

Summer Graduate School
Gaps between Primes and Analytic Number
Theory

July 13, 2015 - July 24, 2015

MSRI, Berkeley, CA, USA

Organizers:

Dimitris Koukoulopoulos (Université de Montréal)

Emmanuel Kowalski (ETH Zuerich)

James Maynard (University of Oxford)

Kannan Soundararajan (Stanford University)

MSRI SUMMER GRADUATE SCHOOL FINAL REPORT
"GAPS BETWEEN PRIMES AND
ANALYTIC NUMBER THEORY"
JULY 13 TO 24, 2015

Organizers and Lecturers

- D. Koukoulopoulos (Montréal)
- E. Kowalski (Zürich, lead organizer)
- J. Maynard (Montréal-Oxford)
- K. Soundararajan (Stanford)

Teaching assistants

- Z. Brady (Stanford)
- B. Löffel (ETH Zürich)

Presentation

The objectives of this summer school were to present in detail the recent achievements of Y. Zhang, J. Maynard and others concerning gaps between primes, and their further applications concerning distribution of primes (including "large" gaps between primes). Almost no formal prerequisites in analytic number theory was expected from the students, and the courses developed from scratch all relevant material. An auxiliary goal was for the school to serve as introduction to the topics of the semester programme in analytic number theory which has been proposed to MSRI for the Spring 2017 semester, and which is organized by C. David, A. Granville, E. Kowalski, Ph. Michel, K. Soundararajan and T. Tao.

The contents of the school were as follows:

- Four 6-hour long minicourses, by the four lecturers:
 - (1) "Introduction to prime number theory, ζ and L-functions, and the prime number theorem" (D. Koukoulopoulos)
 - (2) "The Bombieri-Vinogradov theorem and introduction to sieve theory" (K. Soundararajan)
 - (3) "The methods of Goldston, Pintz and Yıldırım and of Maynard-Tao for gaps between primes" (J. Maynard)
 - (4) "Distribution of arithmetic functions in arithmetic progressions and exponential sums over finite fields" (E. Kowalski).
- Extensive tutoring and exercise sessions led by the two TAs;
- Mini-projects and presentations that were suggested by the lecturers and prepared by small groups of students, with presentations during the last few days of the school. A majority of students participated in one of these, and a few more advanced graduate students presented work from their ongoing PhD theses.

- A presentation and discussion by F. Su (Harvey-Mudd College), current president of the MAA, on “Life as a faculty member”.

Contents of the minicourses

Introduction to prime number theory, ζ and L-functions, and the prime number theorem (D. Koukoulopoulos)

These lectures, held for the most part during the first week, were designed to introduce students to the basic analytic theory of the distribution of primes, up to and including the Siegel-Walfisz Theorem. Not only are these results used as fundamental tools in the other courses, but this was also the occasion to present concretely many basic ideas and methods of analytic number theory, especially for those students with background in other fields.

The presentation used the modern approach recently devised by D. Koukoulopoulos himself (building on ideas of Granville and Soundararajan) that avoids the use of complex function theory beyond its most elementary aspects. Although this more classical approach (due essentially to Hadamard and de la Vallée Poussin) is important in its own right, the choice allowed the lecturer to give essentially full proofs without requiring any “black box”.

The Bombieri-Vinogradov theorem and introduction to sieve theory (K. Soundararajan)

The goal of this lecture series was to present, again in full detail, the proof of the Bombieri-Vinogradov theorem, which is another main ingredient in the recent progress on gaps between primes. The result was also clearly situated in its context as an unconditional alternative to the Generalized Riemann Hypothesis. Both as a prerequisite and as additional topics, the course also introduced sieve methods and some of their applications, especially the large sieve in its arithmetic and analytic forms.

