Spring 2017

EMISSARY

Mathematical Sciences Research Institute

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Analytic Number Theory

Emmanuel Kowalski

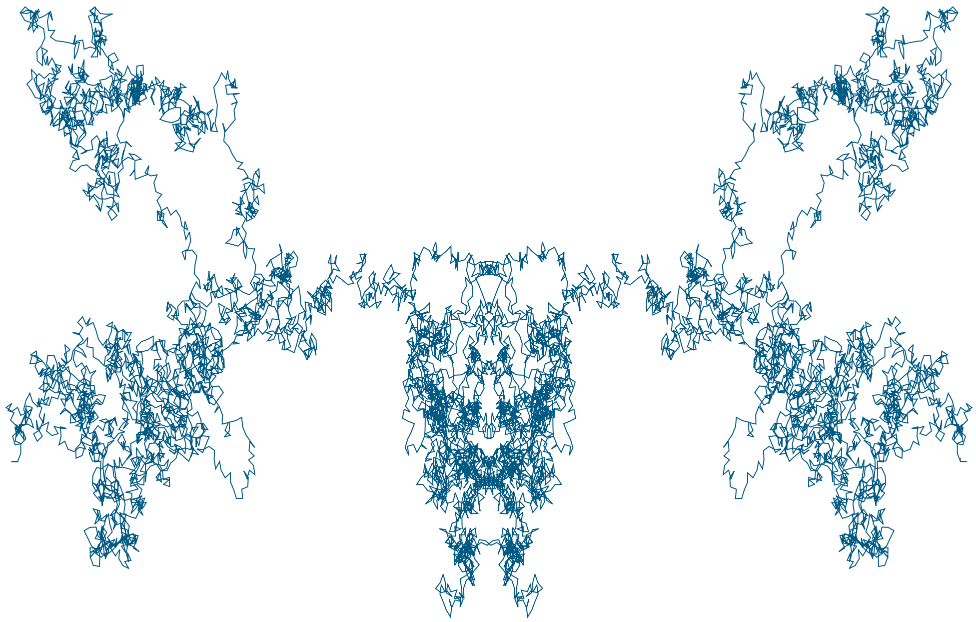
The basic problem in analytic number theory is often to compute how many integers, or primes, or other arithmetic objects (from L-functions to modular forms to elliptic curves to class groups to ...), satisfy certain conditions. Such questions have fascinated mathematicians for centuries; for instance, one can ask:

(a) How many prime numbers are there less than some large given number x? How many $n \leqslant x$ are there such that the interval from n to $n+246$ contains at least two prime numbers?

(b) Fixing integers $k \geqslant 2$ and $r \geqslant 1$, for a large integer n, how many integers n_1, ..., n_r are there such that $n_1^k + \cdots + n_r^k = n$ (Waring's problem)?

(c) How many integers in an interval $x < n \leqslant x + x^{1/2}$ are only divisible by primes smaller than $x^{1/20}$? or larger than $x^{1/1000}$? or both?

(d) How many integers $n \leqslant x$ are such that $n^4 + 2$ is not divisible by the square of any prime?



A Kloosterman path joins the partial sums of $\mathrm{Kl}_2(1;10007)$ over $1 \leqslant x \leqslant k$ when k varies from 0 to 10006. (The figure appears with axes on page 5.)

The techniques that are used in analytic number theory are remarkably varied and cunning (combinatorics, harmonic and complex analysis, probabilistic ideas, ergodic theory, algebraic geometry, representation theory, all — and others — find their use). And the influence of such problems on mathematics often goes well beyond the original motivation of solving them. For instance, complex analysis benefited in the 19th and early 20th century from the efforts devoted to the proof of the Prime Number Theorem, from Riemann to Hadamard and de la Vallée Poussin. Similarly, representation theory took some of its inspiration from Dirichlet's study of primes in arithmetic progressions; much more recently, the ideas of higher-order Fourier analysis were to a large extent pursued in connection with problems about linear equations in primes, in particular in the recent work of Green, Tao, and Ziegler.

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The View from MSRI

David Eisenbud, Director

Time Scales of the Programs

Like annual flowers, new programs and workshops bloom at MSRI each spring and fall. Other Institute activities have different rhythms. Here are some reflections on the long and short time scales.

The Scientific Advisory Committee (SAC) meets twice a year in person, each time for two days, and one agendum each time is a discussion of potential semester-long programs, either submitted from outside or generated from within, that MSRI might run in the future. It's a surprisingly distant future: because of the planning and resources that are necessary, we generally approve programs in place three years ahead of when they will run, and the process of vetting and accepting a program, from letter of intent to final signoff, can add a year or more.

We also take account of "hot topics" with workshops planned a year or less in advance. This spring's Hot Topics workshop, on Motivic Periods, was planned last year, and the Insect Navigation workshop we ran in the fall with the Howard Hughes Medical Institute was planned at even shorter range. The conferences on Critical Issues in Math Education, the Summer Graduate Schools . . . all these are planned more quickly than the semester-long programs.

One planning operation takes surprisingly long (surprising, anyway to me): the National Math Festival. The "first ever" took place in April 2015. We are organizing the second Festival for April 22, 2017, and we've been working on it ever since the close of the first Festival! I believe the work has paid off: You can see what's coming at nationalmathfestival.org. About 25,000 people attended the public day of the first Math Festival — we expect a still larger crowd this time. Come and join the fun if you can!

