions in $\mathbb{Z}$, thinking of the value of T as a coefficient of the polynomial equation $f(T, Y_1, \ldots, Y_m) = 0$, we would have an algorithm to determine for which integer values t of T there exist $y_1, \ldots, y_m \in \mathbb{Z}$ such that the polynomial equation is satisfied. Thus, we would have an algorithm to decide membership in U, leading to a contradiction.



From left to right: Martin Davis, Julia Robinson, Yuri Matiyasevich, and Hilary Putnam, who answered Hilbert's tenth problem in a result that connected number theory with computability theory.

Another Major Question

A question that arises immediately from the answer to Hilbert's problem is whether it is also true that there is no algorithm to determine when a polynomial equation with integer coefficients has solutions in $\mathbb{Q}$. It is not hard to see that if there were an algorithm for solutions in $\mathbb{Z}$, there would be an algorithm for solutions in $\mathbb{Q}$. Indeed, let $g(X_1, \ldots, X_r) \in \mathbb{Z}[X_1, \ldots, X_r]$ and consider looking for solutions in $\mathbb{Q}$. We can rewrite all the variables ranging over $\mathbb{Q}$ as quotients of variables ranging over $\mathbb{Z}$ with a requirement that a variable in the denominator is not equal to 0. (We can use a common denominator to cut down on the number of variables.)

So, in the first approximation, we have the following equation and condition: $g(\frac{Y_1}{Z}, \ldots, \frac{Y_m}{Z}) = 0 \wedge Z \neq 0$. Let $d = \deg g$ and multiply the first equation by Z^d to eliminate the denominators. Let

$$h(Y_1, \ldots, Y_m, Z) = Z^d g(\frac{Y_1}{Z}, \ldots, \frac{Y_m}{Z}) = 0.$$

We now have to address the issue of re-writing $Z \neq 0$ as a polynomial equation and replacing the resulting system of two equations by one. First, we construct a Diophantine definition for the set of non-zero integers. Consider the following equation where all variables range over $\mathbb{Z}$:

$$(2x - 1)(3y - 1) = uw.$$

We claim that this equation has solutions in $\mathbb{Z}$ if and only if $u \neq 0$. First, suppose $u = 0$. Then either $2x - 1 = 0$ or $3y - 1 = 0$. Neither case can occur, if both x and y are integers. Suppose now that $u \neq 0$. Then we can factor $u = u_1 u_2$ so that $(2, u_1) = 1$ and $(3, u_2) = 1$. Next, we solve a congruence $2x \equiv 1 \bmod u_1$. Since $(2, u_1) = 1$, we have that 2 is a unit in the ring $\mathbb{Z}/u_1$ and, therefore, the congruence has a solution x in $\mathbb{Z}$. Thus, there exists $w_1 \in \mathbb{Z}$ such that $2x - 1 = w_1 u_1$. Similarly, since $(3, u_2) = 1$, the congruence $3y \equiv 1 \bmod u_2$ has a solution in $\mathbb{Z}$, and there exists $w_2 \in \mathbb{Z}$ such that $3y - 1 = u_2 w_2$. Putting this together, we have $(2x - 1)(3y - 1) = wu$ where $w = w_1 w_2$. Thus, we can replace the "non-equality" $Z \neq 0$ by the equation $(2X - 1)(3Y - 1) = ZW$.

Now, there is an easy way to combine equations when we are looking for solutions in $\mathbb{Q}$ (or any real field). Given a system of two polynomial equations, $g(\bar{X}) = 0$ and $h(\bar{X}) = 0$, we can replace them by a single polynomial equation: $(g(\bar{X}))^2 + (h(\bar{X}))^2 = 0$. Thus, we have succeeded in replacing a polynomial equation with variables ranging over $\mathbb{Q}$ by an equivalent equation with variables ranging over $\mathbb{Z}$. By "equivalent" we mean that the first equation has solutions in $\mathbb{Q}$ if and only if the second equation has solutions in $\mathbb{Z}$. Hence, if Hilbert's problem over $\mathbb{Q}$ were undecidable, it would be undecidable over $\mathbb{Z}$.

Thus, what can we say regarding Hilbert's question applied to $\mathbb{Q}$? Unfortunately, we do not know much about this problem since it turned out to be quite intractable. At the same time, as often happens with deep problems in mathematics, attempts to approach the problem from different perspectives produced many interesting results and developments. These include Barry Mazur's conjectures on topology of rational points, results concerning big rings contained in $\mathbb{Q}$ (among them a theorem of Bjorn Poonen showing that Hilbert's question has no answer over many big rings), a theorem of Jochen Koenigsmann about constructing a purely universal definition of $\mathbb{Z}$ over $\mathbb{Q}$, and many others.