The methods of Goldston, Pintz and Yıldırım and of Maynard-Tao for gaps between primes (J. Maynard)

The original work of Y. Zhang proving the existence of bounded gaps between primes depended on two essential tools: (1) the method of Goldston, Pintz and Yıldırım for “detecting” small gaps; (2) an extension of the Bombieri-Vinogradov Theorem beyond the range allowed by the large sieve. However, J. Maynard (and independently T. Tao) discovered that a more efficient implementation of the first part could be used to circumvent the second (so that only the Bombieri-Vinogradov Theorem was needed). In fact, the method of Maynard and Tao was able to deduce much stronger statements on bounded intervals containing many primes. All of these topics, including some of the most recent developments concerning large gaps, were discussed in depth in this minicourse.

Distribution of arithmetic functions in arithmetic progressions and exponential sums over finite fields (E. Kowalski)

MSRI GRADUATE SCHOOL

Although the distribution theorems of Y. Zhang on primes in arithmetic progressions are not necessary anymore for proving the existence of bounded gaps, they retain considerable interest in view of the importance of the study of arithmetic functions in arithmetic progressions to large moduli, beyond the range allowed by the large sieve or the Generalized Riemann Hypothesis. This minicourse presented some general problems and results concerning this research area, especially with respect to the multiple divisor functions. In particular, it explained using examples how such questions naturally lead to exponential sums over finite fields, and to the Riemann Hypothesis over finite fields. A survey of the latter was presented, highlighting in particular the statement and application of a general form of Deligne's Theorem, as a quasi-orthogonality statement for certain special functions over finite fields.

Student Presentations

Among the presentations given by the students were the following topics:

- Probabilistic number theory (e.g., Erdős-Kac Theorem);
- The asymptotic ternary Goldbach problem;
- Stepanov's proof of special cases of the Riemann Hypothesis over finite fields;
- Introduction to modular forms;
- Some consequences of the Riemann Hypothesis;
- Introduction to the circle method;
- Primes of the form $x^2 + ny^2$.

as well as some more talks presenting their PhD projects by already advanced graduate students.

Student feedback

The mathematical background and level of the students was very varied, ranging from beginning graduate students in fields unrelated to analytic number theory to very advanced students close to the completion of their PhDs.

From the discussions and interactions of the lecturers and TAs with the students, it seems that the lectures were well followed. The many exercises prepared by the TAs were particularly appreciated, and a number of students requested that the exercise sheets be posted on the web page of the school.

The TAs had extensive interactions with the students during the exercise and discussion sessions that were held most of the afternoons, and that were also used to prepare the student presentations. The following feedback comes from these interactions, as reported by the TAs:

- With respect to the exercise sessions, it was a challenge that the students had very different backgrounds. Some were already familiar with the material and the techniques, while for others almost everything was completely new.

- It was very good that the students could give small talks. Since they were relatively free in choosing the topic, they could choose something which matched their skill level.
- It was important that the exercises covered a variety of topics, so that every student could choose what they needed to practice. It was not expected that all students could solve all exercises, although the best ones probably did most of them.
- For less experienced students, exercises focusing on specific topics were most helpful.
- The discussion sessions were times when students could solve exercises, look up material that they did not understand during the lectures, prepare their short, and ask question to the TAs.
- Although most it was felt that most students learnt a lot, there were a few who, having only recently started their PhD, did seem to be a bit lost.
- The atmosphere in the exercise sessions was very good. In particular, some of the less experienced students felt a bit uncomfortable asking questions in the presence of the professors; some also feared a bit to give a talk in front of the professors, but most of them agreed to present something, even if it was not very advanced, and seemed to have good experiences from this exercise.

The TAs also pointed out that a potential improvement to the way the school was organized would have been to be able to prepared exercises in advance, especially with respect to preparing solutions before the beginning of the school. Unfortunately, such coordination is not easy, especially in a summer school of this kind with four fairly interdependent topics.

