Mathematical Sciences Research Institute

Final Report
on the
Mathematical Sciences Research Institute
2014 Undergraduate Program (MSRI-UP)
supported by
NSA Grant
H98230-13-1-0262
May 2015

2014 Mathematical Sciences Research Institute – Undergraduate Program (MSRI-UP)

Summer Report

Table of Contents

2014 Mathematical Sciences Research Institute – Undergraduate Program (MSRI-UP)

Summer Report

1. Introduction

The eighth Mathematical Sciences Research Institute – Undergraduate Program (MSRI-UP) took place from 21 June to 03 August 2014 at the Mathematical Sciences Research Institute, Berkeley, CA under the leadership of the On-site Director Herbert A. Medina from Loyola Marymount University (LMU) and Co-Directors Duane Cooper, from Morehouse College, Ricardo Cortez from Tulane University, Ivelisse Rubio from the University of Puerto Rico – Río Piedras, and Suzanne Weekes from Worcester Polytechnic Institute. Although the main part of MSRI-UP happened during summer 2014, as is explained below, significant parts of the program happen after the summer.

The 2014 MSRI-UP was funded by grants from the National Security Agency (NSA) and the National Science Foundation (NSF) with onsite staffing, facilities and support provided by the Mathematical Sciences Research Institute (MSRI). It is noteworthy that the staff and students often praised the MSRI facilities during the program, and the students overwhelmingly rated the facilities and academic/research infrastructure very highly in the end-of-program evaluation.

The program was designed for undergraduates who are majoring in mathematics or related science. Eighteen students from universities in California, Connecticut, Georgia, Illinois, Iowa, Kansas, Louisiana, Maine, Massachusetts, New Jersey, New York, and Oregon participated in the six-week research program in mathematics. Eight of the students were women. Participants in MSRI-UP received round-trip travel to Berkeley, CA, room and board for the duration of the program, and a $3,100 stipend.

Five MSRI-UP faculty and staff from the Loyola Marymount University (LMU), Tulane, University of Liège, University of Maryland at College Park, and University of Iowa, were responsible for assisting the students in creating an academic and research environment that would help to achieve the main objective of the program:

> *To identify talented students, especially those from underrepresented groups, who are interested in mathematics and make available to them meaningful research opportunities, the necessary skills and knowledge to participate in successful collaborations, and academic peers and mentors who can advise, encourage and support them through a successful graduate program.*

The objective is designed to contribute significantly toward meeting the program goal of *increasing the number of graduate degrees in the mathematical sciences, especially doctorates, earned by U.S. citizens and permanent residents by cultivating heretofore untapped mathematical talent within the U.S. Black, Hispanic/Latino and Native American communities*.

2. Funding Information

The funding for MSRI-UP can be summarized as follows:

1. National Security Agency $147,097[1]
2. The National Science Foundation $120,721[2]

In addition, the MSRI provided much additional support by providing indispensable excellent administrative assistance and allowing the program to use classrooms, offices, computers, facilitating transportation, etc.

3. Recruitment, Application and Admissions Procedures

The Co-Directors began recruiting for the 2014 MSRI-UP at the Annual SACNAS Conference in San Antonio, TX in October 2013. The Co-Directors and MSRI Associate Director passed out applications, brochures and posters and talked to dozens of students and faculty about the program. The MSRI-UP home page also provided information about and applications for the program.

The Co-Directors disseminated electronic information about the program to hundreds of mathematicians including SACNAS members who belong to the mathematics community and to professors who are active in the undergraduate research community.

The on-line application was managed through mathprograms.org and became available on 15 November 2013. It consisted of four items: a completed student application form,

[1] Grant number H-98230-13-1-0262.

transcripts, a statement of interest, and a letter of recommendation. Applications received by 15 February 2014 received full consideration. The 2014 MSRI-UP received 346 applications.

The Co-Directors Cooper, Cortez, Medina, Rubio and Weekes reviewed each application and evaluated it using four criteria: 1.) the student's grades in mathematics courses; 2.) the student's mathematical background; 3.) the statement of interest; and 4.) the letter of recommendation. In the application review process, the Co-Directors also took note of the applicants to try to achieve a group that was gender balanced, ethnically and geographically diverse, and that had students from across the spectrum of American higher-education institutions.

4. Summary of Participant Demographics

Table 1 details the demographics of the eighteen MSRI-UP students. Achieving this type of diversity and gender balance is important to creating the academic and research environment explained below and to achieving one of the MSRI-UP objectives.

5. Housing and Meals for Students & Teaching Assistants

The eighteen students and the program's two graduate student teaching assistants were provided room and board in the Stern Dormitory at the University of California, Berkeley. They rode a UC Berkeley shuttle up and down the hill to MSRI each weekday.

On weekdays, lunch and afternoon tea for students and program staff was served at MSRI. This may seem like an insignificant detail, but the fact that all the human resources in the program shared at least one meal together on a daily basis as was very important for program cohesiveness and helped to create the mathematical intellectual community that promoted its success. Indeed, many important mathematical and academic conversations took place during lunch and afternoon tea.

6. Pre-Research Training Phase

The eighteen 2013 MSRI-UP students, led by Professor Victor H. Moll, from Tulane University, participated in an intensive training in some techniques in combinatorics and number theory. Professor Moll was assisted by Dr. Eric Rowland, a Postdoctoral fellow at the University of

[2] Grant number 1156499.

Liège, and two graduate students: Leyda Almodóvar from the University of Iowa and Asia Wyatt from the University of

Table 1

2014 Mathematical Sciences Research Institute (MSRI-UP)

Student Data

University	
Northwestern University	1
Bowdoin College	1
Mills College	1
Iowa State University	1
Tulane University	1
University of Chicago	2
Amherst College	1
State University of New York, Potsdam	1
St. Mary's College	1
Kansas State University	1
Brown University	1
University of California, Santa Cruz	1
Princeton University	1
California State University Fullerton	1
Georgia Institute of Technology	1
Mount Holyoake	1
Reed College	1

State in which Students Study	
California	4
Illinois	3
Massachusetts	3
Georgia	1
Iowa	1
Louisiana	1
Maine	1
New Jersey	1
New York	1
Oregon	1
Rhode Island	1

Ethnicity	
Latino	9
African American	3
Asian American	3
White	2
Pacific Islander	1

Gender	
Female	8
Male	10

Graduation Date	
Fall 2014	1
Spring 2015	11
Spring 2016	6

Maryland, College Park. (It is noteworthy that Ms. Almodóvar is an alumna from the 2009 MSRI-UP.)

During the first ten days of MSRI-UP, students received lectures, participated in group discussions and work on problem sets. Prof. Moll not only introduced students to the mathematical field, but also provided them with sufficient background and introduced them to their research project so that students could complete their project during the rest of the program.

The students also participated in a computational two-week computational laboratory/workshop where they learned how to use the *Mathematica* computer algebra system. This portion of the program was led by Dr. Rowland whose knowledge and expertise in using *Mathematica* is significant.

The entire training phase was conducted in MSRI's Baker Board Room. After leaving MSRI in the late afternoon, students continued doing mathematics in the study lounges in Stern during the evening, and the staff was regularly available for consultation often until almost midnight. A copy of the summer schedule can be found in Appendix A.

7. Research Projects and Student Presentations

After the first ten days of the program all students worked exclusively on an undergraduate research project carefully designed and directed by their research director. There were six groups of three students each. Students wrote technical reports and presented the results of their research at the MSRI-UP Student Colloquium the last day of the program. (See Appendix B. for the student presentation schedule on the program's capstone last day.)

At the end of the second week of the program, students had a description of their possible research projects. The students did preliminary reading and literature searches on the projects and turned in a list of their preferred topics and students with whom they would like to work as a group; they also presented a list of students that they thought might not be a good match for them. The staff, including the on-site director, held several discussions and studied different options for the research groups and topic assignments. By Monday of the third week of the program, all groups had been formed and selected a research project.

Each one of the graduate students assisted three of the research groups; Prof. Moll and Dr. Rowland worked with all the groups. The entire staff (including the on-site director) was available all day to meet the students to discuss their projects and assist on preparing their technical reports and oral presentation; the graduate students also often met with research groups at night. In summary, all MSRI-UP students and staff hey worked tirelessly to complete the projects.

During the program, MSRI-UP participants were introduced to some of the techniques that are used while conducting successful research in the mathematical sciences. Indeed, many MSRI-UP students learned to work as part of a research team, to develop an effective faculty advisor-student relationship, to use the computer as a tool, to use the Internet as a resource, to prepare and give oral presentations, and to write technical reports and produce presentations using LaTeX.

The outcome of all the students' and staff's work and dedication resulted in six technical reports and an equal number of oral presentations in the Student Colloquium Series. A complete collection of the abstracts for the students' research projects can be seen in http://www.msri.org/pages/msri-up-2014-research-projects. A collection of all the technical reports will soon be available for downloading from the same page. A sample technical report is included in Appendix C. Sixteen of the eighteen MSRI-UP students will present their research at the 2014 SACNAS Conference in Los Angeles, CA in October 2014. We expect that most of the students will present their research at the Joint Mathematics Meetings in San Antonio, TX in January of 2015.

The program's objectives could be achieved only within a rich and intense academic and research environment. The research environment in MSRI-UP created the richness and intensity and thus drove the program.

8. Evaluation of Student Work

Close interaction with students allowed the staff to give individuals feedback on their work throughout the program. Indeed, research leader, postdoc and graduate assistants gave students written feedback on their homework during the pre-research phase of the program and on the numerous drafts of their technical reports and slides for their oral presentation.

Professor Moll, with input from Dr. Rowland, Ms. Almodóvar and Ms. Wyatt, also wrote an end-of-program candid evaluation for each student in the program. This evaluation will be kept by Prof. Medina in case students need formal assessment of their work in the program, and, more importantly, will constitute a significant portion of future letters of recommendations that Prof. Moll is certain to write for many students.

9. Colloquia Series

The 2013 MSRI-UP hosted seven mathematical scientists (including 3 graduate students) for a colloquia series: Nancy Rodriguez, Stanford University; Trachette Jackson, University of Michigan; Richard Laugesen, University of Illinois, Urbana-Champaign; and Lisa R. Goldberg, University of California, Berkeley were the professional mathematician colloquium speakers.

In addition, three MSRI-UP alumni who are now graduate students in the mathematical sciences presented on their current work. These were Leyda Almodóvar, (MSRI-UP 2009) University of Iowa; Alexander Moll, (MSRI-UP 2008), Massachusetts Institute of Technology; and Gina Pomann, (MSRI-UP 2007), North Carolina State University.

Each MSRI-UP speaker was asked to share with the students the story of "how they got to where they are." This important portion of the presentation was designed to help the MSRI-UP students to "see themselves" in the speaker. This worked very well and all speakers spent a significant amount of time interacting with students after their talk.

10. MSRI-UP Workshops

The program held regular workshops on Friday designed to assist MSRI-UP students in the summer work and, more importantly, in their road to graduate school.

Dr. Colette Patt, Director of Diversity Programs in the Physical Sciences at UC, Berkeley and Prof. Martin Olsson, Vice Chair for Graduate Affairs in the Mathematics Department at UC Berkeley visited MSRI-UP and gave a workshop on applying for graduate school and finding funding for graduate school. The workshop addressed questions/issues such as the significant differences between a master's and a doctoral program, the funding opportunities available for most graduate programs, and the benefits of obtaining a graduate degree. In addition to this basic information, they presented successful techniques for applying to graduate school. They discussed the elements that constitute a good statement of purpose, the types of professors from whom one should seek letters of recommendation, and successful techniques for addressing not-so-stellar semesters. Finally, they also discussed successful strategies for compiling a winning national fellowship application. Their workshop will prove very valuable as the MSRI-UP students move towards applying to graduate school and to fellowships.

Herbert A. Medina conducted a workshop on the GRE Mathematics Subject Exam. Students received information about the test and learned techniques to tackle the problems efficiently; they also worked in sample practice problems and received material and practice exams to prepare for the test.

Professor Medina also led a workshop on the LaTeX typesetting system which students used to write their research reports, prepare their end-of-program presentations, and will use to prepare their research posters to post-summer scientific conferences.

11. Recreational/Cultural Activities

In addition to all the academic activities described above, MSRI-UP students were treated to several cultural/recreational activities designed to enhance team-building and to promote a collaborative environment. These included a visit to the California Academy of Sciences, a trip to Muir Woods (including a nature walk), two trips to San Francisco, a picnic at Tilden Park, a boat tour of San Francisco Bay, an Oakland's Athletics baseball game. These carefully-planned recreational and cultural activities were essential to MSRI-UP's success as they gave students the opportunity to put mathematics aside for a few hours so that they could come back later to their work with renewed vigor; they also helped to build the MSRI-UP mentored community as all staff and visitors participated in the activities with the students.