Forthcoming Workshops

Jan 20–21, 2021: *Connections Workshop: Mathematical Problems in Fluid Dynamics*

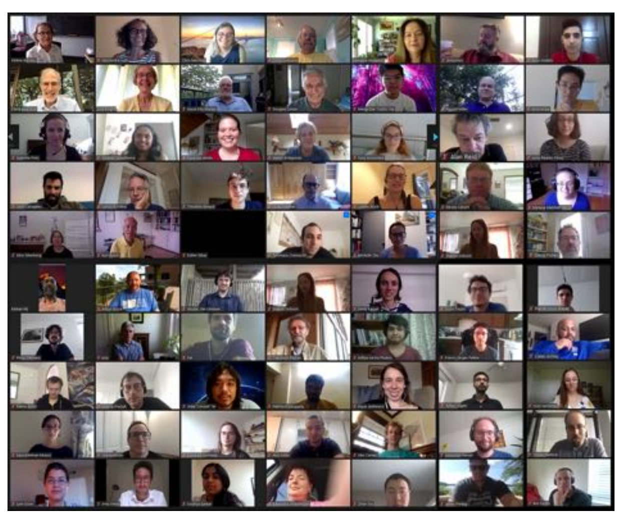
Jan 25–29, 2021: *Introductory Workshop: Mathematical Problems in Fluid Dynamics*

Apr 12–23, 2021: *Recent Developments in Fluid Dynamics*

Apr 28–30, 2021: *CIME 2021 Initiating, Sustaining, and Researching Mathematics Department Transformation of Introductory Courses for STEM Majors*

May 3–7, 2021: *Hot Topics: Topological Insights in Neuroscience*

For more information about any of these workshops, as well as a full list of all upcoming workshops and programs, please see msri.org/workshops. Pending ongoing COVID-19 disruptions, some workshops may be held online.



The Fall 2020 member welcome event.

Joyful Math Online: Join Us for NMF 2020–21

All events will be online this year, so enjoy a front-row seat no matter where you are!

When the world began changing, we changed the National Math Festival. We invite you to share the new plans below with friends and family, and join us online for events spanning from December 2020 through April 2021.

As ever, the NMF is free and open to the public, with activities designed for all ages, from very young children through lifelong learners. We invite you to celebrate the fun, beauty, and power of math through games, puzzles, math, magic, uplifting and intriguing talks, beautiful and engaging films and film panels, kids' book readings, live performances, and much more.

As much as possible, we are making this year's festival as interactive as ever. Like many in the math world, we are reinventing how to be together in community during this time — how to learn together, how to have fun together, how to meet new friends and share common interests. We are grateful to our partners in the math world who are showing us their strategies for being together online; we will keep learning from each other. Times are hard right now and play is a respite. Come, relax, join us. Laugh together in some seriously joyful math play.

We are incredibly honored to be led by the community of math organizations and outstanding individuals who will shape our program. Thank you for all you are doing, alone and together, to bring math to more families during the pandemic.

Three Ways to Get Involved

1. Join the NMF Facebook group, Math Moms and Math Dads: facebook.com/groups/mathmomsandmathdads.

2. Invite the kids in your life to send questions to James Tanton for the new NMF Weekly Puzzle Newsletter: globalmathproject.org/nmf-weekly/.

3. Attend one (or more!) of the 2020–21 Festival events. And take advantage of the opportunity to join the NMF Live Online programming on April 16–18 — all events are free!

The BARKIN/SELISSEN PROJECT performs "Dance of the Diagram" at the 2019 festival.

Students make tesselations with magnetic tiles, courtesy of the National Museum of Mathematics (MoMath).

December 2020

Join us by the fire (so to speak) for some cozy Mathical Book Prize live author readings. We are partnering with the National Council of Teachers of English and the National Council of Teachers of Mathematics to ship Mathical Books ahead of time to kids in Title I schools so they are ready to meet these authors!

Tweens and Teens Take on Mathical
Wednesday, Dec 9, 2020 | 1:00–2:30 pm ET

Solving for M by Jennifer Swender
DK Life Stories: Katherine Johnson by Ebony Joy Wilkins
Slay by Brittney Morris

Little Ones Rule with Mathical
Thursday, December 10, 2020 | 1:00–2:00 pm ET

Pigeon Math by Asia Citro
Cao Chong Weighs an Elephant by Songju Ma Daemicke
Nothing Stopped Sophie by Cheryl Bardoe

January 2021

Join us in January for math, music, and theater as we celebrate math and social justice through hip hop culture, the encouraging rhythms of rap, and the personality, persistence, and overcoming of obstacles of women in mathematics across the years.

Math Rap (Double Feature)
Tuesday, January 12, 2021 | 1:00–2:00 pm ET

Math Concert with the Music Notes
Hip Hop Math with Professor Lyrical

"Witches of Agnesi" Play Performance
Wednesday, January 27, 2021 | 1:00–2:30 pm ET

Live Streaming and *Panel Discussion*:
Susan Gerofsky, University of British Columbia
Moira Chas, Stony Brook University
Nancy Scherich, University of Toronto
Shelly Jones, Central Connecticut State University

February 2021

Come listen! . . . as we hear the 2021 Mathical Book Prize winning authors read from their lively new works of kids' literature.

Come join the Young People's Project for a game show in which students judge mathematicians on their charm and clarity explaining their favorite big ideas in mathematics.

Mathical Book Prize 2021 Winners Announcement
Tuesday, February 9, 2021 | 1:00–2:00 pm ET

Prize-winning authors will read from their lively new works of kids' literature.

What's the Big Idea: Game Show Featuring Students and Mathematicians
Tuesday, February 16, 2021 | 1:00–2:30 pm ET

Hosted by Cliff Freeman and Scarlitte Alameda, The Young People's Project

March 2021

It's almost time for NMF Live Online, coming up in a few short weeks. But first . . . two derring-do math performances! Join Rhys Thomas for juggling as you've never seen it before; and Brady Haran of Numberphile for whatever he thinks up next.

Believe-It-Or-Not, It's Math ($\times 2$)
Wednesday, March 17, 2021 | 1:00–2:30 pm ET

Rhys Thomas, Science Circus
Brady Haran, Numberphile YouTube Channel

April 2021

NMF Live Online!