Organizers		
First Name	Last Name	Institution
Dimitris	Koukoulopoulos	Université de Montréal
Emmanuel	Kowalski	Eidgenössische TH Zürich-Hönggerberg
James	Maynard	University of Oxford
Kannan	Soundararajan	Stanford University
Teaching Assistants		
First Name	Last Name	Institution
Zarathustra	Brady	Stanford University
Benny	Loeffel	Eidgenössische TH Zürich-Hönggerberg

Participants		
First Name	Last Name	Institution
Charles	Alley	University of Illinois at Chicago
Yang	An	Columbia University
Andrea	Arauz	University of California, Riverside
David	Armour	Baylor University
Juan	Auli	San Francisco State University
chandan	biswas	University of Wisconsin-Madison
Chen	Chen	University of Nevada
Kwok Chi	Chim	Technische Universität Graz
Richard	Frnka	Louisiana State University
Zachary	Harrison	Arizona State University
Catherine	Hsu	University of North Carolina
Hang	Huang	University of Wisconsin-Madison
Daniel	Hudson	University of Victoria
Joseph	Hughes	University of California
Monsikarn	Jansrang	Central Michigan University
Aashita	Kesarwani	Tulane University
Kim	Klinger-Logan	University of Minnesota Twin Cities
Frank	Kloster	University of California, Riverside
Watson	Ladd	University of California, Berkeley
Matt	Lam	Harvey Mudd College
Paul	Lewis	Columbia University
Zane	Li	University of California
Misty	Long	Kansas State University
Tianyi	Mao	CUNY, Graduate Center
Nathan	McNew	Dartmouth College
Xianchang	Meng	University of Illinois at Urbana-Champaign
Mehrzad	Monzavi	University of Texas
Ngai fung	Ng	Purdue University
Abhishek	Parab	Purdue University
Hans	Parshall	University of Georgia
Sarah	Peluse	Stanford University
James	Phillips	University of Virginia
David	Richman	University of Michigan
Adrian	Scheerer	Technische Universität Graz
Lee	Troupe	University of Georgia
Ling-Sang	Tse	University of Toronto
Leila	Vaden	University of Oregon
Eric	Wawerczyk	University of Notre Dame
Matthew	Welsh	Rutgers University
Shuntaro	Yamagishi	University of Waterloo
Ruixiang	Zhang	Princeton University

Officially Registered Student Information

Participants		47
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Gender		47
Male	74.47%	35
Female	25.53%	12
Declined to state	0.00%	0

Ethnicity*		53
White	47.17%	25
Asian	35.85%	19
Hispanic	3.77%	2
Pacific Islander	0.00%	0
Black	0.00%	0
Native American	0.00%	0
Mixed	5.66%	3
Declined to state	7.55%	4