Time Scales of the Institute

MSRI held its first programs nearly thirty-five years ago, in the fall of 1982 (they were Nonlinear Partial Differential Equations, organized by A. Chorin, I. M. Singer, S.-T. Yau, and Mathematical Statistics, organized by L. LeCam, D. Siegmund, C. Stone). This is not quite a mathematical lifetime ago: many of the members and postdocs of the programs are now leaders in the field (and such a postdoc, David Donoho, has recently joined MSRI's Board of Trustees). These thirty-five years have been long enough for MSRI to have established itself as an essential part of the mathematical establishment of the United States and of the world. In recent time the number of applications for MSRI's semester programs, workshops and schools has risen steadily, and now far outstrips what we can provide.

On the other hand, thirty-five years is short in the lives of great institutions. MSRI is still highly dependent on just a few sources of funding, and thus vulnerable to changes outside the mathematical community. Given the importance that MSRI has for the community, it seems to me of the greatest importance to lay the groundwork for a broader base of support, one that will ensure the Institute's continued health over the next 35 years and beyond.

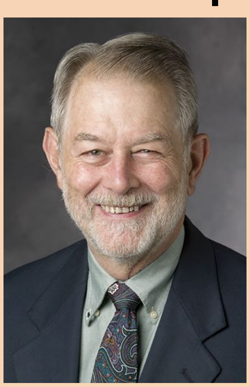
Personal Time Scales — Ars Longa, Vita Brevis

I first came to be Director at MSRI in 1997 — 20 years ago this summer. I retired from the job (and Robert Bryant became Director) in 2007, and after which I taught, and worked for the Simons Foundation, until taking a new term as Director in 2013. Now I've accepted a renewal until 2022. I'm enjoying the job, and there's much I hope to accomplish. But at the end of this next term I will be 75, and, both for MSRI's sake and my own, it will be time to turn over the reins!

My aspirations for the next five years? First and foremost, to continue the breadth and excellence that has characterized MSRI's programs. But, also, to plan for the long-range future by increasing the diversity of MSRI's funding. We've already made progress: In the 2017–18 budget, just passed by the Board of trustees, 48% of the funding will come from sources other than the federal government. This is exactly double the percentage of 2012–13, five years ago, and the total is up as well.

The new funding is made up of endowment, grants from private foundations, and individual giving. Thanks to all those whose gifts and help have made this expansion a reality! Your generosity serves mathematics in a crucial way. I believe that this diversification, if cultivated and developed, will make MSRI sustainable in the long run, so that the Institute can continue to play a central role in the mathematical life of the country and the world.

CME Group–MSRI Prize in Innovative Quantitative Applications



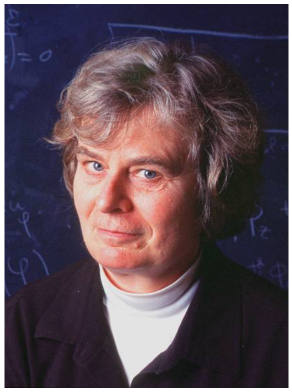
Robert B. Wilson

The eleventh annual CME Group–MSRI Prize in Innovative Quantitative Applications was awarded to Robert B. Wilson, Adams Distinguished Professor of Management, Emeritus, and Professor of Economics (by courtesy), School of Humanities and Sciences, Stanford Graduate School of Business, at a luncheon in Chicago on February 2, 2017.

Dr. Wilson's research and teaching are on market design, pricing, negotiation, and related topics concerning industrial organization and information economics. He is an expert on game theory and its applications. A panel discussion on "Frontiers of Game-theoretic Applications in Economics" was held in conjunction with the award ceremony.

The annual CME Group–MSRI Prize is awarded to an individual or a group to recognize originality and innovation in the use of mathematical, statistical, or computational methods for the study of the behavior of markets, and more broadly of economics. You can read more about the CME Group–MSRI Prize at tinyurl.com/cme-msri.

The New Uhlenbeck Fellowship


Karen Uhlenbeck

As we announced in the previous issue of the Emissary, an anonymous donor has made possible two new named fellowships at MSRI, created in honor of Dr. Karen Uhlenbeck and Dr. Dusa McDuff, both current MSRI trustees.

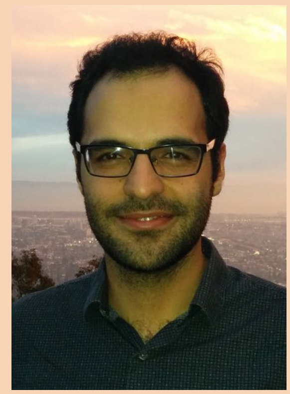
Postdoctoral fellows at MSRI have the unique opportunity to work together with leading researchers in their fields in an environment that promotes creativity and the effective interchange of ideas and techniques. Privately funded fellowships play an important part in providing these opportunities. The first McDuff Fellowship was awarded last fall, and this spring marks the first awarding of the new Uhlenbeck Fellowship.

Karen Uhlenbeck, currently a trustee of MSRI, received her Ph.D. from Brandeis University in 1968. Following appointments at MIT, UC Berkeley, the University of Illinois, and the University of Chicago, she has held since 1987 the Sid W. Richardson Foundations Regents Chair III in Mathematics at the University of Texas at Austin. From 2014–17, she is a visiting Professor at the Institute for Advanced Study in Princeton. She also served as Vice President of the American Mathematical Society from 1987–90.

Karen is a highly distinguished mathematician specializing in differential geometry, nonlinear partial differential equations, and mathematical physics. At the same time, her efforts across the educational spectrum, especially her role as a founder of the IAS/Park City Mathematics Institute, have added vitality to the mathematical scene. Karen's mentoring, both formal (she co-founded the Annual Women in Mathematics Program at the IAS) and informal, is legendary.