Friday, April 16–Sunday, April 18, 2021

Join us for the 2021 National Math Festival, with hands-on, interactive activities for youth of all ages, plenty of booths to stroll around and pick up resources for math play at home or in the classroom, and engaging films and lively math talks to round out the program. Enjoy!

Engaging Math Talks

The End of Space and Time: The Mathematics of Black Holes and the Big Bang, Robbert Dijkgraaf

Changing the "Face" of Mathematics, Erica Graham, Raegan Higgins, Candice Price, Shelby Wilson

Math Is Play!, Emille Davie Lawrence

Numbers through Pictures: A Taste of the Geometry of Numbers, Jesús De Loera

Infinite Powers: The Story of Calculus, Steven Strogatz

Math and the Movies, Joseph Teran

Alfred P. Sloan Foundation Film Screenings / Panels

The Bit Player / Andrea Goldsmith, Mark Levinson, Ronald Rivest, David Tse

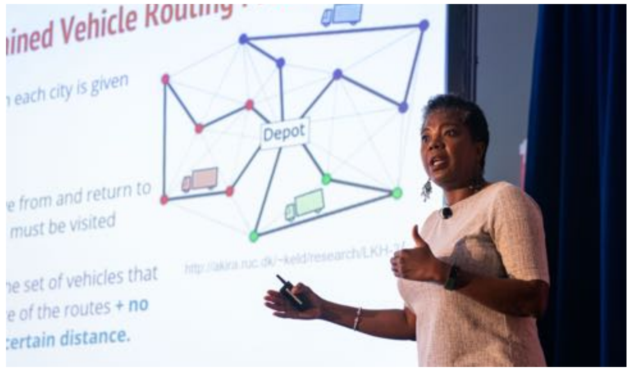
Hidden Figures / Erica Graham, Raegan Higgins, Candice Price, Shelby Wilson

The Man Who Knew Infinity / Kathrin Bringmann, David Eisenbud, Ken Ono

NOVA — Math film TBD

NSF "We Are Mathematics" Video Contest Winners

Secrets of the Surface / Hélène Barcelo, Roya Beheshti Zavareh, Marissa Loving, Marie-Francoise Roy



Suzanne Weekes speaks on computational modeling at the 2019 NMF.

NMF Live Online

Join some of the world's savviest math organizations for a tour-de-force of online fun! Meet math superstars, find your own math star to steer by, and just plain relax and enjoy.

Come prepared to learn, engage, laugh, grow, play, and be part of an exciting math community for kids and adults of all ages. Hands-on, interactive activities are the name of the game. You can also expect puzzles, demos, super-short talks, mathletic events, art-making, and more.

Parents and teachers, pick up resources for math play at home and in the classroom. Kids and grown-ups, roll up your sleeves and dive into easy-access, creative, open math activities for all.

Major Activity Presenters: National Museum of Mathematics (MoMath) • Julia Robinson Mathematics Festival • The Young People's Project • Natural Math and Math-On-A-Stick

. . . and More: Association for Women in Mathematics • Development and Research in Early Math Education (DREME) Network • FUNDAPROMAT • National Association of Mathematicians and the Benjamin Banneker Association • National Council of Teachers of Mathematics • Mathematical Association of America • MSRI and National Science Foundation • NOVA / NOVA Education • National Girls Collaborative Project • ThinkFun • WGBH

The 2021 National Math Festival is organized by the Mathematical Sciences Research Institute (MSRI) in cooperation with the Institute for Advanced Study (IAS) and the National Museum of Mathematics (MoMath). The festival is generously supported by the Simons Foundation, the Alfred P. Sloan Foundation, the National Science Foundation, Schmidt Futures, the Kavli Foundation, and the American Mathematical Society (AMS).

Named Postdocs in RAS — Fall 2020

Strauch

Martin Bobb is the Strauch Postdoctoral Fellow in the Random and Arithmetic Structures in Topology program. He completed his Ph.D. at the University of Texas in 2020, supervised by Jeff Danciger, and is now an NSF Postdoctoral Researcher at the University of Michigan. He works primarily with convex real projective manifolds, a type of geometric structure which generalizes real hyperbolic geometry. His research mainly focuses on teasing out strong geometric and topological information from these manifolds. A deeper understanding of these geometric structures informs their study from the perspective of representation theory and dynamics. *The Strauch fellowship is funded by a generous annual gift from Roger Strauch, Chairman of The Roda Group. He is a member of the Engineering Dean's College Advisory Boards of UC Berkeley and Cornell University, and is also currently the chair of MSRI's Board of Trustees.*

Viterbi

Tommaso Cremaschi is this semester's Viterbi fellow in the Random and Arithmetic Structures in Topology program. He grew up in Rome, and after participating in the Part III program at the University of Cambridge, he obtained his Ph.D. at Boston College in 2019, under the supervision of Ian Biringer and Martin Bridgeman. In his thesis he studied 3-dimensional manifolds of infinite type, that is, 3-manifolds with non-finitely generated fundamental group. For a large class of these manifolds, he proved a hyperbolisation result, akin to Thurston's result, answering a question by Agol. He is very grateful for the opportunity to stay at MSRI, where he hopes to expand his research area, start new collaborations and complete some current projects. *The Viterbi postdoctoral fellowship is funded by a generous endowment from Dr. Andrew Viterbi, well known as the co-inventor of Code Division Multiple Access based digital cellular technology and the Viterbi decoding algorithm, used in many digital communication systems.*