* ethnicity specifications are not exclusive

31 responses

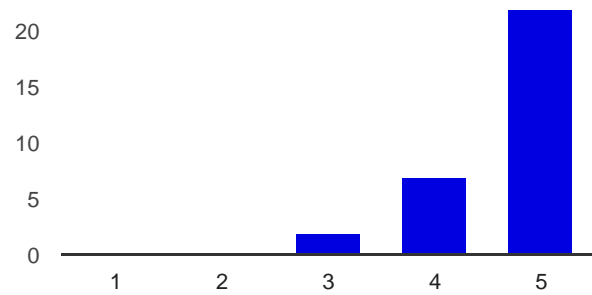
31 responses/47 Participants = 65%

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Summary

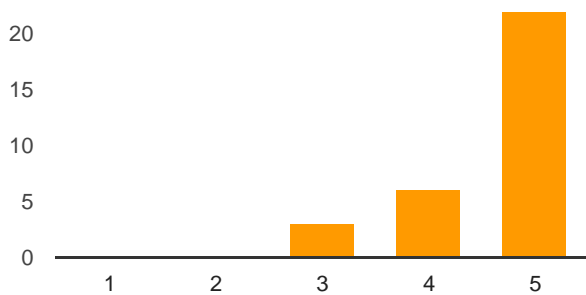
Topic presentation and organization

The various topics within the summer school integrated into a coherent picture



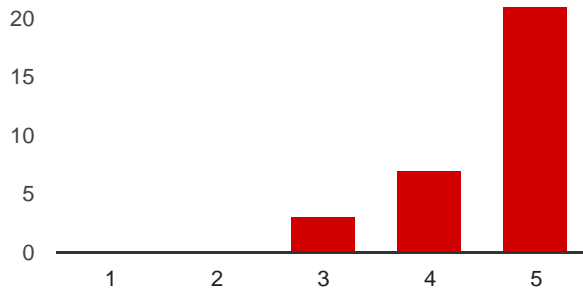
Not at all: 1	0	0%
2	0	0%
3	2	6.5%
4	7	22.6%
Very much: 5	22	71%

The faculty speakers were generally clear and well organized in their presentation



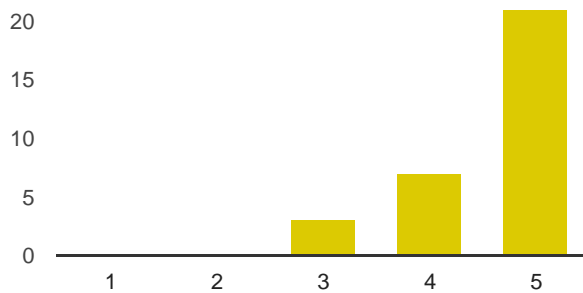
Not at all: 1	0	0%
2	0	0%
3	3	9.7%
4	6	19.4%
Very much: 5	22	71%

The school was intellectually stimulating



Not at all: 1	0	0%
2	0	0%
3	3	9.7%
4	7	22.6%
Very: 5	21	67.7%

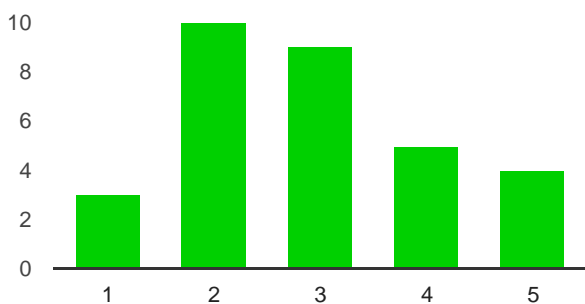
The overall experience of the school was worthwhile



Not at all: 1	0	0%
2	0	0%
3	3	9.7%
4	7	22.6%

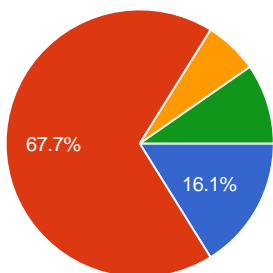
Very: 5 **21** 67.7%

The TA sessions were helpful



Not at all: 1	3	9.7%
2	10	32.3%
3	9	29%
4	5	16.1%
Very much: 5	4	12.9%

The amount of material presented was:



Too much	5	16.1%
Just the right amount	21	67.7%
Not enough	2	6.5%
No opinion	3	9.7%

Additional comments on the topic presentation and organization

Everything was excellent. I just hope we can have a third week to cover the materials left about Zhang's proof (it is hard and long, but you won't have an easy chance to learn and understand it if your thesis is not on any directly relevant topic). I have read both Maynard and Zhang's papers and admire both. And in this summer school I understood both papers much better thanks to the

speakers. But it was yet not enough for the whole picture. That's why I would like an extra week. I am a bit disappointed that faculty did not suggest enough (actually probably none at all) interesting open problems, hard or easy, for us to work on. The projects were like reading and we have to find stuff to work on on our own. Though this is understandable as this topic is fairly new.

Everything was great except for the TA sessions. The TAs seemed unprepared and unable to encourage conversation.

I thought the presentations were really good! It really helped me to understand the material throughout the workshop. It was clear and had right amount of material. I learned a lot and I really enjoyed my overall experience at this summer school!