Karen has been awarded a MacArthur "Genius" Fellowship, the Steele Prize of the American Mathematical Society, and the U.S. National Medal of Science. In 1990, she became the second woman (after Emmy Noether in 1932) to present a plenary lecture at an International Congress of Mathematics. She is a fellow of the American Academy of Arts and Sciences and the U.S. National Academy of Sciences. Karen is also the recipient of seven honorary degrees, most recently from Harvard, Princeton and Brandeis Universities.

. . . and First Uhlenbeck Fellow


Naser Talebizadeh Sardari

Naser Talebizadeh Sardari is the first recipient of the Uhlenbeck postdoctoral fellowship. He is a member of this spring's Analytic Number Theory program.

In his Ph.D. thesis, Naser studied the distribution and lifting properties of solutions of quadratic equations, which have applications in a number of areas of analysis and mathematical physics, and proved a uniform quantitative form of strong approximation for a quadratic form in at least 4 variables that is optimal for at least 5 variables. Currently, Naser's favorite result states that whereas there is a polynomial time algorithm due to Schoof to express integers as sums of two squares, when possible, the analogue problem with imposed congruence conditions is NP-complete, assuming some widely-believed arithmetic conjectures.

Named Positions for Spring 2017

MSRI is grateful for the generous support that comes from endowments and annual gifts that support faculty and postdoc members of its programs each semester. The Clay Mathematics Institute awards its Senior Scholar awards to support established mathematicians to play a leading role in a topical program at an institute or university away from their home institution.

Eisenbud and Simons Professors

Analytic Number Theory
Tim Browning, University of Bristol
Chantal David, Concordia University
Andrew Granville, Université de Montréal
Philippe Michel, École Polytechnique Fédérale de Lausanne
Tamar Ziegler, Hebrew University of Jerusalem

Harmonic Analysis
Guy David, Université Paris-Sud
Ciprian Demeter, Indiana University
Jill Pipher, Brown University
Elias Stein, Princeton University

Chancellor's Professor

Harmonic Analysis
Tatiana Toro, University of Washington

Named Postdoctoral Fellows

Analytic Number Theory
Viterbi: Julia Brandes, University of Gothenburg
Berlekamp: Maria Nastasescu, Brown University
Uhlenbeck: Naser Talebizadeh Sardari, Princeton University

Harmonic Analysis
Huneke: Max Engelstein, Massachusetts Institute of Technology
Strauch: Marina Iliopoulou, University of Birmingham

Clay Senior Scholars

Analytic Number Theory
Manjul Bhargava, Princeton University
David (Roger) Heath-Brown, University of Oxford

Harmonic Analysis
Alexander Volberg, Michigan State University

Analytic Number Theory (continued from page 1)

A Strategy

Suppose we want to count, at least approximately, the solutions to a problem of analytic number theory. Let N be this number; the basic approach to evaluating N often proceeds in the three following steps:

(a) Try to guess the answer, using various heuristic techniques and expected properties of the objects that are considered (for example, since Gauss, we expect that an integer n of large size is prime with probability about $1/(\log n)$). Let G denote this guess.

(b) Try to express naturally the desired number N as the sum of G and some other expression R that might be tractable. If this fails, attempt, at least, to obtain some lower (or upper) bounds of the kind $N \geqslant aG + R$ or $N \leqslant aG + R$ for some number $a > 0$.

(c) If step (b) has succeeded, the remainder R will often look like a sum of quantities that change sign, and the remaining task is to show that these changes of sign are sufficiently chaotic to ensure that R is of size smaller than the guess G.

Very frequently, it is the last step that contains the core of a proof, and for this reason, much work in analytic number theory is devoted to proving estimates for oscillating sums of various origins. The examples we discuss below are also of this type. This does not mean that the first two steps are easy — for instance, much of the recent spectacular progress on gaps between primes is based on exceptionally clever arguments related to these first two stages, due to Goldston–Pintz–Yıldırım, and more recently to Maynard and Tao.

Multiplicative Functions

In the study of the distribution of prime numbers, and of the multiplicative properties of integers, many fundamental questions involve the interplay between addition and multiplication. The Möbius function $\mu(n)$ and its close cousin the Liouville function $\lambda(n)$ illustrate very well how little we understand of this interaction! For a positive integer $n \geqslant 1$, one defines $\lambda(n) = (-1)^k$, where k is the number of prime factors of n, counted with multiplicity. This is an example of an oscillating function. The sum

$$M(x) = \sum_{1 \leqslant n \leqslant x} \lambda(n)$$

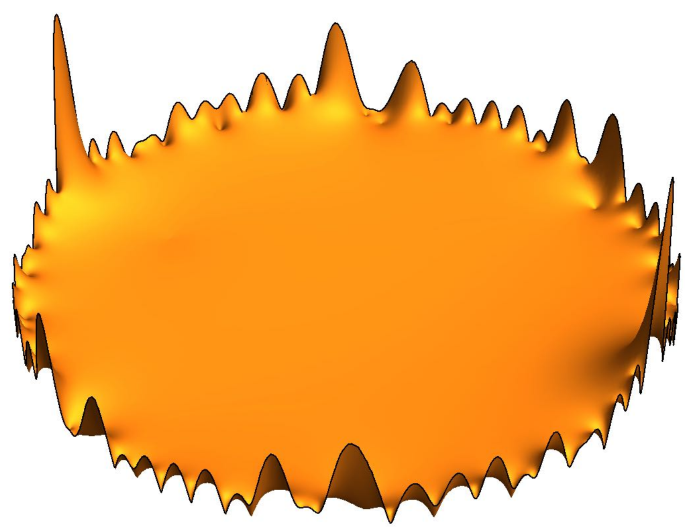
then measures the difference between the number of integers up to x with an even and with an odd number of prime factors. The simple question of estimating its size as x gets large is already one of the great mysteries of number theory. The conjecture that $M(x)/x^\delta$ tends to 0 for any fixed $\delta > 1/2$ is equivalent to the Riemann hypothesis — but this is only known for $\delta = 1$! However weak this seems, the fact that $M(x)/x$ tends to 0 has rich consequences: it is equivalent to the Prime Number Theorem, that the number $\pi(x)$ of prime numbers $p \leqslant x$ is asymptotic to $x/(\log x)$.