12. Program Evaluation During MSRI-UP

Informal formative program evaluation took place from the beginning of MSRI-UP through conversations with students and staff. Formal formative evaluation continued at the beginning of the second week of the program when Prof. Medina met individually with the students in order to solicit feedback on the program. The aim of the meeting was to listen to student concerns, complaints and methods for improving the program. The students' feedback was communicated to the staff so that adjustments could be made in order to improve the program.

Staff meetings were held on a weekly basis. At these meetings, the staff discussed each of the eighteen students in the program. In addition, the staff was in contact throughout the day each and every day of the program so that troubleshooting and program adjustments were made in a real-time basis.

During these meetings Prof. Medina had the opportunity to have more close contact with the students and staff, to listen and address individual concerns and provide individual mentoring to the students. Because of the intensity of pre-research training phase of the program, usually after the first week the students are concerned about the work load; in the meeting they have the opportunity to raise these concerns and to be reassured that, if necessary, adjustments will be made and that the pace will be different once they start their research. In addition to these forums, the staff's close interaction with the students enabled them to gather informal feedback that also led to adjustments to improve the program.

13. End-of-Program Evaluation

Each MSRI-UP student was required to complete a comprehensive, end-of-program, online evaluation. For the first time, MSRI-UP used the assessment instrument developed by the Undergraduate Research Student Self-Assessment (URSSA) project at the University of Colorado, Boulder (http://www.colorado.edu/eer/research/undergradtools.html) that was adapted to REUs in the NSF Division of Mathematical Sciences with the input and collaboration of several REU directors. Program specific questions were added to the common instrument by Prof. Medina so that as much information about MSRI-UP could be gathered. Appendix D contains the end-of-program online assessment instrument with the students' responses.

The results of the end-of-program evaluations are overwhelmingly positive. We highlight the positive nature of the responses by including all of the responses to two questions in Tables 2 and 3.

The five MSRI-UP Directors will hold conversations to evaluate the results of this fall in order to identify issues that need to be addressed and to design necessary adjustments for the 2015 program.

14. Long-Term Summative Evaluation

Measuring the program's effectiveness in achieving the MSRI-UP goal will take several years. In order to do so, the Directors will follow the educational progress of each MSRI-UP student for several years. To facilitate this process, each year MSRI-UP students provide their respective year's Directors their current educational/professional status and contact information that is valid for at least one year. All of this information is kept in a common, online database that can be updated by each of the Directors.

Table 2

Responses to the Following Question from End-of-Program Evaluation

After the research experience at MSRI-UP, do you see yourself participating in other undergraduate research projects or continuing to work on your project from this summer? Please answer "Yes" or "No" and provide a one sentence explanation for your answer.

“Yes, I would like to participate in other summer programs or other projects during the year.”

“Yes. I can see myself working in both other undergraduate research projects, as well as possibly continuing this research.”

“Yes, I would definitely like to continue this project. There is so much more to do!”

“Yes. There are still many interesting open questions from the project which I hope to be able to answer; other research opportunities are also in my foreseeable future.”

“Yes. I plan to apply for REUs in graph theory/discrete math as I did not see my current project as something I want to continue in the future.”

“Yes. I plan on expanding on some of our future goals this school year.”

“Yes, the theorem we’re almost done proving is very interesting.”

“Yes. I’ve come to realize that I enjoy math research, and intend to continue this specific project as well as seek out other opportunities to do research.”

“Yes. I will expand upon this topic for my Senior Thesis.”

“Yes. I look forward to working with my research group during the year to finish up our paper.”

“Yes, I do. This was a great experience for me and I am motivated to continue this path.”

“Yes. After this summer I will begin my senior year as an undergraduate. This program has made me realize that I am capable of research, and I am excited to begin work on my coming senior thesis. I will also continue work on my project from this summer, and hope to see how much more work we can do on this topic.”

“Yes. I see myself continuing working on my project!”

“No. I would rather spend my time studying other areas.”

“Yes”

“Yes. I would very much like to publish a paper; also, I enjoy working on this problem.”

“Yes, I may explore this topic for my thesis this coming year.”

“Yes, will continue to work on this project.”

15. Conclusion

Students, staff and visitors have communicated to the On-site Director that the 2014 MSRI-UP was very successful in achieving its objectives. The Director also saw the transformations, mathemathical maturing, blossoming and development of many MSRI-UP students during the six weeks of the program---transformations that undoubtedly will contribute towards the academic careers of these students. By the end of the program all research teams of students had produced technical reports and given oral presentations that were at a very high level and many were clearly on their way to a first-rate graduate program in mathematics. It is our hope that all of the technical reports will be polished over the coming months and sent to peer-reviewed journals to be considered for publication.

Table 3

Responses to the Following Question from End-of-Program Evaluation

Only answer this question if you have participated in previous undergraduate research projects in the mathematical sciences. How does your research experience in MSRI-UP compare to your other research experience(s) in the mathematical sciences?

“It has been more rigorous than past research experiences, though with much more rewards.”

“This one was much more engaging as far as problem solving/formulation goes. The problems were more well-selected as well.”

“It was much more structured, and the participants were more diverse. I also felt valued.”

“This one was a lot more intense. I enjoyed collaborating with other students which I haven't done before. Overall this was a great experience.”

“This has been, by far, the best research experience I have ever had. I have been involved in 4 different research projects and only now I feel like I know what mathematical research is all about.”

“I liked the actual math of this one better (I was at [REU name redacted] last summer). I think having previous experience prepared me for this summer. I spent less time “spinning my wheels” this summer. My project this summer seemed more important, and I definitely found it more interesting.”

“In the MSRI-UP program I have gotten more of a chance to think independently about a problem and how to go about solving it, or in the least, learning more about methods of solving it.”

“This research experience was much more intensive.”

“MSRI-UP was significantly better than prior research experiences.”

The data that will verify that the MSRI-UP objectives contribute towards the goal of increasing the number of Latinos/Chicanos, African American and Native Americans earning graduate degrees in the mathematical sciences will not be available for several years, but some of the preliminary data is already very promising. Indeed, several MSRI-UP alumni have already earned masters' degrees and several are poised to earn a Ph.D. in the mathematical sciences in the coming 1-3 years. The Directors are committed to continuing to collect data in the years to come to demonstrate the long-term impact of the program.

2014 MSRI-UP Calendar

	Sunday 22-Jun-14	Monday 23-Jun-14	Tuesday 24-Jun-14	Wednesday 25-Jun-14	Thursday 26-Jun-14	Friday 27-Jun-14	Saturday 28-Jun-14
7:40 or 8:10am		Shuttle to MSRI	Shuttle to MSRI	Shuttle to MSRI	Shuttle to MSRI		
8:30 AM		Welcome to MSRI!				8:40 or 9:10am	
9:00 AM		Check in, IDs,	9-11:50, Seminar	9-11:50, Seminar	9-11:50, Seminar	Shuttle to MSRI	9:30-?, "The city"
10:00 AM		Library, etc.				9:45-11:50, LaTeX	San Francisco
11:00 AM		10:45-11:50, Seminar				Workshop	
12:00 PM		11:50, Lunch	11:50, Lunch	11:50, Lunch	11:50, Lunch	11:50, Lunch	Details to follow
1:00 PM							
1:30 PM		1:30-4:30, Seminar/	1:30-4:30, Lab/	1:30-4:30, Lab/	1:30-4:00, Lab/		
2:00 PM		Problem Session/	Problem Session/	Problem Session/	Problem Session/	2-3, Colloquium	
3:00 PM		Lab	Seminar	Seminar	Seminar	Nancy Rodriguez	
4:30 PM						3:25; 3:55; 4:25; etc.	
5:00 PM		4:55, 5:25, 5:55, 6:25	4:55, 5:25, 5:55, 6:25	4:55, 5:25, 5:55, 6:25	4:55, 5:25, 5:55, 6:25	Shuttle to Stern	
	6:15, Progam	Shuttle to Stern	Shuttle to Stern	Shuttle to Stern	Shuttle to Stern		
	Orientation	Evening Study	Evening Study	Evening Study	Evening Study		
	Foothill Rec Rm	Sessions as	Sessions as	Sessions as	Sessions as		
	Building 4	Announced by TAs	Announced by TAs	Announced by TAs	Announced by TAs		

	Sunday 29-Jun-14	Monday 30-Jun-14	Tuesday 01-Jul-14	Wednesday 02-Jul-14	Thursday 03-Jul-14	Friday 04-Jul-14	Saturday 05-Jul-14
7:40 or 8:10am		Shuttle to MSRI	Shuttle to MSRI	Shuttle to MSRI	Shuttle to MSRI		
8:30 AM						H	
9:00 AM		9-11:50, Seminar	9-11:50, Seminar	9-11:50, Seminar	9-11:50, Seminar		
10:00 AM						O	
11:00 AM							11-? Picnic
12:00 PM		11:50, Lunch	11:50, Lunch	11:50, Lunch	11:50, Lunch	L	Tilden Park
1:00 PM							Details to follow
1:30 PM		1:30-4:30, Lab/	1:30-4:30, Lab/	1:30-4:30, Lab/	1:30-4:00, Lab/	I	
2:00 PM		Problem Session/	Problem Session/	Problem Session/	Problem Session/		
3:00 PM		Seminar	Seminar	Seminar	Seminar	D	
4:30 PM							
5:00 PM		4:55, 5:25, 5:55, 6:25	4:55, 5:25, 5:55, 6:25	4:55, 5:25, 5:55, 6:25	4:55, 5:25, 5:55, 6:25	A	
		Shuttle to Stern	Shuttle to Stern	Shuttle to Stern	Shuttle to Stern		
		Evening Study	Evening Study	Evening Study	Evening Study	Y	
		Sessions as	Sessions as	Sessions as	Sessions as		
		Announced by TAs	Announced by TAs	Announced by TAs	Announced by TAs		

2014 MSRI-UP Calendar

	Sunday 6-July	Monday 7-July	Tuesday 8-July	Wednesday 9-July	Thursday 10-July	Friday 11-July	Saturday 12-July
		7:40 or 8:10 Shuttle*	7:40 or 8:10 Shuttle	7:40 or 8:10 Shuttle	7:40 or 8:10 Shuttle		
8:30 AM		to MSRI	to MSRI	to MSRI	to MSRI	8:40, 9:10 or 9:40 Shuttle	
9:00 AM		Work on Projects.	Work on Projects.	Work on Projects.	Work on Projects.	to MSRI	CA Academy
10:00 AM		Group meeting with	Group meeting with	Group meeting with	Group meeting with	Applying to Graduate	of Science,
11:00 AM		staff to be arranged.	staff to be arranged.	staff to be arranged.	staff to be arranged.	School Seminar	Golden Gate
12:00 PM		11:50, Lunch	11:50, Lunch	Walk to BBQ Lunch	11:50, Lunch	11:50, Lunch	Bridge, etc.
12:30 PM				w/ grad students			Details
1:00 PM							to follow
1:30 PM		Work on Projects.	Work on Projects.	SACNAS Conf. Meeting	Work on Projects.		
2:00 PM		Group meeting with	Group meeting with	Work on Projects...	Group meeting with	2-3, Colloquium	
3:00 PM		staff to be arranged.	staff to be arranged.		staff to be arranged.	Trachette Jackson	
4:30 PM		Shuttle to Stern	Shuttle to Stern	Shuttle to Stern	Shuttle to Stern	3:25, 3:55, 4:25, etc.	
5:00 PM		4:25, 4:55, 5:25, 5:55	4:25, 4:55, 5:25, 5:55	4:25, 4:55, 5:25, 5:55	4:25, 4:55, 5:25, 5:55	Shuttle to Stern	

	Sunday 13-July	Monday 14-July	Tuesday 15-July	Wednesday 16-July	Thursday 17-July	Friday 18-July	Saturday 19-July
		7:40 or 8:10 Shuttle	7:40 or 8:10 Shuttle	7:40 or 8:10 Shuttle	7:40 or 8:10 Shuttle		
8:30 AM		to MSRI	to MSRI	to MSRI	to MSRI	8:40, 9:10 or 9:40 Shuttle	
9:00 AM		Work on Projects.	Work on Projects.	Work on Projects.	Work on Projects.	to MSRI	
10:00 AM		Group meeting with	Group meeting with	Group meeting with	Group meeting with	10-12, Former MSRI-UP	Free
11:00 AM		staff to be arranged.	staff to be arranged.	staff to be arranged.	staff to be arranged.	Current Grad. Stud. Pres.	Saturday
12:00 PM		11:50, Lunch	11:50, Lunch	11:50, Lunch	11:50, Lunch	11:50, Lunch	(Oakland A's
12:30 PM							game on
1:00 PM		*SACNAS Material Due!*					Sunday)
1:30 PM		Work on Projects.	Work on Projects.	Work on Projects.	Work on Projects.		
2:00 PM		Group meeting with	Group meeting with	Group meeting with	Group meeting with	2-3, Colloquium	
3:00 PM		staff to be arranged.	staff to be arranged.	staff to be arranged.	staff to be arranged.	Richard Laugesen	
4:30 PM		Shuttle to Stern	Shuttle to Stern	Shuttle to Stern	Shuttle to Stern	3:25, 3:55, 4:25, etc.	
5:00 PM		4:55; 5:25; 5:55; 6:25	4:55; 5:25; 5:55; 6:25	4:55; 5:25; 5:55; 6:25	4:55; 5:25; 5:55; 6:25	Shuttle to Stern	

* There is a large group of grad students at MSRI these two weeks. Their morning session starts at 9:15 on Monday and at 9:30 all other days so plan your shuttle travel accordingly.