McDuff

Nicholas Miller is the McDuff Postdoctoral Fellow in this semester's Random and Arithmetic Structures in Topology program. He grew up in San Diego, where he attended UC San Diego for his undergraduate education. Nicholas received his Ph.D. from Purdue University in 2017 under the supervision of David Ben McReynolds. He then spent two years as a postdoc at Indiana University and is currently a Morrey Visiting Assistant Professor at UC Berkeley. Nicholas is broadly interested in hyperbolic geometry, with an emphasis on arithmetic manifolds. Much of his work has focused on understanding the ramifications that arithmeticity (or lack thereof) has on the geometry of finite volume hyperbolic manifolds. *The McDuff fellowship was established by an anonymous donor in honor of Dusa McDuff. She is an internationally renowned mathematician, a member of the National Academy of Sciences, and a recipient of the AMS Leroy P. Steele Prize (2017). She is also currently a trustee of MSRI.*

Gamelin

Soumya Sankar is the Gamelin postdoctoral fellow in the program on Random and Arithmetic Structures in Topology. She completed her Ph.D. at the University of Wisconsin-Madison in 2020 under the supervision of Jordan Ellenberg. An arithmetic geometer by training, she is interested in statistical questions about invariants that arise in arithmetic and algebraic geometry. She has worked on counting rational points on various moduli spaces and has studied statistical properties of curves and abelian varieties in positive characteristic. Recently, she has been broadening her horizons by working on more algebro-geometric questions as well as on questions in the intersection of number theory and topology. *The Gamelin postdoctoral fellowship was created in 2014 by Dr. Ted Gamelin, Emeritus Professor of the UCLA Department of Mathematics. The Gamelin fellowship emphasizes the important role that research mathematicians play in the discourse of K-12 education.*

Huneke

Yvon Verberne is the Huneke Postdoctoral Fellow for this semester's Random and Arithmetic Structures in Topology program. Yvon received her Ph.D. from the University of Toronto in June of 2020 under the supervision of Kasra Rafi. In January of 2021, she will begin a position at the Georgia Institute of Technology as an NSERC Postdoctoral Scholar under the mentorship of Dan Margalit. Yvon's principal research interests lie in the intersection of low-dimensional topology and geometric group theory, where she mainly focuses on mapping class groups of surfaces. *The Huneke postdoctoral fellowship is funded by a generous endowment from Professor Craig Huneke, who is internationally recognized for his work in commutative algebra and algebraic geometry.*

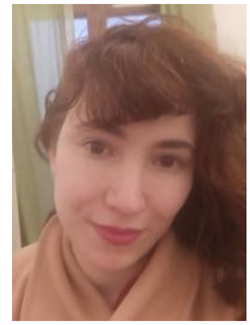
Named Postdocs in DDC — Fall 2020

Viterbi

Philip Dittmann is this semester's Viterbi Postdoctoral Fellow and a member of the Decidability, Definability, and Computability in Number Theory program. Philip was an undergraduate student at TU Darmstadt and a master's student at the University of Cambridge before completing his DPhil studies at the University of Oxford under the supervision of Jochen Koenigsmann in 2018. Since then he has been a postdoctoral researcher, first at KU Leuven, and most recently at TU Dresden, which is also from where he is participating virtually in this semester's activities. Philip's research interests are in the algebra and model theory of fields. Specifically, he is currently working on existential and first-order definability questions in finitely generated, global, and more general fields. *The Viterbi postdoctoral fellowship is funded by a generous endowment from Dr. Andrew Viterbi, well known as the co-inventor of Code Division Multiple Access based digital cellular technology and the Viterbi decoding algorithm, used in many digital communication systems.*

Vincent Della Pietra

Esther Elbaz Saban is this semester's Vincent Della Pietra Postdoctoral Fellow in the Decidability, Definability, and Computability in Number Theory program. At the end of her schooling at the École Normale Supérieure (Paris), she obtained her Master 2 degree under the direction of Zoé Chatzidakis. She then pursued a Ph.D. with Françoise Delon at the Paris Diderot University, during which she focused on the notion of Grothendieck rings in model theory. She developed flexible ideas to construct a structure whose Grothendieck ring is isomorphic to a prechosen ring. During her postdoc at the Ben Gurion University of the Negev, advised by Moshe Kamenski and Assaf Hasson, she used these ideas to construct a structure whose Grothendieck ring is isomorphic to a given quotient of $\mathbb{Z}[X]$. *The Vincent Della Pietra fellowship was established in 2017 by the Della Pietra Foundation. Vincent received his Ph.D. in mathematical physics from Harvard University. He is a partner at Renaissance Technologies, co-founder of the Della Pietra Lecture Series at Stony Brook University, and a board member of PIVOT.*



Call for Proposals

All proposals can be submitted to the Director or Deputy Director or any member of the Scientific Advisory Committee with a copy to proposals@msri.org. For detailed information, please see the website msri.org/proposals.

Thematic Programs

The Scientific Advisory Committee (SAC) of the Institute meets in January, May, and November each year to consider letters of intent, pre-proposals, and proposals for programs. The deadlines to submit proposals of any kind for review by the SAC are **March 1**, **October 1**, and **December 1**. Successful proposals are usually developed from the pre-proposal in a collaborative process between the proposers, the Directorate, and the SAC, and may be considered at more than one meeting of the SAC before selection. For complete details, see tinyurl.com/msri-progprop.

Hot Topics Workshops

Each year MSRI runs a week-long workshop on some area of intense mathematical activity chosen the previous fall. Proposals should be received by **March 1**, **October 1**, and **December 1** for review at the upcoming SAC meeting. See tinyurl.com/msri-htw.