Even the analogue of such a weak statement as $M(x)/x \to 0$ is out of reach if one attempts to study *correlations* of values of the Möbius or Liouville function. For instance, if $k \geqslant 2$ and $0 \leqslant h_1 < \cdots < h_k$ are fixed integers, one conjectures that the sums

$$\sum_{1 \leqslant n \leqslant x} \lambda(n + h_1) \cdots \lambda(n + h_k) \qquad (*)$$

should also be $o(x)$ as x gets large. In other words (for $k = 2$, $(h_1, h_2) = (0, 1)$) to know the value of $\lambda(n)$ should not allow us to guess anything about the value of $\lambda(n+1)$. What could look more natural? As before, even a small amount of cancellation would have important consequences, and yet this remains open as soon as $k \geqslant 2$. However, a recent breakthrough of Matomäki and Radziwiłł in the study of short sums of multiplicative functions (see the article of K. Soundararajan) has led to spectacular progress — even for much more general functions, thanks to the idea of "pretentious" functions mimicking the Möbius function, due to Granville and Soundararajan.

In particular, in joint work with Tao, Matomäki and Radziwiłł have proved that the quantities in $(*)$ are $o(x)$ as soon as one can average, even in the slightest way, over the parameters $(h_1, \ldots, h_k)$. Another result of Tao, restricted to $k = 2$, is the logarithmically averaged form of $(*)$; among other ingredients, it has led to the solution of the Erdős discrepancy problem.



Visualizing the peaks in the circle method: The circle method extracts a precise guess to the number of solutions to Waring's problem by analyzing these peaks.

The Circle Method

When attempting to solve polynomial equations with integral coefficients, one of the basic methods is the circle method, which uses the elementary formula

$$\int_0^1 \exp(2i\pi\alpha(n-m))\,d\alpha = \begin{cases} 0 & \text{if } n \neq m \\ 1 & \text{if } n = m \end{cases}$$

to detect the equality of two integers. From there, the number N of solutions to Waring's problem of representing $n \geqslant 1$ as a sum of r integer k-th powers is

$$N = \int_0^1 S(\alpha)^r \exp(-2i\pi n\alpha)\,d\alpha,$$

where $S(\alpha)$ is the oscillatory sum

$$\sum_{1\leqslant m\leqslant n^{1/k}} \exp(2i\pi\alpha m^k).$$

The *circle method*, first developed by Ramanujan, Hardy, and Littlewood, extracts from this formula a very precise guess G by evaluating the part of the integral corresponding to values of α that are close to a rational number a/q with small denominator. This is illustrated by the figure on the preceding page, where the peaks along the circle correspond to such values of α, where the function $S(\alpha)$ is much more predictable.

To show that N is indeed close to G is relatively easy if the number r of variables is allowed to be large, since a large r accentuates the contributions of those peaks. When r gets smaller, it becomes increasingly difficult to control the contribution of the other values of α. In recent years, striking progress has been made in this area by Wooley through his method of "efficient congruencing." Of special note, a crucial conjecture of Vinogradov on the optimal size of

$$\int_{[0,1]^n} \Big| \sum_{1\leqslant m\leqslant x} \exp(2i\pi(\alpha_1 m + \cdots + \alpha_n m^n)) \Big|^{2s} d\alpha_1 \cdots d\alpha_n$$

was proved by Wooley for $n = 3$, and in general by Bourgain, Demeter, and Guth, who used methods of harmonic analysis — illustrating once again the vitality of the connections between analytic number theory and other areas of mathematics. The circle method and this result are discussed in more detail in the Harmonic Analysis program article by L. Guth on page 8 of this issue.

Trace Functions

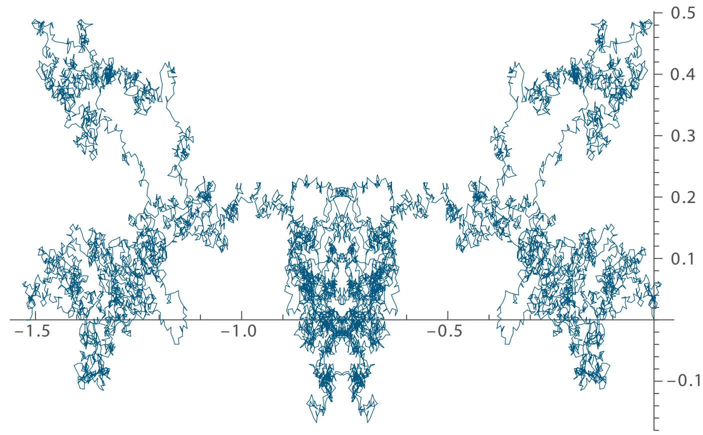
As our last example, we wish to highlight the truly remarkable interplay between algebraic geometry and exponential sums over finite fields. This is currently studied extensively (by Fouvry, Kowalski, Michel, and Sawin, among others) in the general context of "trace functions," which are functions over finite fields similar to the classical functions of mathematical physics. They appear extremely frequently in problems of analytic number theory, and their algebro-geometric origin often holds the key to analytic applications.