The 4 lecturers were fantastic. It was an honor to have the chance to learn from them. In the future I think a more structured discussion hour would be helpful.

The latter lectures by Dimitris Koukoulopoulos were at a level and speed a bit beyond the audience.

The lectures eventually moved faster than I was able to keep up with, but I used the online video recordings to go back and learn things I didn't catch the first time around during lecture. Really glad to have that resource! Some TA sessions could have benefitted from a more structured presentation style; it sometimes felt like the TAs did not prepare that much for the problem sessions.

The topics worked well together and there was much interplay between different talks. I feel that the whole two weeks were well planned and prepared.

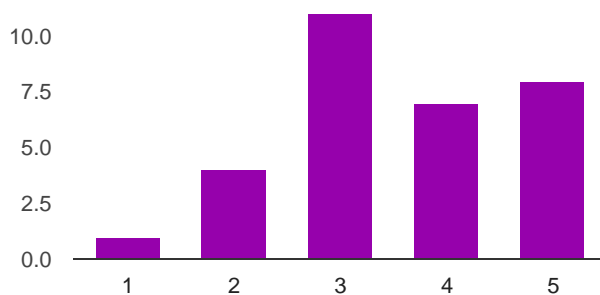
Sound and Kowalski were extremely well prepared and clear. I learned quite a bit from their lectures.

The TA's were not well prepared. Some exercises were not correct, with mistakes that were not simple typos. The lectures were good.

It would be helpful if the lecturers would announce the topics a day in advance so we could skim the the material to prepare.

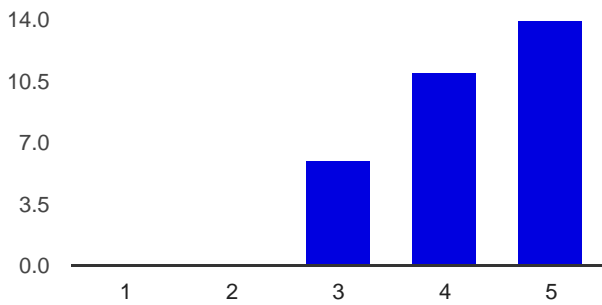
Personal assessment

I was well prepared to benefit from the school



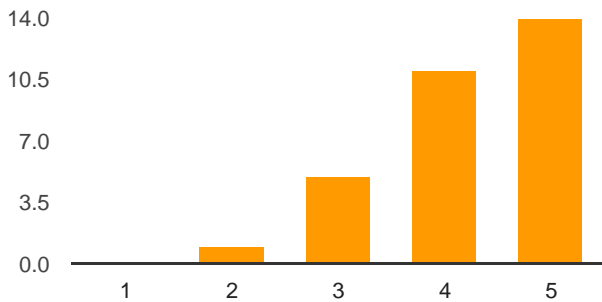
Not at all: 1	1	3.2%
2	4	12.9%
3	11	35.5%
4	7	22.6%
Very: 5	8	25.8%

My interest in the subject matter was increased by the school



Not at all: 1	0	0%
2	0	0%
3	6	19.4%
4	11	35.5%
Very much: 5	14	45.2%

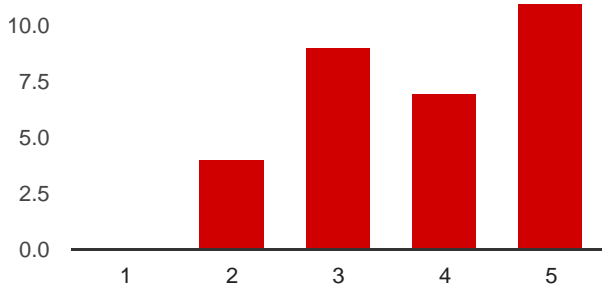
The school helped me meet people with similar scientific interests