Summer Graduate Schools

Every summer MSRI organizes several two-week long summer graduate workshops, most of which are held at MSRI. Proposals must be submitted by **March 1**, **October 1**, and **December 1** for review at the upcoming SAC meeting. See tinyurl.com/msri-sgs.

Call for Membership

MSRI invites membership applications for the 2021–22 academic year in these positions:

Research Members	by December 1, 2020
Postdoctoral Fellows	by December 1, 2020

In the academic year 2021–22, the research programs are:

Universality and Integrability in Random Matrix Theory and Interacting Particle Systems
Aug 16 – Dec 17, 2021
Organized by Ivan Corwin, Percy Deift, Ioana Dumitriu, Alice Guionnet, Alexander Its, Herbert Spohn, Horng-Tzer Yau

The Analysis and Geometry of Random Spaces
Jan 18 – May 27, 2022
Organized by Mario Bonk, Joan Lind, Steffen Rohde, Eero Saksman, Fredrik Viklund, Jang-Mei Wu

Complex Dynamics: From Special Families to Natural Generalizations in One and Several Variables
Jan 18 – May 27, 2022
Organized by Sarah Koch, Jasmin Raissy, Dierk Schleicher, Mitsuhiro Shishikura, Dylan Thurston

MSRI uses **MathJobs** to process applications for its positions. Interested candidates must apply online at mathjobs.org. For more information about any of the programs, please see msri.org/programs.

Random and Arithmetic Structures in Topology

(continued from page 1)

topology, and other fields. Recently, new perspectives opened with the study of randomness in various settings that we describe below.

Settings for Randomness

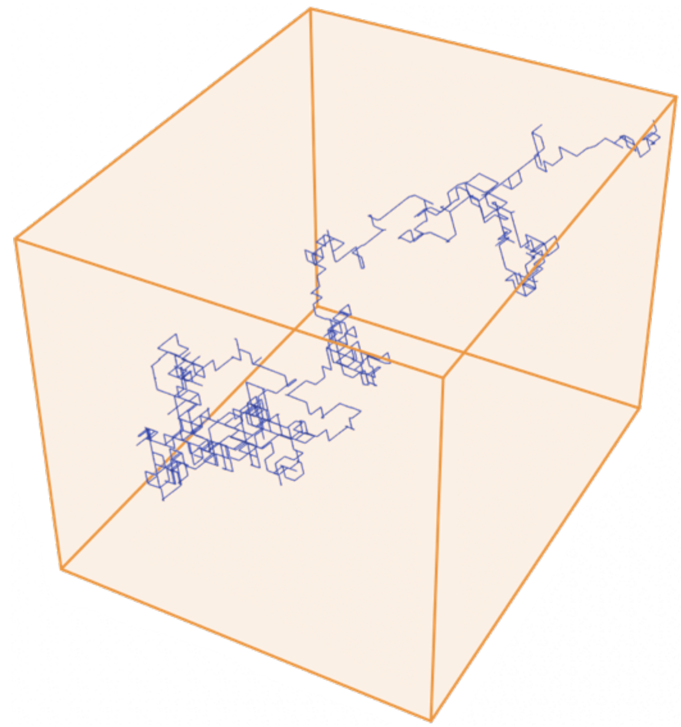
Measure theoretic invariants. The notion of randomness in group theory can be seen vividly in the powerful notion of invariant random subgroups (IRS): these are conjugacy-invariant probability measures on the space of subgroups in a given group G. Amongst other things, this provides a framework to study invariants of locally symmetric manifolds of non-compact type. Indeed, IRS can be regarded as generalizations, of both normal subgroups and lattices, which makes this notion particularly rich.

Closely related to IRS is the so-called Benjamini–Schramm space, which is defined to be the space of all Borel probability measures on the space of isometry classes of pointed proper metric spaces equipped with the Gromov–Hausdorff topology. The recently emerged interest in the investigation of this space in the form of Benjamini–Schramm convergent sequences of manifolds and the behavior of their geometric and topological invariants gives a strong connection to (metric) geometry.

Measure theory arises more directly in the concept of measure equivalence relation and cost. Entropy for group actions provides a numerical invariant, which in turn sometimes has an interpretation in terms of topological invariants constructed from the group von Neumann algebra over the group ring. The more recent concept of sofic entropy can be studied for a large class of groups — the so-called sofic groups — which comprises all residually finite and all amenable groups.

Random walks on groups. Informally, a random walk on a group describes a path that consists of a succession of random group elements. (See the figures on the cover and at right.) Understanding these random walks has had remarkable success in understanding many structural aspects of a group, for example, amenability (to name but one).

Arithmetic manifolds. Arithmetic groups and manifolds have long been studied because their investigation combines tools from number theory, geometry, and topology. In high rank these capture all lattices, but in the case of hyperbolic manifolds, examples of non-arithmetic manifolds are easy to construct; in particular, in dimension three they are abundant. In this setting, the solution to the virtual fibering conjecture yields a classification of closed hyperbolic 3-manifolds, at least up to passing to finite covers, and so we have a good understanding of such manifolds. On the other hand, in the study of random hyperbolic 3-manifolds, it has been increasingly important to analyze the expected geometric and topological properties and uncover the asymptotic behavior of invariants in towers of covers. This approach aims at understanding the large-scale geometric and topological invariants of such manifolds and the relation to arithmetic properties using tools from random walks on groups,



A random walk in three dimensions. Informally, a random walk on a group describes a path that consists of a succession of random group elements; this concept can contribute to the understanding of the structural elements of a group.

group sieving, geometric representation theory, L^2-invariants, homology with twisted coefficients, and global analysis.