A typical example, which is ubiquitous in the theory of quadratic forms, modular forms, and L-functions, is the Kloosterman sum $\mathrm{Kl}_2(a;p)$, defined for a prime number p and an integer a coprime to p by

$$\mathrm{Kl}_2(a;p) = \frac{1}{\sqrt{p}} \sum_{1\leqslant x\leqslant p-1} \exp\Big(\frac{2i\pi}{p}(ax+\bar{x})\Big),$$

where $\bar{x}$ denotes the inverse of x modulo p.

The example of a Kloosterman path in the figure below (also shown on the cover) illustrates the oscillatory nature of this sum: it shows the polygonal path joining the partial sums of $\mathrm{Kl}_2(1;10007)$ over $1 \leqslant x \leqslant k$ when k varies from 0 to 10006.



A Kloosterman path: The polygonal path joins the partial sums of $\mathrm{Kl}_2(1;10007)$ over $1 \leqslant x \leqslant k$ when k varies from 0 to 10006.

Obtaining a good estimate for the Kloosterman sums is crucial to many applications. The best possible individual bound is $|\mathrm{Kl}_2(a;p)| \leqslant 2$, which was proved by A. Weil in the 1940s using the Riemann hypothesis for *curves* over finite fields. Deeper properties, concerning the variation of the sum with a, involve much more sophisticated algebraic geometry, and especially Deligne's most general form of the Riemann hypothesis over finite fields.

Zhang's sensational proof of the existence of infinitely many primes within intervals of bounded length hinged, for instance, on the proof of estimates like

$$\Big| \sum_{1\leqslant y\leqslant p-2} \mathrm{Kl}_2(y;p)\mathrm{Kl}_2\Big(\frac{y}{y+1};p\Big) \Big| \leqslant Cp^{1/2},$$

which first appeared, with a proof by Birch and Bombieri, in a paper of Friedlander and Iwaniec. In fact, we can illustrate the breathtaking efficiency of algebraic geometry with the following estimate, following from the Riemann hypothesis over finite fields and work of Katz: for any $k \geqslant 2$, if the shifts (h_i) are distinct modulo p, we have

$$\Big| \sum_{1\leqslant a\leqslant p-1} \mathrm{Kl}_2(a+h_1;p)\cdots \mathrm{Kl}_2(a+h_k;p) \Big| \leqslant C_k p^{1/2}$$

for some constant $C_k \geqslant 0$ — compare this statement to the conjectures concerning the sum (∗). Estimates of "sums of products" like these were used, for instance, by Kowalski and Sawin to "explain" the statistical shape of paths like those in the Kloosterman path figure.

Focus on the Scientist: Jill Pipher

This spring, Jill Pipher is one of the Eisenbud Professors in the Harmonic Analysis program at MSRI. Jill was born in Harrisburg, PA. An interest in music led her to Indiana University to study piano. She transferred to UCLA, where her coursework ranged from Russian literature to epistemology. A course in set theory and logic set her on the path to a career in mathematics.

Jill Pipher

At UCLA, Jill completed her B.A. and continued for a Ph.D. with a dissertation supervised by John Garnett. After serving as an L.E. Dickson instructor and Assistant Professor at the University of Chicago, she moved to Brown University, where she is the Elisha Benjamin Andrews Professor of Mathematics.

Her fundamental contributions to both analysis and cryptography have been widely recognized: she was elected a fellow of the American Mathematical Society (inaugural class), was an invited speaker at the International Congress of Mathematicians in 2014, and was elected Fellow of the American Academy of Arts and Sciences in 2015. She has been named Vice President for Research at Brown, effective Summer 2017.

Jill is an applied mathematician. In 1999, together with Jeff Hoffstein and Joe Silverman, she co-founded NTRU Cryptosystems, Inc., to market cryptographic algorithms, NTRUEncrypt and NTRUSign. These are based on the shortest vector problem in lattices. Today, NTRU is a part of Security Innovation, Inc. Jill holds several related patents.

She is also a theoretician. In mathematical analysis, Jill's work lies at the heart of the theory of solutions of elliptic partial differential equations (PDE) on non-smooth domains. Her expertise in harmonic analysis has been essential in the development of the modern perspective on the study of boundary regularity of solutions of elliptic PDEs on the natural class of domains whose boundaries have Lipschitz regularity. One of the main themes of this spring's Harmonic Analysis program is rooted in these contributions. One of her lectures delivered a few years ago vividly illustrated how Carleson measures, a classical tool in harmonic and complex analysis, had transformed the field. A member of the audience summarized the talk: "Give Jill a Carleson measure, and she will change the world."

Jill has also exercised a remarkable influence on the mathematical community at large. She was a driving force behind the formation of the Institute for Computational and Experimental Research in Mathematics (ICERM) at Brown University and served as its founding director from 2010 to 2016. In the fall of 2011, as ICERM was launching its first semester program, Jill simultaneously began a term as president of the Association for Women in Mathematics (AWM). Her energy and commitment are incredible. As AWM president, she guided a renewed focus on supporting research of women in Mathematics, and was instrumental in the launching of a new series of Biennial Research Symposia.