Not at all: 1	0	0%
2	1	3.2%
3	5	16.1%

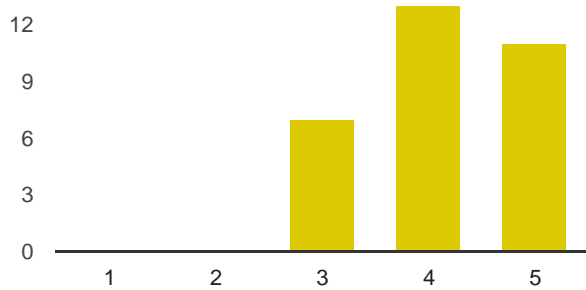
4 **11** 35.5%
 Very much: 5 **14** 45.2%

It is likely that I will work in the area of the school subject in the future



Not at all: 1 **0** 0%
 2 **4** 12.9%
 3 **9** 29%
 4 **7** 22.6%
 Very: 5 **11** 35.5%

How would you evaluate your interaction with other participants?



not satisfactory: 1 **0** 0%
 2 **0** 0%
 3 **7** 22.6%
 4 **13** 41.9%
 above satisfactory: 5 **11** 35.5%

Additional comments on your personal assessment

The summer school was great! Since it is a very important topic, I thought that there would be more grad students attending. It would have been better if there were more attendees, since then it would be easier to find other students at your level to discuss the math with.

I am very satisfied. You can see I did not prepared well to benefit much as this was what I was working on a year ago, not now. However the clear presentation and interaction with others inspired me a lot in this direction, which was unexpectedly amazing.

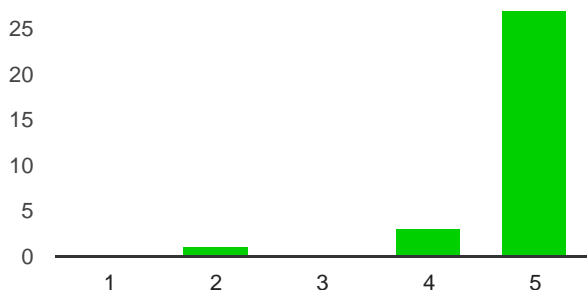
I had very little experience in analytic number theory before this summer school, and I think as a result of that I fell behind in the second week. But I am very much interested in this field after learning [some of] the material from this summer school.

Most of the others were very agreeable and I look forward to seeing them at other conferences.

This topic is somewhat tangential to my field but overall I think that the experience was quite helpful.

MSRI Venue

I found the MSRI staff helpful



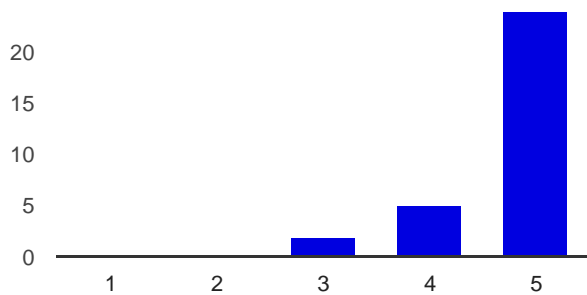
Not at all: 1	0	0%
2	1	3.2%
3	0	0%
4	3	9.7%
Very: 5	27	87.1%

The MSRI physical facilities were conducive for such a school



Not at all: 1	0	0%
2	1	3.2%
3	2	6.5%
4	3	9.7%
Very: 5	25	80.6%

The MSRI computer facilities were adequate for such a school



Not at all: 1	0	0%
2	0	0%
3	2	6.5%
4	5	16.1%
Very: 5	24	77.4%

Additional comments on the MSRI venue

loved the library! and the views, of course

Great location.

Having to walk 5 minutes for a cigarette was annoying, but this is my only complaint.

I don't know how MSRI got the venue at that location, but they are very lucky. The surroundings are beautiful, the building is great, and the library is spectacular in the number of books and journals.