Weeks 3 and 4

2014 MSRI-UP Calendar

	Sunday 20-July	Monday 21-July	Tuesday 22-July	Wednesday 23-July	Thursday 24-July	Friday 25-July	Saturday 26-July
		7:40 or 8:10 Shuttle*	7:40 or 8:10 Shuttle	7:40 or 8:10 Shuttle	7:40 or 8:10 Shuttle		
8:30 AM		to MSRI	to MSRI	to MSRI	to MSRI	8:40, 9:10 or 9:40 Shuttle	
9:00 AM		Work on Projects.	Work on Projects.	Work on Projects.	Work on Projects.	to MSRI	
10:00 AM		Group meeting with	Group meeting with	Group meeting with	Group meeting with	Mathematics Subject	To be
11:00 AM	BART	staff to be arranged.	staff to be arranged.	staff to be arranged.	staff to be arranged.	GRE Workshop	Determined
12:00 PM	at 11:40	11:50, Lunch	11:50, Lunch	11:50, Lunch	11:50, Lunch	11:50, Lunch	Suggestions?
12:30 PM							
1:00 PM	Oakland						
1:30 PM	Athletics	Work on Projects.	Work on Projects.	Work on Projects.	Work on Projects.		
2:00 PM	Game	Group meeting with	Group meeting with	Group meeting with	Group meeting with	2-3, Colloquium	
3:00 PM		staff to be arranged.	staff to be arranged.	staff to be arranged.	staff to be arranged.	Lisa Goldberg	
4:30 PM		Shuttle to Stern	Shuttle to Stern	Shuttle to Stern	Shuttle to Stern	3:25, 3:55, 4:25, etc.	
5:00 PM		4:25, 4:55, 5:25, 5:55	4:25, 4:55, 5:25, 5:55	4:25, 4:55, 5:25, 5:55	4:25, 4:55, 5:25, 5:55	Shuttle to Stern	

	Sunday 27-July	Monday 28-July	Tuesday 29-July	Wednesday 30-July	Thursday 31-July	Friday 1-August	Saturday 2-August
		7:40 or 8:10 Shuttle	7:40 or 8:10 Shuttle	7:40 or 8:10 Shuttle	7:40 or 8:10 Shuttle	7:40 or 8:10 Shuttle	
8:30 AM		to MSRI	to MSRI	to MSRI	to MSRI		
9:00 AM		Work on Projects.	Work on Projects.	Work on Projects.	Work on Projects.	Final Presentations	
10:00 AM		Group meeting with	Group meeting with	Group meeting with	Group meeting with		Packing
11:00 AM		staff to be arranged.	staff to be arranged.	staff to be arranged.	staff to be arranged.		Day
12:00 PM		11:50, Lunch	11:50, Lunch	Walk to BBQ Lunch	11:50, Lunch	11:50, Lunch	
12:30 PM				w/ grad students			
1:00 PM						Final Presentations	
1:30 PM		Work on Projects.	Work on Projects.	Work on Projects.	Work on Projects.		
2:00 PM		Group meeting with	Group meeting with	Group meeting with	Group meeting with		
3:00 PM		staff to be arranged.	staff to be arranged.	staff to be arranged.	staff to be arranged.		
4:30 PM		Shuttle to Stern	Shuttle to Stern	Shuttle to Stern	Shuttle to Stern	3:55, 4:25, etc.	
5:00 PM		4:55; 5:25; 5:55; 6:25	4:55; 5:25; 5:55; 6:25	4:55; 5:25; 5:55; 6:25	4:55; 5:25; 5:55; 6:25	Shuttle to Stern	
						6:00 Final Program Dinner at Local Restaurant	

Sunday
03-Aug
Departure
Day

Weeks 5 and 6

2014 MSRI-UP Final Presentations

Berkeley, California, 01 August 2014

9:05–9:40: ***Sequences of p-adic valuations of polynomial functions: an analysis of non-p-regularity and erratic behavior***

Alyssa Byrnes, Tulane University; Isabelle Nogues, Princeton University; Amber Yuan, University of Chicago

Abstract: In the field of number theory, the p-adic valuation is a useful device in studying the divisibility of an integer by powers of a given prime p. This paper centers on 2-adic valuations of quadratic polynomials in $\mathbb{Z}[x]$. In particular, the existence and properties of roots of such polynomials modulo 2^l, are determined and assessed. Polynomials of particular interest are those that yield non 2-regular sequences in $\mathbb{Q}_2$. Such sequences are represented in a novel infinite tree form, and patterns in such trees are analyzed to classify the sequences by their structure and non 2-regular properties. Such classification is further refined through an algebraic analysis of the polynomials at hand.

9:45–10:20: ***Calatan numbers modulo*** 2^α

David Cervantes, State University of New York at Potsdam; Erica Musgrave, St. Mary's College; Gianluca Pane, Brown University

Abstract: Catalan numbers have been studied since the eighteenth century due to their frequent appearance in fields such as set theory, graph theory and combinatorics. However, there are still few results about their properties modulo prime powers. In particular, this project examines the number of residues obtained by viewing Catalan numbers modulo powers of 2. For example, it is known that no Catalan number is equivalent to 3 modulo 4. Similarly, it can be shown that $C(n)$ is not congruent to 1 modulo 8, for n great than or equal to 2. Can other residues that do not occur for higher powers of 2 be characterized? As these higher powers of two are analyzed, it can be seen experimentally that more and more residues are not attained. Data has been produced to conjecture that as α gets large, the proportion of residues modulo 2^α attained by some Catalan number is between 0.125 and 0.27.

10:30–11:05: ***Arithmetic properties of infinite products***

Hadrian Quan, University of California, Santa Cruz; Fernando Roman, Kansas State University; Michole Washington, Georgia Institute of Technology

Abstract: The work discussed here develops methods to evaluate certain infinite products in closed form. These are finite products of values of the Gamma function. Presented here are infinite products of rational functions $R(n)$ raised to the power of some sequence M_n. The sequences satisfy certain regularity conditions as either a ℓ-periodic or k-automatic. Of particular interest is the regular paperfolding sequence considered by J.P. Allouche. Also included are some results on the p-adic valuation of partial products of these types, which also contain some patterns of interest.

(continued on back)

1:00–1:35: ***On p-adic valuations of generalized Fibonacci sequences***

Joseph Chavoya, California State University, Fullerton; Alphonso Lucero, Iowa State University; Sean Reynolds, University of Chicago

Abstract: We study the p-adic valuations of generalized Fibonacci sequences, focusing on the particular sequence given by $S_n = F_n + 2L_n$, where F_n and L_n are the Fibonacci and Lucas sequences, respectively. Analyzing this sequence, we create a closed form formula for certain p, as well as formulate conjectures regarding sequences appearing from studying $\nu_p(S_n)$.

1:45–2:20: ***On p-adic limits of combinatorial sequences***

Alexandra Michel, Mills College; Andrew Miller, Amherst College; Robert Joseph Rennie, Reed College

Abstract: Methods for determining p-adic convergence of sequences which are expressible in terms of products of factorials are established. The Catalan sequence is investigated, using these methods, for p-adically convergent subsequences. An infinite class of convergent subsequences of Catalan numbers is found for every prime, and the limits of these subsequences are evaluated.

2:30–3:05: ***On the divisibility and valuations of the Franel numbers***

Abraham Schulte, Northwestern University; Samantha VanSchalkwyk, Mount Holyoke College; Adela Yang, Bowdoin College

Abstract: The Franel numbers are the sums of the cubes of binomial coefficients. This sequence is of great interest. They are the first power for which the sums are not defined by a closed form formula. Primes may be partitioned with respect to the p-adic valuations of Franel numbers: those whose valuation is always 0, those whose valuation is equal to the number of occurrences of a particular digit in base-p, and those which fall into neither category. Furthermore, the 2-adic valuations of the Franel numbers have interesting properties. This talk explores the properties of these numbers.

2014 MSRI-UP Staff

Leyda Almodóvar, University of Iowa, Graduate Student Assistant
Herbert A. Medina, Loyola Marymount University, Program Director
Victor H. Moll, Tulane University, Research Leader
Eric Rowland, University of Liège, Postdoctoral Fellow
Asia Wyatt, University of Maryland, College Park, Graduate Student Assistant

On p-adic valuations of the generalized Fibonacci sequences

Joseph Chavoya California State University Fullerton

Alphonso Lucero Iowa State University

Sean Reynolds University of Chicago

August 2014

Abstract

We study the p-adic valuations of generalized Fibonacci sequences, focusing on the particular sequence given by $S_n = F_n + 2L_n$, where F_n and L_n are the Fibonacci and Lucas sequences, respectively. Analyzing this sequence, we create a closed form formula for certain p, as well as formulate conjectures regarding sequences appearing from studying $\nu_p(S_n)$.

1 Introduction

The Fibonacci and Lucas numbers are well-known sequences given by a second order recurrence that share many identities. Only the initial conditions differ. The Fibonacci numbers, F_n, start with $(0, 1)$ and the Lucas numbers, L_n, with $(2, 1)$. These two initial conditions form a basis for $\mathbb{Z}^2$.

Definition 1.1. Linear combinations of these two sequences, that is, the sequences of the form $f_n = aF_n + bL_n$, are called here *generalized Fibonacci sequences*, denoted by f_n. These satisfy $f_n = f_{n-1} + f_{n-2}$ with the initial conditions $(f_0, f_1) = a(0, 1) + b(2, 1)$.

In order to properly study the powers of primes that divide these generalized Fibonacci numbers and the properties that arise from them, we make extensive use of the p-adic valuation.

Definition 1.2. The *p-adic valuation* of an integer n, denoted by $\nu_p(n)$, is the highest power of p that divides n.

The *p-adic metric*, denoted $|\cdot|_p$, of a number x is defined such that $|x|_p = p^{-\nu_p(x)}$. In particular, define $|0|_p = 0$ for all primes p.

Proposition 1.3 (Properties of $\nu_p(n)$). *For a prime p and integers a, b*

$$\nu_p(ab) = \nu_p(a) + \nu_p(b).$$

If $\nu_p(a) \neq \nu_p(b)$,

$$\nu_p(a+b) = \min(\nu_p(a), \nu_p(b)).$$

Wall [7] shows that the Fibonacci sequence is periodic modulo m for all $m \in \mathbb{N}$. Furthermore, he shows that any natural number is a factor of some Fibonacci number.

Theorem 1.4 (Wall). *For every $m \in \mathbb{N}$, $F_n \bmod m$ forms a periodic sequence.*

Theorem 1.5 (Wall). *For every $m \in \mathbb{N}$, there exists an index n such that $F_n \equiv 0 \bmod m$.*

The p-adic valuations of the Fibonacci and Lucas numbers are well understood. The next result appears in Lengyel [6].

Theorem 1.6 (Lengyel). *Let*

$\alpha(p) =$ *the smallest n such that $F_n \equiv 0 \bmod p$,*

$\pi(p) =$ *the period length of F_n modulo p, and*

$\eta(p) = \nu_p(F_{\alpha(p)})$.

Then, for $p \neq 2$ or 5,

$$\nu_p(F_n) = \begin{cases} \nu_p(n) + \eta(p) & n \equiv 0 \bmod \alpha(p) \\ 0 & \textit{otherwise}, \end{cases}$$

and

$$\nu_p(L_n) = \begin{cases} \nu_p(n) + \eta(p) & \pi(p) \neq 4\alpha(p) \textit{ and } n \equiv \frac{\alpha(p)}{2} \bmod \alpha(p) \\ 0 & \textit{otherwise.} \end{cases}$$

For $p = 2$,

$$\nu_2(F_n) = \begin{cases} 0 & n \equiv 1, 2 \bmod 3 \\ 1 & n \equiv 3 \bmod 6 \\ \nu_2(n) + 2 & n \equiv 0 \bmod 6, \end{cases} \quad \textit{and} \quad \nu_2(L_n) = \begin{cases} 0 & n \equiv 1, 2 \bmod 3 \\ 2 & n \equiv 3 \bmod 6 \\ 1 & n \equiv 0 \bmod 6. \end{cases}$$

Finally, for $p = 5$,

$$\nu_5(F_n) = \nu_5(n) \text{ and } \nu_5(L_n) = 0.$$

Bloom [1] provides a characterization of the period length of generalized Fibonacci sequences modulo m.