Anosov representations. Anosov representations were developed by Labourie in his investigation of Hitchin representations, and they are now regarded as a natural higher rank analogue of convex cocompact representations into rank one Lie groups. Another such analogue, which is closely related when the group is hyperbolic, are the discrete subgroups of $\mathrm{PSL}(d,\mathbb{R})$ which act convex cocompactly on properly convex domains in projective spaces. Tools from the thermodynamic formalism have been profitably used to study the dynamics of Anosov representations and to analyze deformation spaces of Anosov representations. In the study of Anosov representations, one experiences a beautiful interplay between ideas from geometry, topology, Lie theory, and dynamics.

The Virtual Reality

We now describe how we attempted to adapt to these new circumstances. There are currently six members of the program located in Berkeley: Research Professors Martin Bridgeman and Richard Canary, Research Members Michelle Chu and Kasia Jankiewicz, and Postdoctoral Members Nicholas Miller and Soumya Sankar. The virtual activities, however, have been very well attended and we estimate that somewhere between 150 and 200 mathematicians are participating in the program on a semi-regular basis. In order to make these activities as accessible as possible to worldwide participants, almost all take place between 9am and noon Pacific time.

Introductory workshops. After a long period of uncertainty, planning for the semester began in July. In consultation with early career mathematicians in the program, it became clear that they would find an introductory workshop especially valuable. The organizing committee for this workshop consisted of Martin Bridgeman, Richard Canary, Michelle Chu, Tommaso Cremaschi, James Farre, and David Fisher. We decided to have six three-hour mini-courses, with topics spanning the different foci of the program, each of which was accompanied by a one-hour talk by an early career mathematician.

Given that the activities were virtual, and considering the constraints on the timing of the talks, we spread the workshop over three weeks. Mini-courses were given by Ian Biringer (geometric structures on manifolds), Indira Chatterji (property T and aTmenability from a geometric viewpoint), Alexander Fuman (rigidity phenomena via ergodic theory), Fanny Kassel (Anosov representations), Lola Thompson (arithmetic and spectral geometry), and Giulio Tiozzo (random walks on weakly hyperbolic groups). One-hour talks were given by Martin Bobb, Michelle Chu, Kasia Jankiewicz, Nicholas Miller, Soumya Sankar, and Yvon Verberne. Talk attendance varied between 50 and 100 participants.

Research and career meetings. We are holding a weekly Research Seminar on Mondays featuring two 45-minute talks (usually by members of the program), a Program Associate Seminar on Tuesdays (with talks largely by graduate students who are program associates), and a Postdoctoral Research Seminar (with talks usually given by early career mathematicians). Alessandra Iozzi and and Nicholas Miller are organizing the Research Seminar, Tommaso Cremaschi and James Farre are organizing the Program Associate Seminar, and Tommaso Cremaschi, Soumya Sankar, and Yvon Verberne are organizing the Postdoctoral Research Seminar.

Tommaso Cremaschi and Wesley Calvert (from the concurrent Decidability, Definability, and Computability program) are organizing a weekly seminar to address career issues for early career mathematicians. We also scheduled three other three-hour mini-courses which will be spread over the remainder of the semester, by David Fisher (commensurators, irreducible subgroups, and applications), Tsachik Gelander (invariant random subgroups and lattices), and Maria Beatrice Pozzetti (higher rank Teichmüller–Thurston theories).

Prompts for informal interactions. As is customary with MSRI programs, the program began with a series of five-minute talks (over Zoom) where members introduced themselves by giving brief presentations about their research interests. The MSRI mentoring program, an integral part of the program that offers early career mathematicians regular meetings with assigned senior researchers, was also moved online with each postdoc assigned two mentors whom they meet virtually each week.

The most difficult parts of the typical MSRI semester to replace are the informal interactions that happen between talks and at tea. While the result is inevitably imperfect, we have tried to recreate this atmosphere as much as possible. To this end, Martin Bridgeman, Soumya Sankar, and Yvon Verberne created a seminar entitled "This week I am thinking about . . . ," where each participant is given ten minutes to discuss a question they are pondering, a difficulty they have encountered, or a discovery they have recently made. We have also encouraged, and attempted to facilitate, the creation of online reading groups. The senior local members have also organized biweekly social activities for all local members. Finally, we are holding nearly daily informal, online teas. After experimenting with several formats, we are currently using Gather.town as the forum for our teas.

A Silver Lining

One silver lining in all this was the impressive response of the junior members of the program who took the lead in much of the organization, which was deeply appreciated by all involved. We hope that soon MSRI programs will return to having lively in person interaction as a focus, but we also hope that future programs can learn from our experiments.

Named Positions, Fall 2020

MSRI is grateful for the generous support that comes from endowments and annual gifts that support faculty and postdoc members of its programs each semester.