I found it very difficult to work at msri. The library is very nice but either we should be allowed to take books into area's with chalkboards and where we can eat and drink coffee while reading, or there should be chalkboards in the library and we should be allowed to bring coffee into the library. Also the quality of the coffee was very low. Furthermore, there is no cellphone reception

which made it difficult for me to stay and work as i was trying to stay in contact with my family.

It is great. People were telling me the material in the library was not comprehensive enough. I personally found it fine. It was a bit annoying that we only had 2 computers with access to most electronic scientific literature. But other aspects of the whole venue are next to perfect.

Mathematicians turn coffee into theorems. It was great to have some, but i think there should be more of a variety because the coffee served was pretty bad. I also would have liked to have coffee in the lecture hall during the presentations.

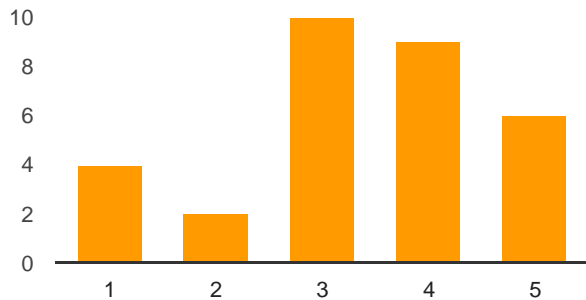
I am in love with the facilities at MSRI. The venue is beautiful.

MSRI was amazing, especially the library. I hope to come back.

Beautiful! Thank you for hosting!

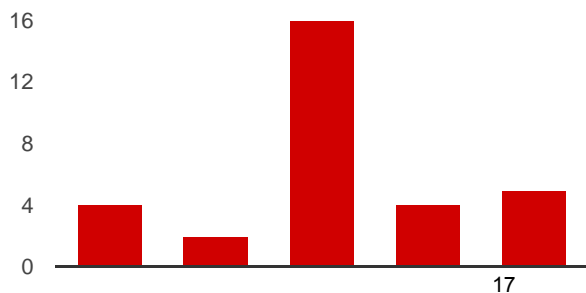
Accommodation and Food

The summer school accommodation



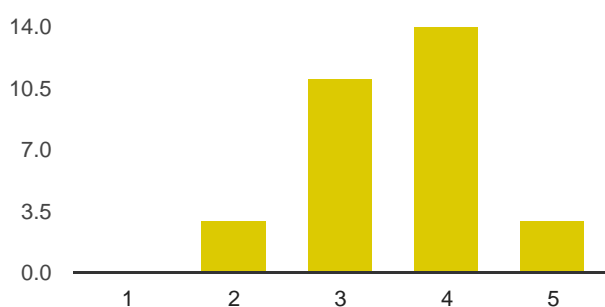
not satisfactory: 1	4	12.9%
2	2	6.5%
3	10	32.3%
4	9	29%
above satisfactory: 5	6	19.4%

The food at the dormitories



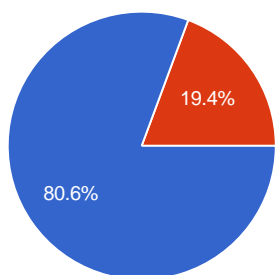
not satisfactory: 1	4	12.9%
2	2	6.5%
3	16	51.6%
4	4	12.9%
above satisfactory: 5	5	16.1%

The food provided at MSRI



not satisfactory: 1	0	0%
2	3	9.7%
3	11	35.5%
4	14	45.2%
above satisfactory: 5	3	9.7%

Did you prefer the lunch provided at MSRI in week one or week two



Week 1	25	80.6%
Week 2	6	19.4%

Additional comments on accommodation and food

The dorms are fine. It was a bit annoying that we don't have indoor restrooms. Also the soundproof was poor so I can hear people talking loudly late in the night in our lounge. I believe

they moved when I complained to them. Other aspects were good. The lunch in week 2 was much worse than the lunch in week 1 and I personally think the standard in week 2 should be improved. The food we had in week 1 was amazing though. They should at least have fresh water for week 2.