Theorem 1.7 (Bloom)**.** *If a generalized Fibonacci sequence* f_n *has a term* f_N *such that* $m \mid f_N$*, then the period of* $f_n \bmod m$ *is equal to the period of* $F_n \bmod m$*.*

However, little is known about the p-adic valuations of the generalized Fibonacci numbers. Given that any generalized Fibonacci sequence can be expressed as $f_n = aF_n + bL_n$, we expect there to be similar results regarding their p-adic valuation. In the next section, we apply identities for F_n and L_n and Proposition 1.3 in order to explicitly calculate $\nu_p(aF_n + bL_n)$.

2 Formulas for generalized Fibonacci sequences

Using the identity $L_n = F_{n-1} + F_{n+1}$, the following is true:

Theorem 2.1.

$$aF_n + bL_n = \begin{cases} 2aF_{n+1} & \text{if } a = b \\ (2a+k)F_{n+1} + kF_{n-1} & \text{if } a < b \\ (2b)F_{n+1} + lF_n & \text{if } a > b. \end{cases}$$

where $k = b - a$ *and* $l = a - b$.

Proof. Case 1: If $a = b$, then

$$\begin{aligned} aF_n + bL_n &= aF_n + aL_n \\ &= aF_n + aF_{n-1} + aF_{n+1} \\ &= aF_{n+1} + aF_{n+1} \\ &= 2aF_{n+1}, \end{aligned}$$

as needed.

Case 2: If $a < b$, then $b = a + k$ for some $k \in \mathbb{Z}$ and

$$\begin{aligned} aF_n + bL_n &= aF_n + (a+k)L_n \\ &= aF_n + (a+k)F_{n-1} + (a+k)F_{n+1} \\ &= aF_n + (a)F_{n-1} + (a)F_{n+1} + kF_{n-1} + kF_{n+1} \\ &= aF_{n+1} + (a)F_{n+1} + kF_{n-1} + kF_{n+1} \\ &= 2aF_{n+1} + kF_{n-1} + kF_{n+1} \\ &= (2a+k)F_{n+1} + kF_{n-1}, \end{aligned}$$

as needed.

Case 3: If $a > b$, then $a = b + l$ for some $l \in \mathbb{Z}$ and

$$\begin{aligned} aF_n + bL_n &= (b+l)F_n + bL_n \\ &= bF_n + bF_{n-1} + bF_{n+1} + lF_n \\ &= bF_{n+1} + bF_{n+1} + lF_n \\ &= 2bF_{n+1} + lF_n, \end{aligned}$$

as needed. □

Example 2.2. If $a = b = 1$, then

$$\begin{aligned} F_n + L_n &= F_n + L_n \\ &= F_n + F_{n-1} + F_{n+1} \\ &= F_{n+1} + F_{n+1} \\ &= 2F_{n+1}. \end{aligned}$$

Example 2.3. If $a = 2$ and $b = 1$, it follows that

$$\begin{aligned} 2F_n + L_n &= F_n + F_{n-1} + F_{n+1} + F_n \\ &= F_{n+1} + F_{n+1} + F_n = 2F_{n+1} + F_n. \end{aligned}$$

(Note that for this particular example, it follows that $2F_{n+1} + F_n = F_{n+2} + F_{n+1} = F_{n+3}$).

Making this observation, it is possible to represent the p-adic valuations of generalized Fibonacci sequences using the well-known p-adic valuation of the Fibonacci sequences.

Corollary 2.4.

$$\nu_p(aF_n+bL_n) = \begin{cases} \nu_p(2aF_{n+1}) & \text{if } a = b \\ \min(\nu_p((2a+k)F_{n+1}), \nu_p(kF_{n-1})) & \text{if } a < b \text{ and } \nu_p((2a+k)F_{n+1}) \neq \nu_p(kF_{n-1}) \\ \min(\nu_p((2b)F_{n+1}), \nu_p(lF_n)) & \text{if } a > b \text{ and } \nu_p((2b)F_{n+1}) \neq \nu_p(lF_n). \end{cases}$$

where $k = b - a$ and $l = a - b$.

Note that this representation only provides clear insight in very few cases, i.e. $a = b$.

Example 2.5. If $a = b$, then

$$\nu_2(aF_n + bL_n) = \nu_2(2aF_{n+1}) = \nu_2(aF_{n+1}) + 1.$$

3 Reduction Modulo Powers of Primes

Proving formulas for p-adic valuations for generalized Fibonacci sequences required a technique called *Reduction Modulo Powers of Primes*. Taking a sequence modulo powers of p for some prime p, is enough to determine the p-adic valuation of the sequence. This technique relied on 3 principles:

Proposition 3.1. *The Fibonacci second order recurrence holds under modular arithmetic.*

Proposition 3.2. *All generalized Fibonacci sequences are periodic modulo m, for all natural numbers m.*

Proposition 3.3. *For an integer x and a prime p, $x \equiv 0 \bmod p^n$ and $x \not\equiv 0 \bmod p^{n+1}$ imply $\nu_p(x) = n$.*

Consider $\{f_n\}$ modulo p. If there are indices n where $f_n \not\equiv 0 \bmod p$, then $\nu_p(f_n) = 0$. Similarly, $\{f_n\}$ is reduced modulo p^2 and indices where p^2 does not divide f_n but p divides f_n are found, and consequently, $\nu_p(f_n) = 1$. This process is repeated until $f_n \not\equiv 0 \bmod p^k$ for all $n \in \mathbb{N}$. From Proposition 3.1 and Proposition 3.2, it can be concluded that this process will hold for all indices n. An illustration of this technique is provided in the proof of the following theorem.

Theorem 3.4. *Let $S_n = F_n + 2L_n$,*

$$\nu_2(S_n) = \begin{cases} 0 & n \equiv 1, 2 \bmod 3 \\ 1 & n \equiv 3 \bmod 6 \\ 2 & n \equiv 0 \bmod 6. \end{cases}$$

Proof. By Proposition 3.1, the recursion formula $S_n = S_{n-1} + S_{n-2}$ holds modulo m, for all $m \in \mathbb{N}$. Now consider $S_n \bmod 2^\alpha$.

n	0	1	2	3	4	5	6	7	8	9	10	11	12	13
S_n	4	3	7	10	17	27	44	71	115	186	301	487	788	1275
$S_n \bmod 2$	0	1	1	0	1	1	0	1	1	0	1	1	0	1
$S_n \bmod 2^2$	0	3	3	2	1	3	0	3	3	2	1	3	0	3
$S_n \bmod 2^3$	4	3	7	2	1	3	4	7	3	2	5	7	4	3

When $\alpha = 1$ notice that if $n \equiv 1$ or $2 \bmod 3$, then $S_n \not\equiv 0 \bmod 2$, and by Proposition 3.3, $\nu_2(S_n) = 0$. For $\alpha = 2$, notice that if $n \equiv 3 \bmod 6$, $\nu_2(S_n) = 1$ by Proposition 3.3. Finally, when $\alpha = 3$, it is clear that $\nu_2(S_n) = 2$ for $n \equiv 0 \bmod 6$ since 2^3 does not divide any term of S_n.

Now, because the recursion formula is maintained, it is clear that there is periodicity, and so the sequence will repeat modulo m if any two terms repeat consecutively. For S_n modulo 2 the period is 3, modulo 2^2 the period is 6, and modulo 2^3 the period is 12. □

4 2-adic Valuations

Based off of Theorem 1.6, the 2-adic valuations of Fibonacci and Lucas numbers are special cases in which an explicit formula for $\nu_2(F_n)$ and $\nu_2(L_n)$ can be determined. This is shown in the fact that $\nu_2(L_n)$ is 0,1, or 2 depending on n. In order to determine $\nu_2(aF_n + bL_n)$, a and b must be known to apply Theorem 1.6 to get a formula for $\nu_2(aF_n + bL_n)$. In the following theorem's we present specific cases of a and b where an explicit formula for $\nu_2(aF_n + bL_n)$ could be determined. Note that in all of these theorems, we relate $\nu_2(aF_n + bL_n)$ to $\nu_2(L_n)$. For other values of a and b, $\nu_2(aF_n + bL_n)$ is unknown.

In choosing arbitrary values for a, b, $\nu_2(aF_n + bL_n)$ was calculated and the following pairs of (a, b) gave explicit formulas similar to $\nu_2(L_n)$:

$$\begin{array}{cccc} (1,2) & (3,2) & (4,1) & (5,1) \\ (1,3) & (3,5) & (4,3) & (5,2) \\ (1,6) & (3,6) & (4,5) & (5,3) \\ (1,7) & (3,7) & (4,7) & (5,6) \\ (1,9) & (3,9) & (4,9) & (5,10) \\ (1,10) & (3,10) & (4,11) & (5,13). \end{array}$$

For the column where $a = 4$ and b odd, the following theorem was developed.

Theorem 4.1. *For odd, positive integer b and $k \geq 2$*

$$\nu_2(2^k F_n + bL_n) = \nu_2(L_n) = \begin{cases} 0 & n \equiv 1, 2 \bmod 3 \\ 2 & n \equiv 3 \bmod 6 \\ 1 & n \equiv 0 \bmod 6. \end{cases}$$

Proof. Consider $\{2^k F_n + bL_n\}$ modulo powers of 2.

n	0	1	2	3	4	5	6	7	8	9	10	11
$2^k F_n$ mod 2	0	0	0	0	0	0	0	0	0	0	0	0
bL_n mod 2	0	1	1	0	1	1	0	1	1	0	1	1
$2^k F_n + bL_n$ mod 2	0	1	1	0	1	1	0	1	1	0	1	1

Since $2^k F_n$ is a multiple of 2, $2^k F_n \equiv 0 \bmod 2$. Also, $b \equiv 1 \bmod 2$ so bL_n mod 2 has the same periodic structure as L_n mod 2. Therefore, for $n \equiv 1, 2 \bmod 2$, $\nu_2(2^k F_n + bL_n) = 0$. For this section choose $b \equiv 3 \bmod 4$. When $b \equiv 1 \bmod 4$, the coming results follow similarly.

n	0	1	2	3	4	5	6	7	8	9	10	11
$2^k F_n$ mod 4	0	0	0	0	0	0	0	0	0	0	0	0
L_n mod 4	2	1	3	0	3	3	2	1	3	0	3	3
bL_n mod 4	2	3	1	0	1	1	2	3	1	0	1	1
$2^k F_n + bL_n$ mod 4	2	3	1	0	1	1	2	3	1	0	1	1

Since $2^k F_n$ is a multiple of 4, $2^k F_n \equiv 0 \bmod 4$. From the table for $n \equiv 0 \bmod 6$, $\nu_2(2^k F_n + bL_n) = 1$. Now all that is left to check is $2^k F_n + bL_n$ mod 8 for $n \equiv 3 \bmod 6$.

For this section choose $b \equiv 3 \bmod 8$. When $b \equiv 1, 5, 7 \bmod 8$, the following results follow similarly. Also, if $k \geq 3$, then $2^k F_n \equiv 0 \bmod 8$ and the result would follow trivially, so assume $k = 2$.

n	0	1	2	3	4	5	6	7	8	9	10	11
$F_n \bmod 8$	0	1	1	2	3	5	0	5	5	2	7	1
$2^k F_n \bmod 8$	0	4	4	0	4	4	0	4	4	0	4	4
$L_n \bmod 8$	2	1	3	4	7	3	2	5	7	4	3	7
$bL_n \bmod 8$	6	3	1	4	5	1	6	7	5	4	1	5
$2^k F_n + bL_n \bmod 8$	6	7	5	4	1	5	6	3	1	4	5	1

Again, the concern is only for $n \equiv 3 \bmod 6$. Here it is observable that $2^k F_n + bL_n$ is not equivalent to 0 mod 8. So $\nu_2(2^k F_n + bL_n) \leq 2$ for all n. Thus the formula for $\nu_2(2^k F_n + bL_n)$ holds.

□

Theorem 4.2. *For $a \in \mathbb{Z}$,*

$$\nu_2(aF_n + 2L_n) = \begin{cases} \nu_2(L_n) + 1 & a \equiv 0 \bmod 8 \\ \nu_2(L_{n+3}) & a \equiv 1 \bmod 2 \\ \nu_2(L_{n+1}) + 2 & a \equiv 10 \bmod 32 \\ \nu_2(bL_{n+2}) + 2 & a \equiv 14 \bmod 32 \\ \nu_2(L_{n+4}) + 2 & a \equiv 18 \bmod 32 \\ \nu_2(L_{n+5}) + 2 & a \equiv 22 \bmod 32. \end{cases}$$

Note that for $a \equiv 2, 4, 6, 12, 20, 24, 26, 28,$ or $30 \bmod 32$, $\nu_2(aF_n + 2L_n)$ is indeterminate, and must be evaluated on a case-by-case basis.