Jill has inspired so many of us, both professionally and personally. She is a powerful and innovative researcher, whose scientific work spans the gamut from overarching theory to industrial application. Her leadership has sparked the creation of a scientific institute and a private company, and has guided the development of other institutions. She has done all this, while raising a family that includes three children and two grandchildren. The term "gamut" — whose origins lie in the musical scale with which Jill's odyssey began — could not be more apt.

— Tatiana Toro and Michael Christ

Forthcoming Workshops

May 1–5, 2017: *Recent Developments in Analytic Number Theory*

May 15–19, 2017: *Recent Developments in Harmonic Analysis*

Jun 7–9, 2017: *Careers in Academia (at the American Institute of Mathematics, San Jose)*

Aug 17–18, 2017: *Connections for Women: Geometry and Probability in High Dimensions*

Aug 21–25, 2017: *Introductory Workshop: Phenomena in High Dimensions*

Aug 31–Sep 1, 2017: *Connections for Women Workshop: Geometric and Topological Combinatorics*

Sep 5–8, 2017: *Introductory Workshop: Geometric and Topological Combinatorics*

Oct 9–13, 2017: *Geometric and Topological Combinatorics: Modern Techniques and Methods*

Nov 13–17, 2017: *Geometric Functional Analysis and Applications*

Nov 29–Dec 1, 2017: *Women in Topology*

Summer Graduate Schools

Jun 12–23, 2017: *Subfactors: Planar Algebras, Quantum Symmetries, and Random Matrices*

Jun 26–Jul 7, 2017: *Soergel Bimodules*

Jul 10–21, 2017: *Séminaire de Mathématiques Supérieures 2017: Contemporary Dynamical Systems (Montreal)*

Jul 10–21, 2017: *Positivity Questions in Geometric Combinatorics*

Jul 17–28, 2017: *Nonlinear Dispersive PDE, Quantum Many Particle Systems, and the World Between*

Jul 24–Aug 4, 2017: *Automorphic Forms and the Langlands Program*

To find more information about any of these workshops or summer schools, as well as a full list of all upcoming workshops and programs, please visit msri.org/scientific.

Public Understanding

National Math Festival

The 2017 National Math Festival arrives Saturday, April 22, in Washington, DC, hosted by MSRI in cooperation with the Institute for Advanced Study (IAS) and the National Museum of Mathematics (MoMath). This free, public event features a full day of math activities for all ages from 10am–7pm, including lectures, hands-on demos, art, films, performances, athletics, games, and stories at the Washington Convention Center. The 2017 Festival is supported by donations from private foundations, corporations, and individuals, including the Simons Foundation.

A full schedule of all 2017 Festival events is online at nationalmathfestival.org or as a mobile app, NMF 2017.

Featured Presenters

Stephon Alexander • American Mathematical Society • Association for Women in Mathematics (AWM) • The Bridges Organization • Eugenia Cheng • Alissa Crans • Christopher Danielson • Emille Davie Lawrence • Robbert Dijkgraaf • Maria Droujkova • Marcus du Sautoy • Elwyn and Jennifer Berlekamp Foundation • Events DC • *FIRST* • First 8 Studios at WGBH • James Gardner • Gathering 4 Gardner • The George Washington University, Department of Mathematics • Paul Giganti • Herbert Ginsburg • Rebecca Goldin • Brady Haran • George Hart • Elisabeth Heathfield • Joan Holub • Ithaca College, Department of Mathematics • Julia Robinson Mathematics Festival • Scott Kim • Marc Lipsitch • Po-Shen Loh • Math for Love • MSRI • MathPickle • Talea L. Mayo • Mark Mitton • Michael Morgan • Colm Mulcahy • National Aeronautics and Space Administration (NASA) • National Museum of Mathematics (MoMath) • Natural Math • NOVA • NOVA Labs • Laura Overdeck • Stephanie Palmer • Matt Parker • Andrea Razzaghi • Mark Rosin • Science Cheerleader • Raj Shah • Travis Sperry • Clifford Stoll • James Tanton • Richard Tapia • ThinkFun • Janel Thomas • Mariel Vazquez • Talitha Washington • The Young People's Project • Zala Films • Mary Lou Zeeman

Bay Area Mathematical Olympiad

Each spring, MSRI hosts the award ceremony for the Bay Area Mathematical Olympiad, following two exams (one for students in 8th grade and under, and one for students in 12th grade and under), each taken by hundreds of Bay Area students, with five proof-type math problems to be solved in four hours. This year, Matthias Beck (San Francisco State University) spoke to the competitors on reciprocity instances in combinatorics.

Harmonic Series Concerts

MSRI hosts the free, public Harmonic Series concerts throughout the year, highlighting the links between music and mathematics. In 2016–17, performances were held featuring the Del Sol String Quartet; violinist (and University of Kansas math professor) Purnaprajna Bangere with Amit Kavthekar, tabla; pianist David Saliamonas; and jazz vocalist Torbie Phillips with Jason Martineau (piano), Jeff Denson (bass), and Kelly Park (percussion).

Math Circles News

Diana White, Director

New Associate Director, Dr. Jane Long

The National Association of Math Circles (NAMC) is excited to formally announce Dr. Jane Long, an associate professor from Stephen F. Austin State University, as the new Associate Director. Her primary focus is on leading the Math Circle–Mentorship and Partnership program, with which she has served as a mentor since its inception in 2015. She is founder and director of the East Texas Math Teachers' Circle, which began meeting in 2013. Dr. Long regularly visits Math Circles for students and teachers as an invited facilitator and frequently engages in outreach activities related to problem solving for people of all ages and levels. Dr. Long earned a Ph.D. in algebraic topology from the University of Maryland at College Park in 2008.