The dorms at Foothills are very nice. The cafeteria food is just that, cafeteria food. The first catering service for lunch seemed to think more about what would satisfy the majority of the people. They provided salad for every meal in case the main course did not look appealing, their sandwich strategy was "build your own" (which I appreciated more than the ready made sandwiches in week 2), and they even made Coq au Vin on the Friday. The second week was closer to cafeteria food and had less variety.

Both weeks were equally fine in my opinion.

I am a grown man. I had to sleep in a room with three other men, without a refrigerator or microwave. There was no privacy and it was impossible to do work because it was so uncomfortable. I could not even sit up in my bed or roll over without hitting the ceiling. The food made me feel ill and generally was of the lowest quality, further impeding my ability to work. By the second week i had stopped eating in the dining hall except to drink some coffee, which i had no way to prepare in my dorm suite. Again, the food at the dorms made me ill. It was completely unacceptable. Future accommodations should include private rooms, with at least a common and easily accessible refrigerator, microwave, and coffee maker or electric kettle. Also, most of my roommates were morning people, while i enjoy staying up late to work and talk about problems. It would be nice if participants could be grouped according to their preferred working hours so as not to disturb each other.

It got rather noisy sometimes where I was staying. My room in the dormitory was on the ground floor by a street and there were people talking near my window when I wanted to go to bed...

The dorm beds are awful, but I guess that's to be expected. The food at dinner (in the dorm cafeteria) seemed like it had been cooked hours before hand (which, considering that when I looked at the menus online lunch and dinner at the cafeteria were the same, that may well have been the case). Worst of all though, was the fact that the dorm room was infested with ants (before we arrived). It was very unpleasant, especially with ants crawling all over the walls while I tried to take a shower. Overall the accommodations really detracted from what should've been a fantastic two weeks.

The lunch menu was slightly better the first week, though I thought that the lunch for the second week was much healthier and less oily.

lunch both weeks was quite good.

The vegetarian options during week 2 were not very good. The lunch was often something unhealthy.

Sleeping in the dorms was a complete nightmare. There were several high school camps staying there, and almost every night a different group would have a huge party with loud music in the rec room across from our suite. These parties routinely went to 11 PM or later. There were nights where even with ear-plugs I could hear every word of the music they were playing. If there wasn't music, then it was just high school kids screaming for hours and hours (somehow.) Several times I went down and asked the adults in charge if they could turn down the music (or

at least not point the speakers right at our window) because people were trying to sleep, and my reasonable requests were met with rudeness. I complained to the front desk people and they said that they would reprimand the groups, but nothing changed. There were so many camp groups that even if they could somehow identify what group it was from my complaints, the next night there would just be another horrible party with different people. It was so bad that I made the multiple-hour long public transportation journey home to my apartment in the middle of the program to try to catch up on sleep. Not being able to get enough sleep really impacted how much I was able to get out of the program because it's hard to think when you're sleep-deprived. The residents at the dorm were very loud late at night.

week 2 lunch at MSRI is much worse.

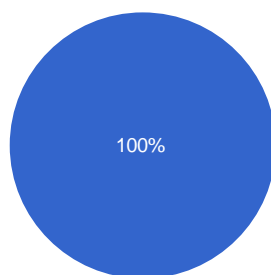
The two caterers were about the same to me. I thought the first day (make your own sandwiches) and barbeque were subpar.

Thank you for completing this survey

We welcome any additional comments or suggestions you may have to improve the overall experience for future participants.

Thanks for the investigation! One last thing: the 8:40am hill line is usually packed. Maybe we can have a shuttle for us exclusively next time?

Better coffee and better food are the main things that need improvement. Once again, the food in the dorms made me ill. I could not eat it. This led to me spending considerable money trying to find acceptable food, which i then had no way to store in my room because i did not have a refrigerator.



Option 1 12 100%

Number of daily responses