Example 4.3. Taking $a = 3$, it follows that $\nu_2(3F_n + 2L_n) = \nu_2(L_{n+3})$.

Example 4.4. Taking $a = 10$, it follows that $\nu_2(10F_n + 2L_n) = \nu_2(4L_{n+1})$.

Example 4.5. Taking $a = 2$, it follows from Corollary 2.4 that

$$\nu_2(aF_n + 2L_n) = \nu_2(2F_n + 2L_n) = \nu_2(4F_{n+1}) = \nu_2(F_{n+1}) + 2.$$

Theorem 4.6. *For $b \in \mathbb{Z}$,*

$$\nu_2(F_n + bL_n) = \begin{cases} \nu_2(L_{n+3}) & b \equiv 2 \bmod 4 \\ \nu_2(L_{n-1}) + 1 & b \equiv 3 \bmod 16 \\ \nu_2(L_{n-2}) + 1 & b \equiv 9 \bmod 16 \\ \nu_2(L_{n+2}) + 1 & b \equiv 7 \bmod 16 \\ \nu_2(L_{n+1}) + 1 & b \equiv 13 \bmod 16. \end{cases}$$

This theorem partially characterizes the 2-adic valuations of generalized Fibonacci sequences of the form $F_n + bL_n$. These particular values for b were chosen because they reveal an explicit formula. All of these formulas are related to $\nu_2(L_n)$. We suspect that the other values of b mod 16 will have explicit formulas related to $\nu_2(F_n)$ but it could not be determined in general.

5 3-adic Valuations

Some results from the specific case $S_n = F_n + 2L_n$ have been simple to extend to all generalized Fibonacci sequences f_n, and are presented as such.

Theorem 5.1. *There does not exist a generalized Fibonacci sequence f_n such that $\nu_3(f_n) = 0$ for all n.*

Proof. We begin by reducing $f_n = aF_n + bL_n$ modulo 3.

Note $L_2 = 3$, so if $a \equiv 0 \bmod 3$, the proof is complete.

If $b \equiv 0 \bmod 3$, then the proof is complete, as it is well-known that every natural number is a factor of some Fibonacci number.

We are now left with 4 cases, in particular $(a, b) = (1,1), (1,2), (2,1),$ and $(2,2)$.

$$(1,1) \text{ and } (2,1) : F_3 + L_3 = 2 + 4 = 6.$$

$$(1,2) \text{ and } (2,1) : F_1 + 2L_1 = 1 + 2 = 3.$$

□

Given Lengyel's formulas for $\nu_p(F_n)$ and $\nu_p(L_n)$, similar behavior is expected from the sequence $\nu_p(S_n)$, and so we look at it in relation to $\nu_p(n)$. Curiously, there are too many zero terms between the non-zero terms, and so remove remove them as follows.

Definition 5.2. For simplicity, we define $\{\nu_p^*(S_l)\} = \{\nu_p(S_n) \mid \nu_p(S_n) \neq 0\}$. Similarly, $\{\nu_p^*(l)\} = \{\nu_p(n) \mid \nu_p(n) \neq 0\}$.

Example 5.3. The case for $p = 3$: $\nu_3(S_n) \neq 0$ for $n \equiv 1 \bmod 4$, which can be see by examining the period modulo 4 and noticing that the only zero terms in $\nu_3(S_n) \bmod 4$ are when $n \equiv 1 \bmod 4$, and so the sequence $\{\nu_3^*(S_l)\}$ consists of every term in $\{\nu_3(S_n)\}$ where the index is congruent to 1 modulo 4.

Comparing the graphs of $\nu_3^*(S_l)$ and $\nu_3^*(l)$ it becomes evident that the latter is a shifted version of the former, up to some degree of accuracy.

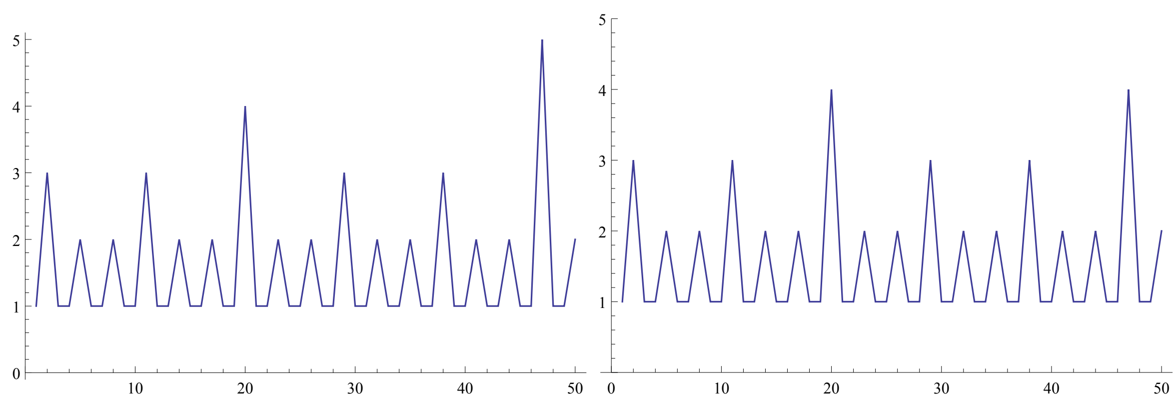


Figure 1: On the left, the first 50 terms of the sequence $\{\nu_3^*(S_l)\}$. On the right, the first 50 terms of the sequence $\{\nu_3^*(l + 23)\}$.

Notice that both of the preceding graphs are precisely equal, except for the 47^{th} term, which is off by 1. More factors can be found that yield increasingly higher accuracy, the first few of which, ignoring multiplicity, are 23, 1805, 174578, 351725. Interestingly, 351725 is accurate up to at least the first 2 million terms in the sequence.

Conjecture 5.4. Fix a prime $p \neq 2$ such that $\nu_p(S_n) \neq 0$. Then, for all $m \in \mathbb{N}$, there exists some K_m such that $\nu_p^*(S_l) = \nu_p^*(l + K_m)$ for all $l \leq m$.

The base-3 expansions of those first terms mentioned are as follows:

$$\begin{aligned} 23 &= 212_3 \\ 1805 &= 2110212_3 \\ 174578 &= 22212110212_3 \\ 351725 &= 122212110212_3. \end{aligned}$$

These seem to be converging to some 3-adic number.

Now considering shifting factors that return the same accuracy, it can be seen that the numbers 23, 104, and 266 all share an accuracy of 46 non-zero terms. Considering all shifting factors that return an accuracy of at least 46 non-zero terms, we see that there appears to be trend in how accurate a particular shifting factor is.

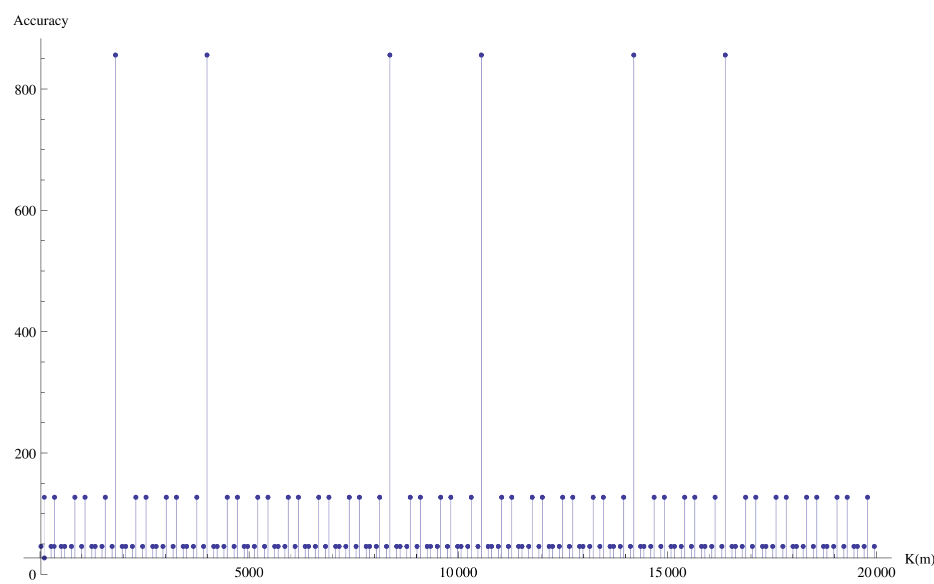


Figure 2: Here we mark the x-axis with the shifting factors of the generalized Fibonacci sequence $F_n + 2L_n$, while the y-axis denotes the accuracy of the shift.

In the figure above, it becomes clear that all shifting factors between 0 and 20,000 repeat the same accuracies, namely 46, 127, and 870. Note that beyond 20,000, these numbers continue to show a similar pattern (differing only by a slow growth), but become increasingly more difficult to compute. Though the idea has not undergone much testing, this trend contributes to the belief that the values of K_m can be determined using a closed-form formula.

Conjecture 5.5. Consider the sequence $\{K_m\}$. If $p = 3$, then $K_m \to c$, where c is some 3-adic number, as $m \to \infty$.

This begs the question: why is 3 special? This leads into the next conjecture.

Conjecture 5.6. (1) Existence of n_i

$$\text{For all natural numbers i, there exists an } n_i \text{ such that } 3^i \mid S_{n_i}.$$

(2) Convergence under 3-adic Metric

$$n_i \to \gamma \text{ as } i \to \infty \text{ for some 3-adic number } \gamma.$$

Attempted proofs for Conjecture 5.6 are as follows.

5.6.1 Binet's formulas for F_n and L_n:

Binet's formulas are as follows:

$$F_n = \frac{\varphi^n - (-\varphi)^{-n}}{\sqrt{5}} \text{ and } L_n = \varphi^n + (-\varphi^{-n}), \text{where } \varphi = \frac{1+\sqrt{5}}{2}.$$

We show that $\sqrt{5} \notin \mathbb{Q}_3$.

First, if $\sqrt{5}$ was in $\mathbb{Z}_3$, then $x^2 \equiv 5 \equiv 2 \bmod 3$. Consideration of $x \equiv 0, 1, 2 \bmod 3$ shows that there are no solutions. So, $\sqrt{5} \notin \mathbb{Z}_3$.

Now, if it were an element of $\mathbb{Q}_3$, then one would see $\nu_3(x^2) = 2\nu_3(x) < 0$. However, notice that $\nu_3(x^2) = \nu_3(5) = 0$, thus providing a contradiction.

As such, Binet's formulas could not be used since there was no ability to discuss $\frac{1+\sqrt{5}}{2}$.

5.6.2 Periods of F_n and L_n Modulo 3^i:

It is known that F_n and L_n are periodic modulo m for any integer m. The goal was to find an index n_i such that $F_n + 2L_n \equiv 0 \bmod 3^i$ for any i, which is equivalent to saying $\{n_i\}$ exists. However, the index where the linear combination yielded a zero modulo powers of 3 could not be determined. Therefore it was not definitive that there exists zeros of $F_n + 2L_n \bmod 3^i$.

5.6.3 The Identity $L_n = F_n + 2F_{n-1}$:

Using the well-known formula above, we have

$$\begin{aligned} S_n &= F_n + 2L_n \\ &= F_n + 2(F_{n+1} + F_{n-1}) \\ &= F_n + 2(F_n + F_{n-1} + F_{n-1}) \\ &= 3F_n + 4F_{n-1}. \end{aligned}$$

Taking the 3-adic valuation of both sides yields

$$\begin{aligned} \nu_3(S_n) &= \nu_3(3F_n + 4F_{n-1}) \\ &= \min(\nu_3(F_n) + 1, \nu_3(F_{n-1})) \text{ if } \nu_3(F_n) + 1 \neq \nu_3(F_{n-1}). \end{aligned}$$

It is well-known that any 3 consecutive Fibonacci numbers are pairwise coprime, and so it is clear that if $3 \mid F_n$, then $3 \nmid 4F_{n-1}$, and thus one can only show that there exist indices such that $\nu_3(S_n) = 0$.