Tatiana Shubin Shares Navajo Math Circles

At the invitation of the Tulsa Math Teachers' Circle and the Tulsa Girls' Math Circle, mathematician Tatiana Shubin, a co-founder and co-director of the Navajo Nation Math Circles program, screened the film, *Navajo Math Circles*, to an enthusiastic Circle Cinema audience in early February. Members of the Tulsa Girls' Math Circle worked with Shubin to sort various polyhedra into three groups — finding Platonic solids, irregular solids, and hypercubes — and derive Euler's constant (See the photo on the cover.)

Since its premiere in January 2016, *Navajo Math Circles* has screened at venues nationally and in Canada, and recently at Stanford University, and at seven film festivals, including Sedona, Arivaca, Arizona International, the American Indian FF and Vision Maker. The film, sponsored by MSRI and Vision Maker Media, premiered on PBS in September 2016 and will continue to be broadcast on local public television stations through 2020 (www.navajomathcirclesfilm.com).

What are Math Circles? and NAMC?

Math Circles are a unique form of K–12 outreach designed to improve students' content knowledge and spark their mathematical interest and identity. In authentic, outside-the-classroom experiences, students engage with mathematics as an open-ended discipline involving exploration, conjecture, and communication with others. Students work on problems — often tackled in groups with a mentor — that can be approached with minimal mathematical background, but lead to deep and advanced mathematical concepts. The number of active Math Circles is ever-changing; currently there are about 180 across the United States.

The National Association of Math Circles (NAMC) is a program of MSRI designed to seed the creation of new Math Circles; provide resources, training, and support to Math Circles across the United States; and catalyze rigorous research and evaluation efforts for Math Circles. To learn more, and to be added to a list to receive updates on future training programs and grant opportunities, please email namc.manager@msri.org.

Fourier Analysis and Diophantine Equations

Larry Guth

Fourier analysis was initially developed in the early 1800s to study problems in mathematical physics. Fourier's fundamental insight was that a basically arbitrary function could be decomposed as a linear combination of complex exponentials with different frequencies. More informally, an arbitrary function can be written as a sum of sine waves with different frequencies. Fourier and others used this insight to study the time evolution of solutions to physically motivated partial differential equations, like the heat equation or the wave equation. This direction of research has remained fruitful ever since, and it is one of the active topics of the Spring 2017 Harmonic Analysis semester at MSRI.

In the early 1900s, mathematicians realized that Fourier analysis is also relevant to Diophantine equations in number theory. Just recently, Jean Bourgain and Ciprian Demeter developed a new theory called decoupling, which makes important progress on Diophantine equations. This theory uses insights that developed (partly) from trying to understand problems about physically motivated partial differential equations from the Fourier analysis angle. This decoupling approach to Diophantine equations is another active topic in this semester's program.

The Fourier–Diophantine Connection

The connection between Fourier analysis and Diophantine equations begins with the following observation. If m is an integer, then the integral

$$\int_0^1 e^{2\pi i m x} dx$$

is equal to 1 if $m = 0$, and vanishes if m is not zero. We can use this fact to express the number of solutions to a Diophantine equation as an integral. Suppose that $P(a_1, \dots, a_r)$ is a polynomial in r variables with integer coefficients. Then the integral

$$\int_0^1 \sum_{1 \leqslant a_i \leqslant A} e^{2\pi i P(a_1,\dots,a_r)x} dx \qquad (*)$$

is equal to the number of solutions to the Diophantine equation $P(a_1, \dots, a_r) = 0$ with $1 \leqslant a_i \leqslant A$. The last formula involves a sum over many variables, but in some special cases, it can be simplified and written as a moment of a sum in fewer variables. For example, consider the polynomial

$$P(a_1, \dots, a_6) = a_1^3 + a_2^3 + a_3^3 - a_4^3 - a_5^3 - a_a^3.$$

In this special case, the integral ($*$) reduces to

$$\int_0^1 \left| \sum_{1 \leqslant a \leqslant A} e^{2\pi i a^3 x} \right|^6 dx. \qquad (\dagger)$$

This integral is the sixth moment of the trigonometric sum

$$\sum_{a=1}^{A} e^{2\pi i a^3 x},$$

which is a sum of waves of different frequencies (1, 8, 27, ...). To understand the sixth moment, we need to understand how these waves interact: How much constructive interference is there, and how much do the different waves cancel each other? Let us make a few elementary observations to start to get a sense of the problem.

By the triangle inequality, we have immediately

$$\left| \sum_{a=1}^{A} e^{2\pi i a^3 x} \right| \leqslant A.$$

This bound is sharp when $x = 0$, because at this point, all the complex exponentials are positive real. In the physics literature, this would be called "pure constructive interference." But we would like to show that at most points x, the trigonometric sum is much smaller than A. The most classical tool to do this is the idea of orthogonality, which dates back to even before Fourier's work. The functions $\{e^{2\pi i n x}\}_{n \in \mathbb{Z}}$ are orthogonal on $[0, 1]$, and so

$$\int_0^1 \left| \sum_{1 \leqslant a \leqslant A} e^{2\pi i a^3 x} \right|^2 dx = \sum_{1 \leqslant a \leqslant A} \int_0^1 \left| e^{2\pi i a^3 x} \right|^2 dx = A.$$

We can get a first bound on the sixth moment of our trigonometric sum by applying the triangle inequality and then orthogonality:

$$\int_0^1 \left| \sum_{1 \leqslant a \leqslant A} e^{2\pi i a^3 x} \right|^6 dx \leqslant A^4 \int_0^1 \left| \sum_{1 \leqslant a \leqslant A} e^{2\pi i a^3 x} \right|^2 dx = A^5.$$