Chern, Eisenbud, and Simons Professors

Richard Canary, University of Michigan
Martin Bridgeman, Boston College
Julia Knight, University of Notre Dame
David Fisher, Indiana University
Yves Benoist, Centre National de la Recherche Scientifique
Valentina Harizanov, George Washington University
Mahan Mj, Tata Institute of Fundamental Research
Florian Pop, University of Pennsylvania
Tsachik Gelander, Weizmann Institute of Science
Zoé Chatzidakis, Centre National de la Recherche Scientifique
Fanny Kassel, Institut des Hautes Études Scientifiques
Roman Sauer, Karlsruhe Institute of Technology

Named Postdoctoral Fellows

Strauch: Martin Bobb, University Michigan
Della Pietra: Esther Elbaz, Ben Gurion University of the Negev
Huneke: Yvon Verberne, Georgia Institute of Technology
Gamelin: Soumya Sankar, Ohio State University
McDuff: Nicholas Miller, University of California, Berkeley
Viterbi: Tommaso Cremaschi, Univ. of Southern California
Viterbi: Philip Dittmann, TU Dresden

Clay Senior Scholars

The Clay Mathematics Institute awards its Senior Scholar awards to support established mathematicians to play a leading role in a topical program at an institute or university away from their home institution.

Uri Bader, Weizmann Institute of Science
François Loeser, Université de Paris VI (Pierre et Marie Curie)

Focus on the Scientists

Valentina Harizanov

Valentina Harizanov is a lead organizer of this semester's program on Decidability, Definability, and Computability in Number Theory. She is one of the leading figures in computable structure theory.

Computable structure theory focuses on recursion-theoretic complexity in familiar kinds of mathematical structures like groups, fields, and vector spaces. Some of the results explain bounds on complexity in terms of definability. Other results show, by delicate constructions, that definability cannot explain all such bounds. Isomorphic copies of a given structure differ in complexity. Even among computable copies, the complexity of the images of a given relation differ in complexity.

Valentina Harizanov

In her thesis, Valentina considered the set of all Turing degrees of images of the relation in computable copies of the structure — the degree spectrum — and constructed examples illustrating the possibilities. Many researchers today are studying degree spectra for structures and isomorphisms, as well as relations. Valentina's book-length expository paper, "Pure computable model theory," was published in 1998, although the first version was written some years earlier. Research in computable structure theory had been carried out mainly by groups in the Soviet Union, with a few isolated researchers in the U.S., Australia, and elsewhere. Valentina's paper included many Russian results and provided an accessible introduction to computable structure theory, encouraging more westerners to take it up.

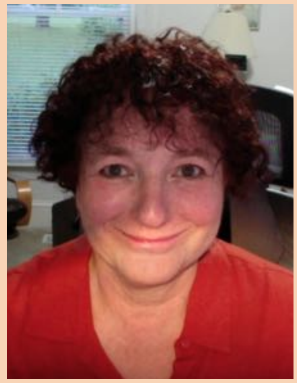
Valentina is from Yugoslavia. She received her Ph.D. from the University of Wisconsin–Madison in 1987 under the supervision of Terry Millar, and is currently a full professor at George Washington University. Although her primary research area is computable structure theory, she has branched out into some very different areas, in particular knot theory and quantum computing. At George Washington, she was a co-director of the Center for Quantum Computing, Information, Logic, and Topology.

Valentina reads widely within mathematics, and has an amazing ability to remember what she has read and use it later. More than once, there has been a group of collaborators, including Valentina, working in my office, stuck. One of the group would be standing at the blackboard, on which we had recorded what we wanted to show, but couldn't. Others would be scribbling on scratch paper. Valentina would be leafing through some book or paper. After a while, Valentina would point to a specific result, saying, "I think this is what we need." And it would be.

— Julia F. Knight

Alexandra Shlapentokh

Alexandra (Sasha) Shlapentokh is a lead organizer of this semester's program on Decidability, Definability and Computability in Number Theory. She is recognized as one of the main experts in the subject of definability in number theory, especially on problems about definability in global fields, infinite algebraic extensions, and generalizations of Hilbert's tenth problem (HTP). Her work is characterized by a mixture of technical power and ingenuity.



Sasha Shlapentokh

Sasha graduated from University of Pennsylvania in 1983 and received her Ph.D. from New York University, under the supervision of Harold Shapiro, in 1988. Currently, she is a professor at East Carolina University.

Although it is not possible to accurately describe in such a short note all the outstanding contributions that Sasha has made to the field, let me briefly mention here two remarkable and recent directions of her research. A central problem on which Sasha's work has been a game-changer is HTP for rings of integers of number fields. After the work of B. Poonen and G. Cornelissen, T. Pheidas, and K. Zahidi, she established a general elliptic curve criterion for this problem that remains the most promising strategy. Recently, she noticed that the general case can be reduced to the case of relative quadratic extensions — the proof of this remarkable observation fits in a couple of lines, yet it took about 40 years to be discovered!

Since the work of Julia Robinson, the question of definability of algebraic integers and undecidability of algebraic extensions of the rationals has been a central one, but progress has been quite difficult in the setting of infinite degree. Sasha introduced the notion of q-boundedness, which allowed her to address these questions for a very wide class of infinite extensions. These results shed some light, for the first time, on an otherwise hopelessly general and difficult problem.

The impact of Sasha's work goes far beyond proving theorems. Her book *Hilbert's Tenth Problem: Diophantine Classes and Other Extensions to Global Fields* has certainly helped to define the field around undecidability in number theory. In addition, it is no surprise that she is a lead organizer of this MSRI program, as she is often a driving force in the organization of conferences and scientific programs on the topic. Energetic, creative, and friendly in her own particular way, the community of definability and undecidability in number theory would not be the same without Sasha Shlapentokh.