Theorem 5.7. *If the sequence $\{n_i\}$ exists and converges to some 3-adic number γ, then $S_\gamma = 0$ in the 3-adics.*

Proof. By definition, S_γ is infinitely divisible by 3, which is precisely 0 in the 3-adics. □

Theorem 5.8. *Suppose that sequences $\{n_j\}$ and $\{n_k\}$ exist where n_j is the first index such that $3^j \mid S_{n_j}$, and n_k is the first index such that $\nu_3(S_{n_k}) \geq k$. Both of these sequences converge to the same limit as $\{n_i\}$, some 3-adic number γ.*

Proof. It is clear that in the limiting case, both S_{n_j} and S_{n_k} are infinitely divisible by 3. Thus, in the 3-adics, they are equivalent to 0, and thus it is clear that they must also converge to γ. □

Theorem 5.9. *Suppose that one can always find a shifting factor K_m for $p = 3$, then the sequence n_i exists and converges to γ.*

Proof. It is clear that $\nu_3^*(n)$ can be made arbitrarily large by considering powers of 3. So, by considering terms $n = 3^j - K_m$, notice that $\nu_3^*(S_n) = j$ for some arbitrary j. However, $\nu_3^*(S_n)$ is just the removal of the zero terms from $\nu_3(S_n)$, and so $\nu_3(S_n)$ can clearly be made arbitrarily large, and thus, by Theorem 5.8, the proof is complete. □

6 Prime Characterizations

It is well-known that no Lucas number is divisible by 5, and so $\nu_5(L_n) = 0$. Primes that return a p-adic valuation for the sequence $S_n = F_n + 2L_n$ can also be found, and a partial list is as follows.

Theorem 6.1. *If* $p \in \{13, 19, 29, 37, 41, 47, 53, 61, 89, 97, 107\}$, *then* $\nu_p(S_n) = 0$.

Proof. For all $p \in \{13, 19, 29, 37, 41, 47, 53, 61, 89, 97, 107\}$, $S_n \bmod p$ is periodic with period according to the following table.

p	13	19	29	37	41	47	53	61	89	97	107
$\pi(p)$	28	18	14	76	40	32	108	60	44	196	72

Calculating $\nu_p(S_n)$ up to $n = \pi(p)$ shows that because none of the terms are 0, $\nu_p(S_n) \leq 0$, and thus $\nu_p(S_n) = 0$. □

What remains curious are the primes that always return a p-adic valuation of 0. Checking first few terms in the On-Line Encyclopedia of Integer Sequences (OEIS) [5], it became clear that those primes had a particularly interesting property: either $5^q p \pm 6$ was prime for some q, or $p = 5^q p_0 \pm 6$ for some q and some other prime p_0.

However, it became clear that this seemed to be true for any prime p, which leads to the following conjecture.

Conjecture 6.2. If $p \geq 5$ is prime, then at least one of the following holds for some $q \in \mathbb{N}$.

1. At least one of $5p^q \pm 6$ is prime, or
2. p is of the form $p = 5p_0^q \pm 6$ for some other prime p_0.

This has been tested for the first 10 million primes. Curiously, most of the exponents q are quite small, indeed less than 10. Only few, comparatively, require exponents much higher.

It appears as though this can be extended to the following:

Conjecture 6.3. Given primes p_0 and $\tilde{p}$ such that $\tilde{p} \pm 1$ and p_0 are relatively prime, so $(\tilde{p} \pm 1, p_0) = 1$, then there exists some integer $q \geq 0$ such that

$$\tilde{p}^q p_0 \pm (\tilde{p} \pm 1)$$

is prime. Furthermore, every prime can be expressed in this form for some p_0 and $\tilde{p}$.

7 Completeness of Lucas Numbers

Are there any natural numbers which cannot be expressed in terms of Lucas numbers? If we were to allow multiplicity, then the statement is trivial, as 1 is a Lucas number. What happens if this is disallowed? Brown [2] provides information on sequences which can be used to express every natural number in a non-trivial manner.

Definition 7.1. A sequence $\{b_n\}$ is *complete* if every natural number can be expressed by summing the terms of a subsequence $\{b_{n_i}\}$. That is,

$$k = \sum_{i=0}^{\infty} \delta_i b_i,$$

where $\delta_i = 0$ or 1.

Theorem 7.2 (Brown)**.** *Without loss of generality, assumed the sequence $\{a_n\}$ is nondecreasing. $\{a_n\}$ is complete if, and only if, the following criteria are met:*

1. $a_0 = 1$
2. *The partial sums $s_{k-1} \geq a_k + 1$ for all $k \geq 1$, where $k \in \mathbb{N}$*

Corollary 7.3 (Brown)**.** *If $a_0 = 1$ and $2a_n \geq a_{n+1}$ for all $n \geq 1$, where $n \in \mathbb{N}$, then the sequence $\{a_n\}$ is complete.*

Theorem 7.4. *The Lucas numbers, L_n, are complete.*

Proof. Let $\{L'_n\}$ be the set of Lucas numbers arranged in nondecreasing order. In this case, we can define $L'_n = L'_{n-1} + L'_{n-2}$ for all natural numbers $n \geq 4$.

The first requirement of Corollary 7.3 is clearly true, and so the only trouble is the second requirement. The first cases where the recurrence does not hold follow as such:

$$2L'_0 = 2 = L'_1$$
$$2L'_1 = 4 \geq 3 = L'_2$$
$$2L'_2 = 6 \geq 4 = L'_3.$$

Now, given any natural number $k \geq 4$, it is clear that $L'_k \geq L'_{k-1}$. It follows that

$$L_k \geq L_{k-1}$$
$$2L_k \geq L_{k-1} + L_k$$
$$2L_k \geq L_{k+1}.$$

This, together with the first cases where the recurrence does not hold, proves that the second requirement is satisfied. Now, this shows that $\{L'_n\}$ is complete. However, the ordering on the set does not affect the completeness, and thus it follows that $\{L_n\}$ is complete. □

We have shown that every natural number can be represented by the Lucas numbers, but is there anything interesting about those representations? For the Fibonacci numbers, Zeckendorf [8] was able to provide a characterization of the representations for the Fibonacci sequence.

Theorem 7.5 (Zeckendorf)**.** *Any natural number has a unique representation of the form*

$$n = \sum_{i=0}^{\infty} \varepsilon_i F_i,$$

where $\varepsilon_i = 0$ *or* 1*, and* $\varepsilon_i \varepsilon_{i+1} = 0$*.*

It was later shown by Daykin [4] that the Zeckendorf represenation was also a characterization of the Fibonacci sequence.

Theorem 7.6 (Daykin)**.** *If* $\{a_n\}$ *is a sequence with unique Zeckendorf representations, then* $\{a_n\}$ *is strictly increasing and* $\{a_n\} = \{F_n\}$*.*

What has yet to be seen is whether or not there is a Zeckendorf representation in the Lucas numbers. The proof that these representations do exist follows similarly to the proof of Zeckendorf's Theorem given in [3].

Theorem 7.7. *Every natural number n has a Zeckendorf representation in the Lucas numbers.*

Proof. We proceed by induction. If $n = 1, 2, 3$, or 4, then the representation is clear, as those are all Lucas Numbers. Now, take $k = 5$. Then we have

$$n = 5 = 1 + 4 = L_1 + L_3.$$

Suppose that every natural number $n \leq k$ has a Zeckendorf representation.

Now, for $n = k + 1$, we have two possibilities.

Case 1: If $k + 1$ is a Lucas number, then the proof is complete.

Case 2: If $k + 1$ is not a Lucas number, then there exists some j such that $L_k < k + 1 < L_{j+1}$. Consider $a = k + 1 - L_j$. Because $a < k$, a has a Zeckendorf representation in the Lucas numbers.

$$L_j + a = k + 1$$

$$L_j + a < L_j + L_{j-1}$$

$$a < L_{j-1}.$$

From this, it is clear that the Zeckendorf representation of a in the Lucas numbers does not contain L_{j-1}. As such, the Zeckendorf representation of $k + 1$ is the representation of a in the Lucas numbers plus L_j. Thus, every natural number has a Zeckendorf represenation in the Lucas numbers. □

It is clear from Theorem 7.6 that despite the fact that there is a Zeckendorf representation in the Lucas numbers, it won't be unique. For example, we could take $12 = L_1 + L_5 = L_0 + L_2 + L_4$.

Furthermore, there is no Zeckendorf representation for generalized Fibonacci sequences as we could take $(f_0, f_1) = (1, 3)$, and thus there is no representation of 2.

8 Acknowledgments

This work was carried out during the 2014 Mathematical Sciences Research Institute Undergraduate Program (MSRI-UP) which is funded by the National Science Foundation (grant No. DMS-1156499) and the National Security Agency (grant No. H-98230-13-1-0262).

We would like to thank Dr. Herbert Medina, Dr. Victor Moll, Dr. Eric Rowland, and Asia Wyatt for their invaluable help with our research and work. The first author would like to thank Dr. Scott Annin from California State University Fullerton for the inspiration, knowledge, and support he has provided. The second author would like to thank Dr. Eric Weber from Iowa State University for providing mentorship and support during the author's undergraduate career. The third author would like to thank Dr. Baoping Liu from the University of Chicago for his unyielding support and excellent tutelage.

References

[1] D. M. Bloom, "On Periodicity in Generalized Fibonacci Sequences," *Amer. Math. Monthly* **72** (1965) 557–560.

[2] J. L. Brown, Jr., "Note on Complete Sequences of Integers," *Amer. Math. Monthly* **68** (1961) 243-252

[3] J. L. Brown, Jr., "Zeckendorf's Theorem and Some Applications," *Fibonacci Quart.* **6** (1964) 163–168

[4] D. E. Daykin, "Representation of Natural Numbers as Sums of Generalized Fibonacci Numbers," *J. Lond. Math. Soc.* **35** (1960) 143–160

[5] OEIS Foundation Inc. (2011), The On-Line Encyclopedia of Integer Sequences, `https://oeis.org/A023219`

[6] T. Lengyel, "The Order of the Fibonacci and Lucas Numbers," *Fibonacci Quart.* **33** (1995) 234–239.

[7] D. D. Wall, "Fibonacci Series Modulo m," *Amer. Math. Monthly* **67** (1960) 525–532.

[8] E. Zeckendorf, "Représentation des nombres naturels par une somme des nombres de Fibonacci ou de nombres de Lucas," *Bull. Soc. Roy. Sci. Liège* **41** (1972) 179–182

Instrument Analysis

Sign out
My home page
Edit my profile
Privacy policy
Legal statement
User forum
Tips and tricks
Help

Instrument dashboard

On this page, you can view and download a summary of results from one or more SALG instruments. You can also view results across instruments.

Results displayed for the following instrument:

ID	Open / Close	Instructor	Course	Semester	Description	N	
66364	Wed Jul 30, 2014 / Sat Aug 02, 2014	Herbert Medina	MSRI-UP 2014	Summer 2014	Research Experience for Undergraduates at MSRI in Berkeley, CA.	18	download

If you'd like to aggregate data across instruments, you can add another instrument to this analysis.

Cross-tabulate questions ?

Frequency distributions of scale results

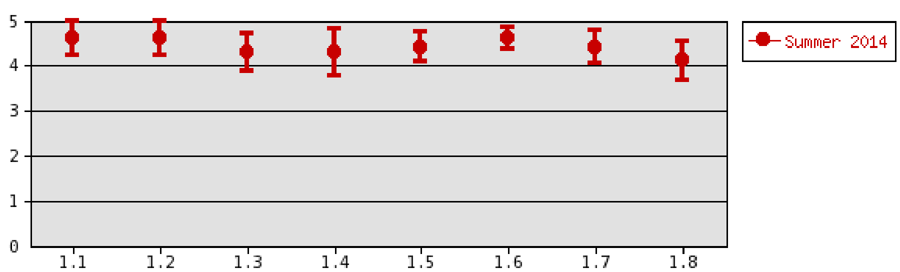
The table below lists the percentage of students responding in each category, along with the mean and number of responses for that item. If you'd like an more detailed analysis, click on the 'details' link to the right of that item.

Gains in THINKING AND WORKING LIKE A MATHEMATICIAN OR A STEM PROFESSIONAL: APPLICATION OF KNOWLEDGE TO RESEARCH WORK.

1. How much did you GAIN in the following areas as a result of your most recent research experience?	1:no gains	2:a little gain	3:moderate gain	4:good gain	5:great gain	9:not applicable	Mean	N	
1.1 Analyzing data for patterns.	0%	6%	0%	17%	72%	6%	4.6	17	details
1.2 Figuring out the next step in a research project.	0%	6%	6%	17%	72%	0%	4.6	18	details
1.3 Problem-solving in general.	0%	6%	11%	33%	50%	0%	4.3	18	details
1.4 Formulating a research question that could be answered with data.	6%	0%	17%	17%	61%	0%	4.3	18	details
1.5 Identifying limitations of research methods and designs.	0%	0%	11%	33%	56%	0%	4.4	18	details
1.6 Understanding the theory and concepts guiding my research project.	0%	0%	0%	44%	56%	0%	4.6	18	details
1.7 Understanding the connections among mathematical disciplines.	0%	0%	17%	28%	56%	0%	4.4	18	details
1.8 Understanding the relevance of research to my coursework.	0%	6%	22%	33%	39%	0%	4.1	18	details

Summary of scale results

The graphic below lists the mean and confidence interval (±3 times the standard error) for each item.