— Hector Pasten

Puzzles Column

Joe P. Buhler and Tanya Khovanova

1. We are given eight unit cubes. The faces of each cube are colored either blue or red. One third of all of those faces are red. We build a $2 \times 2 \times 2$ cube out of these cubes so that exactly one third of the unit cubes' faces visible on the faces of the larger cube are red. Prove that you can use these cubes to build a $2 \times 2 \times 2$ cube whose faces are entirely red.

Comment: Problems 1 and 5 are due to Alexander Shapovalov, a truly prolific problem composer for many different math competitions.

2. Let A be an $n \times n$ matrix, and B the 90-degree clockwise rotation of that matrix (so that, for example, $B_{1,n} = A_{1,1}$). Find, and prove, a relationship between $\det(A)$ and $\det(B)$.

3. Alice tosses 99 fair coins and Bob tosses 100. What is the probability that Bob gets more heads than Alice?

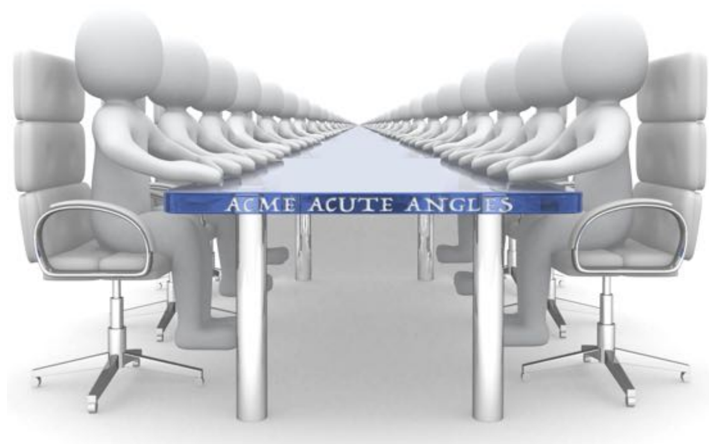
Comment: This appears in the new MAA book *Bicycle or Unicycle*, by Dan Velleman and Stan Wagon, containing 105 puzzles. Many of those appeared in Wagon's long-running and unique "Problem of the Week" emails, and some have appeared here.

4. The Board of Directors of the Acme Acute Angles Company has grown too large. It has 50 members, and they have agreed to the following reduction protocol:

The Board will vote on whether or not to reduce its size. If a majority vote "yes," the newest member is ejected from the Board. If that happens a new yes/no vote is taken, and this continues until half or more of the surviving members vote "no," at which point the protocol ends, and the Board is fixed.

Suppose that each member places the highest priority on personally remaining on the Board, but, given that, agrees that the smaller the Board, the better. To what size will this protocol reduce the Board?

Comment: This puzzle appeared in Peter Winkler's new weekly email, "Mind-Benders for the Quarantined!" hosted at the MoMath (National Museum of Mathematics) web site. Sign-up is free, and a new conundrum appears each Sunday in your inbox. Peter has a charming way of coping with a diverse audience: a small hint appears in your email each Tuesday, a stronger hint on Thursday, and a solution on Saturday.



Modified from: Illustration 30106518 © Lpstudio | Dreamstime.com

5. An isosceles triangle is divided into four smaller isosceles triangles. Is it true that you can always find a pair of congruent triangles among them?

6. Eight out of sixteen coins are heavier than the rest, and weigh 11 grams each. The other eight coins weigh 10 grams each. We do not know which coin is which, but one coin is conspicuously marked as a "Special" coin. Can you figure out whether the special coin is heavier or lighter using a balance scale at most three times?

Comment: The 16-coin problem apppeared on the 2019 Russian Math Olympiad, and is due to Konstantin Knop, a true master of coin-weighing problems. We heard a rumor that the same thing can be done with 18 coins, but do not know whether this is possible.

Send your thoughts to the authors at puzzles@msri.org. Solutions will be posted online by January 2021.

MSRI Receives Presidential Mentoring Award

MSRI is honored to have been selected by the White House for a Presidential Award for Excellence in Science, Mathematics, and Engineering Mentoring (PAESMEM). PAESMEM recognizes the critical roles mentors play in the academic and professional development of the future STEM workforce.

Director David Eisenbud thanks all who have made this award possible: "The work that our institute has done to reach out to groups underrepresented in mathematics at all levels, from early precollege students through women and minorities recruited into our advanced programs, requires the work of many individuals who have demonstrated intense, lifelong devotion to the goals that the PAESMEM program recognizes. To MSRI, this Presidential Award is a deeply moving tribute to those individuals and their contributions."

Colleagues, administrators, and members of the general public nominate individuals and organizations for exemplary mentoring sustained over a minimum of five years. Since 1995, PAESMEM has honored more than 315 individuals and organizations that broaden participation in the STEM pipeline.

Mathematical Sciences Research Institute

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MSRI Announces Support for New Graduate Fellowships

MSRI is pleased to announce new financial support for current graduate students to take part in our research programs, thanks to the generosity of MSRI donors Marie Vitulli, Kristin Lauter, Stephen Della Pietra and Pamela Hurst-Della Pietra, and The Salgo-Noren Foundation. Advanced graduate students in a field can often function on an almost equal footing with the postdocs, so MSRI hosts a number of graduate students, with their advisors, in each program. Some of these students return as distinguished researchers. These Program Associates have been welcome during periods when their thesis advisors are in residence, but these opportunities have been largely self-funded. With this additional support, graduate students will receive financial support to remain in residence at MSRI for the entire semester with their advisor, fully integrated into the semester's research program. Candidates may be nominated by their advisor, who must hold a membership in one of MSRI's semester-long programs.