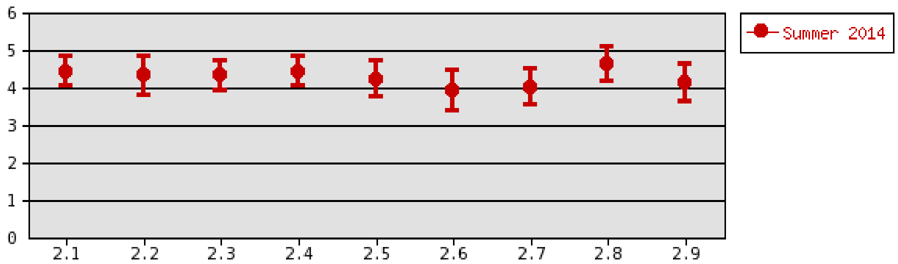
PERSONAL GAINS RELATED TO RESEARCH WORK

2. How much did you GAIN in the following areas as a result of your most recent research experience?	1:no gains	2:a little gain	3:moderate gain	4:good gain	5:great gain	9:not applicable	Mean	N	
2.1 Confidence in my ability to do research.	0%	6%	6%	33%	56%	0%	4.4	18	details
2.2 Confidence in my ability to contribute to mathematics.	6%	6%	0%	33%	56%	0%	4.3	18	details
2.3 Comfort in discussing mathematical concepts with	0%	0%	22%	28%	50%	0%	4.3	18	details

others.									
2.4 Comfort in working collaboratively with others.	0%	0%	22%	17%	61%	0%	4.4	18	details
2.5 Confidence in my ability to do well in future math courses.	0%	6%	17%	22%	44%	11%	4.2	16	details
2.6 Ability to work independently.	6%	6%	17%	33%	33%	6%	3.9	17	details
2.7 Developing patience with the slow pace of research.	0%	11%	17%	33%	39%	0%	4.0	18	details
2.8 Understanding what everyday research work is like.	6%	0%	0%	22%	72%	0%	4.6	18	details
2.9 Taking greater care in conducting procedures in the lab or field.	0%	6%	11%	28%	33%	22%	4.1	14	details

Summary of scale results

The graphic below lists the mean and confidence interval (±3 times the standard error) for each item.

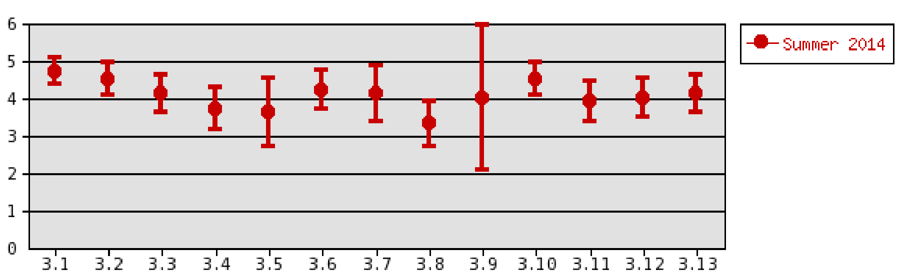


Gains in SKILLS

3. How much did you GAIN in the following areas as a result of your most recent research experience?	1:no gains	2:a little gain	3:moderate gain	4:good gain	5:great gain	9:not applicable	Mean	N	
3.1 Writing mathematical reports or papers.	0%	6%	0%	17%	78%	0%	4.7	18	details
3.2 Making oral presentations.	0%	6%	11%	11%	72%	0%	4.5	18	details
3.3 Defending an argument when asked questions.	0%	11%	17%	22%	50%	0%	4.1	18	details
3.4 Explaining my project to people outside my field.	6%	6%	28%	28%	28%	6%	3.7	17	details
3.5 Preparing a mathematics poster.	6%	6%	11%	11%	17%	50%	3.6	9	details
3.6 Keeping a detailed lab notebook.	0%	0%	17%	17%	28%	39%	4.2	11	details
3.7 Conducting observations in the lab or field.	0%	6%	11%	6%	28%	50%	4.1	9	details
3.8 Using statistics to analyze data.	6%	0%	33%	17%	6%	39%	3.3	11	details
3.9 Calibrating instruments needed for measurement.	0%	6%	0%	0%	11%	83%	4.0	3	details
3.10 Working with computers.	0%	11%	0%	17%	72%	0%	4.5	18	details
3.11 Understanding journal articles.	6%	6%	11%	39%	33%	6%	3.9	17	details
3.12 Conducting database or internet searches.	6%	0%	17%	39%	33%	6%	4.0	17	details
3.13 Managing my time.	6%	6%	6%	44%	39%	0%	4.1	18	details

Summary of scale results

The graphic below lists the mean and confidence interval (±3 times the standard error) for each item.



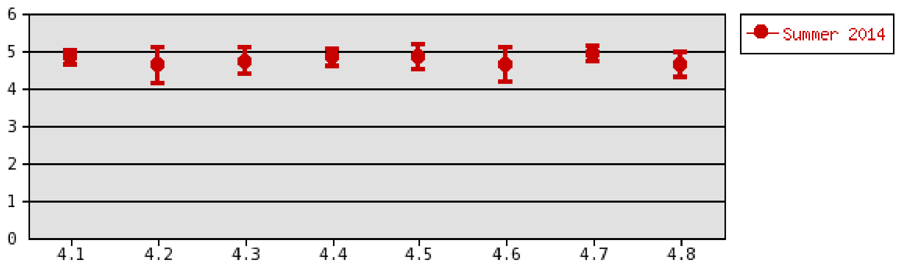
The following questions ask about your overall research experience and about any changes in your attitudes or behaviors as a researcher.

4. During your research experience HOW MUCH did you:	1:none	2:a little	3:some	4:a fair amount	5:a great deal	9:not applicable	Mean	N

4.1 Engage in real-world mathematics research	0%	0%	0%	22%	78%	0%	4.8	18	details
4.2 Feel like a mathematician.	6%	0%	6%	6%	83%	0%	4.6	18	details
4.3 Think creatively about the project.	0%	6%	0%	11%	83%	0%	4.7	18	details
4.4 Try out new ideas or procedures on your own.	0%	0%	6%	6%	89%	0%	4.8	18	details
4.5 Feel responsible for the project.	0%	6%	0%	6%	89%	0%	4.8	18	details
4.6 Work extra hours because you were excited about the research.	0%	11%	0%	11%	78%	0%	4.6	18	details
4.7 Interact with mathematicians from outside your school.	0%	0%	6%	0%	94%	0%	4.9	18	details
4.8 Feel a part of a mathematics community.	0%	0%	11%	17%	72%	0%	4.6	18	details

Summary of scale results

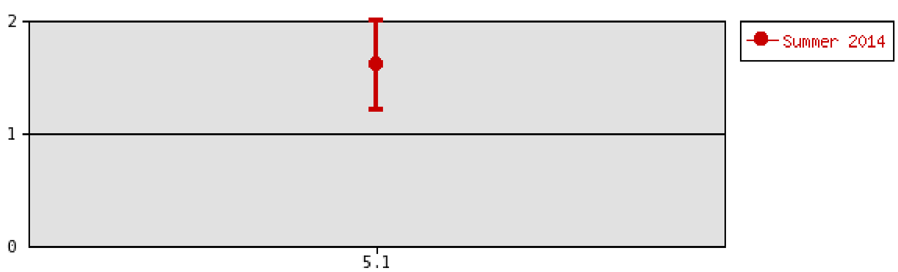
The graphic below lists the mean and confidence interval (±3 times the standard error) for each item.



5. Previous mathematical sciences research experience	1:Never participated	2:1 summer	3:2 summers	4:3 summers	: :	Mean	N	
5.1 How many times have you participated in SUMMER research, excluding this one?	61%	28%	6%	6%		1.6	18	details
5.2 How many times have you participated in academic year research in the mathematical sciences?	Enter codes for text answers					--	15	details
5.3 After the research experience at MSRI-UP, do you see yourself participating in other undergraduate research projects or continuing to work on your project from this summer? Please answer "Yes" or "No" and provide a one sentence explanation for your answer.	Enter codes for text answers					--	18	details
5.4 Only answer this question if you have participated in previous undergraduate research projects in the mathematical sciences. How does your research experience in MSRI-UP compare to your other research experience(s) in the mathematical sciences?	Enter codes for text answers					--	9	details

Summary of scale results

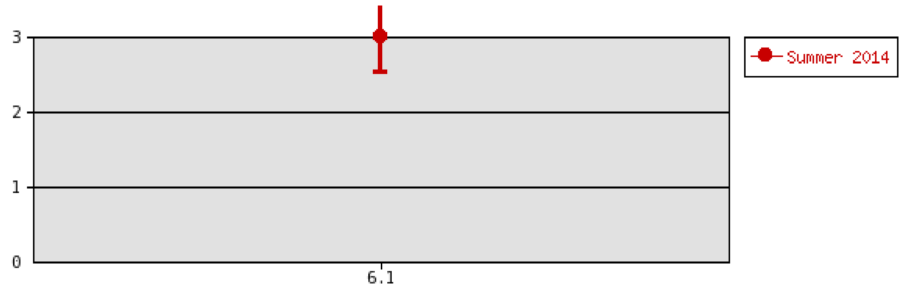
The graphic below lists the mean and confidence interval (±3 times the standard error) for each item.



6. Stipend	1:Not at all important	2:Slightly important	3:Important	4:Very important	: :	Mean	N	
6.1 How important was the stipend or money you were paid in allowing you to do research?	17%	6%	39%	39%		3.0	18	details

Summary of scale results

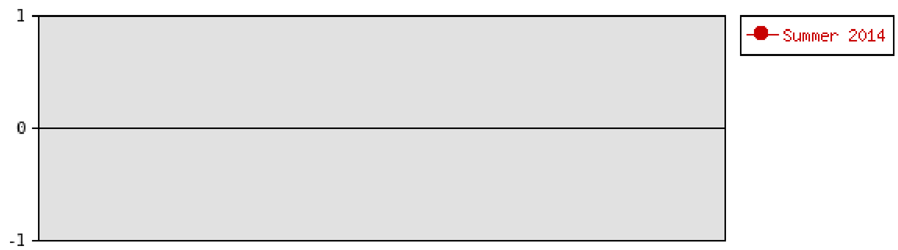
The graphic below lists the mean and confidence interval (±3 times the standard error) for each item.



The following questions about the guidance and support you received during the program.

7. Please answer each of the following about help, guidance and support received during the different parts/phases of the program.	1:too much, I would have liked to do more on my own.	2:the correct amount	3:not enough	: : :	Mean	N	
7.1 The help, support, feedback and encouragement received from the program staff during the first two weeks of the program was	6%	94%	0%		--	18	details
7.2 The help, support, encouragement from the TAs in the evening study sessions was	11%	61%	28%		--	18	details
7.3 The help, support, guidance and encouragement from the staff received by our research group was	17%	83%	0%		--	18	details
7.4 The help, guidance and support in preparing our research paper was	0%	100%	0%		--	18	details
7.5 The help, guidance and support in preparing our final presentation was	0%	94%	6%		--	18	details

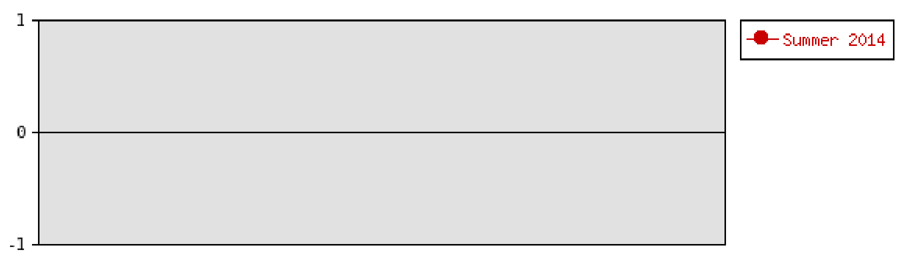
Summary of scale results
The graphic below lists the mean and confidence interval (±3 times the standard error) for each item.



Mathematical Level of MSRI-UP

8. Please answer the following about the mathematical level of the program.	1:way too challenging	2:challenging and hence at the right level	3:challenging, but could have been more so	4:not very challenging	5:too easy.	:	Mean	N	
8.1 The mathematical level of the first two weeks of the program was	0%	67%	28%	0%	6%		--	18	details
8.2 The computational level of our group's research project was	0%	72%	6%	22%	0%		--	18	details
8.3 The mathematical level of our group's research project was	0%	89%	11%	0%	0%		--	18	details

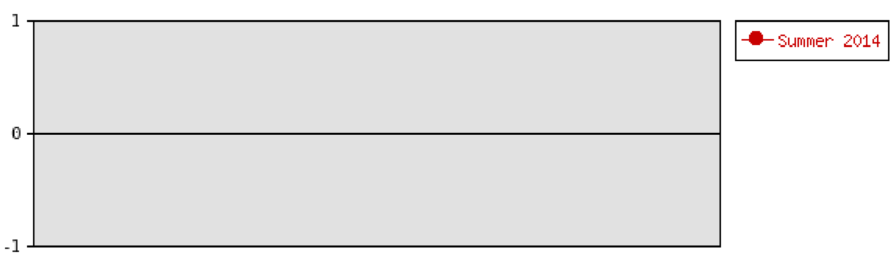
Summary of scale results
The graphic below lists the mean and confidence interval (±3 times the standard error) for each item.



These questions ask you how successful you feel MSRI-UP was at achieving certain objectives/goals.

9. MSRI-UP has been __________	1:very successful	2:successful	3:somewhat successful	4:unsuccessful	5:completely unsuccessful	:	Mean	N	
9.1 at providing me with a rich research experience in the mathematical sciences.	89%	11%	0%	0%	0%		--	18	details
9.2 at familiarizing me with mathematics experimentation and research protocols/ techniques.	72%	28%	0%	0%	0%		--	18	details
9.3 at increasing my desire and motivation to pursue a graduate degree in the mathematical sciences.	78%	17%	0%	0%	6%		--	18	details
9.4 at in making me aware of strategies that will maximize the likelihood of admission to graduate programs best suited to my goals/needs/aspirations.	67%	17%	17%	0%	0%		--	18	details
9.5 in making me aware of programs/strategies that will help me to secure financial support for graduate school.	44%	33%	22%	0%	0%		--	18	details
9.6 in assisting me to begin to build a network of faculty mentors and peers that can assist me with future educational and career plans.	61%	22%	11%	6%	0%		--	18	details
9.7 through the colloquium series, at giving me a glimpse of active areas in the mathematical sciences.	56%	28%	11%	6%	0%		--	18	details

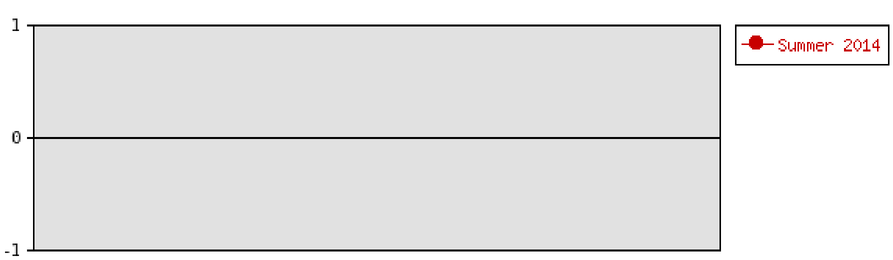
Summary of scale results
The graphic below lists the mean and confidence interval (±3 times the standard error) for each item.



Friday morning workshops

10. Looking towards next year's program, please tell us how you feel about keeping/not keeping each of the Friday morning workshops in this year's program.	1:definitely keep	2:keep, but don't feel strongly about it	3:don't need so don't do this one next year	:	:	:	Mean	N	
10.1 LaTeX	61%	33%	6%				--	18	details
10.2 Graduate school & fellowship application	94%	6%	0%				--	18	details
10.3 MSRI-UP alumni now current graduate students	50%	39%	11%				--	18	details
10.4 GRE Mathematics subject exam	94%	6%	0%				--	18	details
10.5 Is there another workshop that you'd like to suggest for next year's program?	Enter codes for text answers						--	8	details

Summary of scale results
The graphic below lists the mean and confidence interval (±3 times the standard error) for each item.



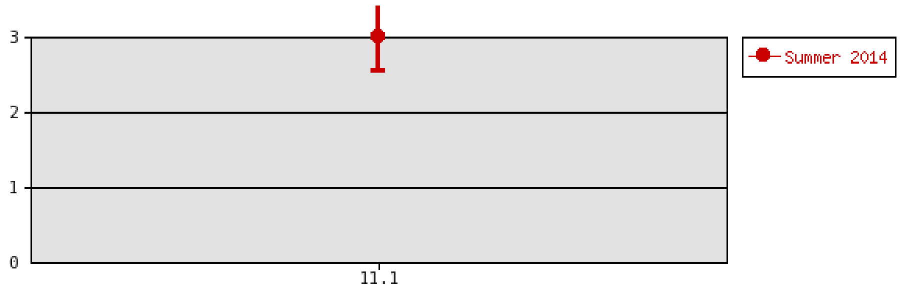
Importance of Location

11. How important was it in your decision to participate in MSRI-UP that the program was at the Mathematical Sciences Research Institute in Berkeley, CA.	1:Not at all important	2:Slightly important	3:Important	4:Very important	:	:	Mean	N	
11.1 Provide level of importance in your decision to choose this program.	6%	33%	17%	44%			3.0	18	details

11.2 Comment on the adequacy of MSRI for this program. e.g., classroom, offices, library, computers, wifi, afternoon tea, etc.	Enter codes for text answers	--	17	details

Summary of scale results

The graphic below lists the mean and confidence interval (±3 times the standard error) for each item.

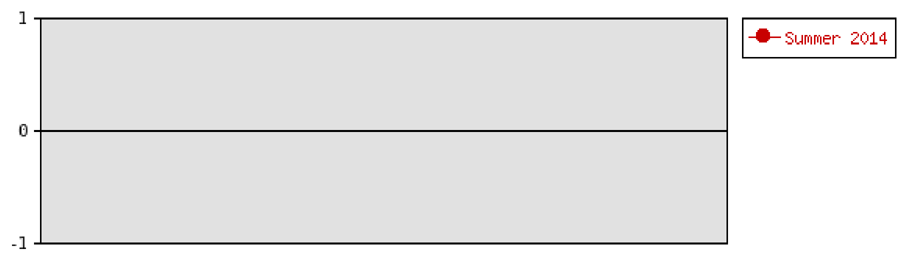


Satisfaction with results

12. Please state how satisfied you are with the results of your work this summer.	1:very satisfied	2:satisfied	3:not satisfied	: : :	Mean	N	
12.1 Research results	72%	28%	0%		--	18	details
12.2 Written technical report	50%	44%	6%		--	18	details
12.3 Oral presentation	56%	39%	6%		--	18	details

Summary of scale results

The graphic below lists the mean and confidence interval (±3 times the standard error) for each item.

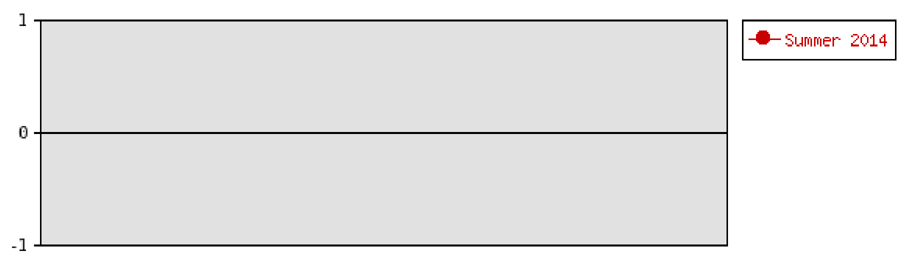


Overarching Comments

13. Please tell us anything else you'd like about the following.	1:No	2:Yes	9:not applicable	: : :	Mean	N	
13.1 Do you feel that MSRI-UP has positively influenced your outlook on your academic future?	6%	94%	0%		--	18	details
13.2 If you answered "Yes" to the previous, comment on how. If you answered "No", please explain.	Enter codes for text answers				--	18	details
13.3 Is there anything else you'd like to tell us?	Enter codes for text answers				--	8	details

Summary of scale results

The graphic below lists the mean and confidence interval (±3 times the standard error) for each item.



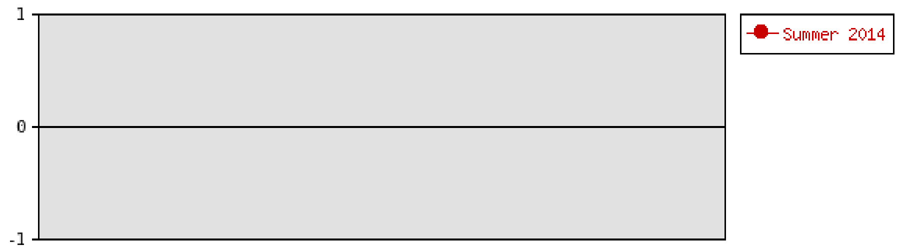
Some demographic information

14. Gender	1:Male	2:Female	3:Decline to answer	: : :	Mean	N	
14.1 What is your gender?	56%	44%	0%		--	18	details

14.2 Other	Enter codes for text answers	--	--

Summary of scale results

The graphic below lists the mean and confidence interval (±3 times the standard error) for each item.

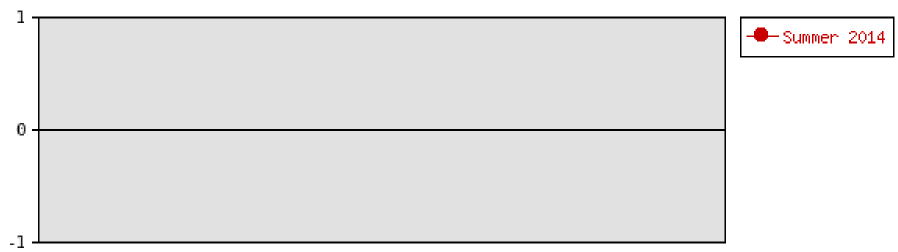


.

15. Race	1:Native American	2:Asian American	3:African American	4:White	5:Other	6:Decline to answer	Mean	N	
15.1 What is your race?	0%	11%	11%	39%	33%	6%	--	18	details
15.2 Other	Enter codes for text answers						--	6	details

Summary of scale results

The graphic below lists the mean and confidence interval (±3 times the standard error) for each item.

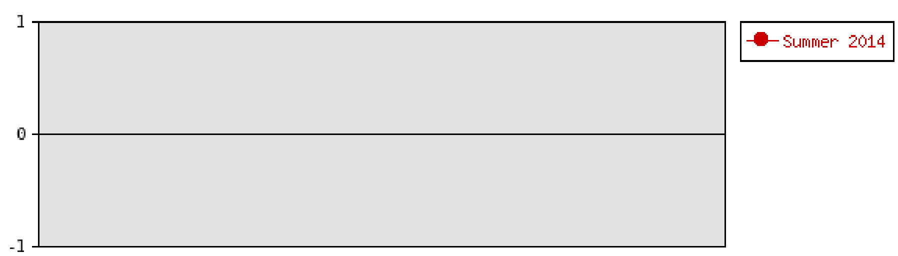


.

16. Ethnicity	1:Hispanic	2:Non-Hispanic	3:Decline to answer	:	:	:	Mean	N	
16.1 What is your ethnicity?	50%	44%	6%				--	18	details

Summary of scale results

The graphic below lists the mean and confidence interval (±3 times the standard error) for each item.



.

17. What year are you in college?	1:Freshman/rising sophomore	2:Sophomore/rising junior	3:Junior/rising senior.	4:Senior	5:Other	:	Mean	N	
17.1 I am a:	0%	33%	61%	6%	0%		--	18	details

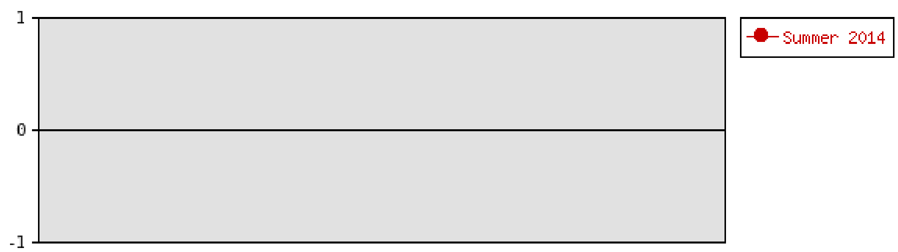
Summary of scale results

The graphic below lists the mean and confidence interval (±3 times the standard error) for each item.

.

18. GPA	1:3.5 - 4.0	2:3.0 - 3.49	3:2.5 - 2.99	4:2.0 - 2.49	5:Below 2.0	6:Don't know	Mean	N	
18.1 What is your GPA?	72%	22%	0%	0%	0%	6%	--	18	details
18.2 What is your GPA in your math courses?	78%	17%	0%	0%	0%	6%	--	18	details

Summary of scale results
The graphic below lists the mean and confidence interval (±3 times the standard error) for each item.



.

19. Other demographics	1:Yes	2:No	3:Decline to answer	:	:	:	Mean	N	
19.1 Are you a first generation college student?	44%	56%	0%				--	18	details
19.2 Are you a student at a 2-year community college?	0%	100%	0%				--	18	details
19.3 Do you have a disability?	0%	94%	6%				--	18	details

Summary of scale results
The graphic below lists the mean and confidence interval (±3 times the standard error) for each item.