Now let us switch to the number theory side and make an educated guess of the order of magnitude of this integral. Recall that the integral ($\dagger$) is equal to the number of solutions of the Diophantine equation

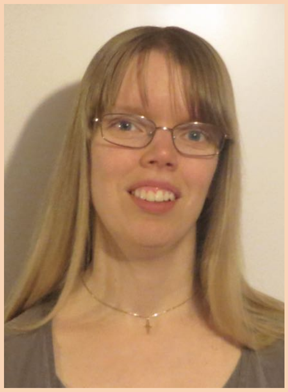
$$a_1^3 + a_2^3 + a_3^3 = a_4^3 + a_5^3 + a_6^3,$$

with $1 \leqslant a_i \leqslant A$. Each side of this equation lies somewhere between 3 and $3A^3$. Both sides are complicated. In order to make a rough guess of the number of solutions, let us imagine that each side is a random integer in the range $[3, 3A^3]$. The probability that the two sides are equal is then on the order of A^{-3}. There are A^6 possible choices of $a_1, \dots, a_6$, and so we might guess that there are on the order of A^3 solutions.

Focus on the Scientist: Kaisa Matomäki

Kaisa Matomäki is a key member of this spring's Analytic Number Theory program at MSRI. Kaisa is part of the exciting new generation of analytic number theorists who are radically transforming the field.

Kaisa was born in Nakkila, Finland, and completed her undergraduate and masters degrees at the nearby University of Turku. She obtained her doctorate in 2009 (for work on applications of sieve methods) under the supervision of Glyn Harman at Royal Holloway, University of London, before moving back to the University of Turku, where she currently holds an Academy Research Fellowship.



Kaisa Matomäki

Kaisa's work is characterized by a virtuosic technique which allows her to brush aside formidable technical difficulties and get straight to the heart of a problem. While she has a number of impressive publications, perhaps her standout result is the recent spectacular work with Radziwiłł on multiplicative functions in short intervals (*Annals of Math.*, 2016). A central object in number theory is the Liouville function which is a completely multiplicative function taking the value 1 on numbers with an even number of prime factors, and -1 on numbers with an odd number of prime factors. The Liouville function is expected to behave much like a random sequence of signs ± 1, but little is known in this direction. A special case of their work establishes that for almost all starting points n, the Liouville function takes about as many positive values as negative in the interval $[n, n+h]$, provided only that h tends to infinity with n.

This milestone result has already spurred further progress in the field — it forms a crucial ingredient in Tao's proof of a logarithmic version of the Chowla conjecture, and which in turn led to his resolution of the Erdős discrepancy problem. The Chowla conjecture, which asserts that consecutive values of the Liouville function are uncorrelated, appeared as intractable as the twin prime conjecture; now, after the work of Kaisa, Radziwiłł, and Tao, it no longer seems such a distant dream.

Kaisa and Radziwiłł were recognized for their breakthrough with the 2016 SASTRA Ramanujan Prize. Kaisa has also received the 2016 Academy of Finland's Award for Scientific Courage, and the 2016 Väisälä Prize. She lives in Lieto, Finland, with her husband Pekka and two young children, ages one and four.

— Kannan Soundararajan

This is an open problem of number theory (and of Fourier analysis). Conjecturally, the integral (†) is bounded by CA^3. The best known bounds are better than the A^5 that we obtained, but still significantly larger than A^3. In order to do better, we need better estimates about the cancellation in the trigonometric sum. The estimate from orthogonality tells us that the set of $x \in [0,1]$ where

$$\left| \sum_{1 \leqslant a \leqslant A} e^{2\pi i a^3 x} \right| \geqslant A^{3/4}$$

has measure at most $A^{-1/2}$. But the conjectured bound on the sixth moment implies that this set has measure at most $A^{-3/2}$. So we need to show that constructive interference is actually much rarer than what orthogonality tells us.

Constructive Interference and Orthogonality

The issue of constructive interference plays an important role in partial differential equations from physics. Let us discuss the wave equation as an example. The wave equation models a variety of kinds of waves, including sound waves traveling in air. Using Fourier's insight, we can write any solution to the wave equation as a linear combination of pure sine waves, traveling in different directions. The places where these waves interfere constructively are places where the original solution is big: places where the sound waves are focusing. Estimating how much focusing occurs is an important problem in the study of the wave equation. The Strichartz inequality is a fundamental result in this area from the 1970s.

Suppose we consider a solution to the wave equation in three spatial dimensions, $u(x,t)$, with initial data $u_0(x)$. The statement of the Strichartz inequality involves the frequencies of the waves in the decomposition. To simplify the statement of the inequality, let us assume that the Fourier transform of the initial data, $\hat{u}_0$, is supported in the unit ball and also that $\int_{\mathbb{R}^3} |u_0(x)|^2 dx = 1$. In this case, Strichartz's inequality says that

$$\int_{\mathbb{R}^3 \times \mathbb{R}} |u(x,t)|^4 \leqslant C,$$

where C is a constant independent of the solution. This tells us that the set of $(x,t) \in \mathbb{R}^3 \times \mathbb{R}$ where $|u(x,t)| > \lambda$ has measure at most $C\lambda^{-4}$. This bound is much stronger than what follows by orthogonality alone. (In fact, just using orthogonality does not give any finite bound for the measure of this set.)

The Strichartz inequality grew out of work by Stein, Fefferman, and others on the "restriction" theory of the Fourier transform. This theory involves the Fourier transform on $\mathbb{R}^d$ for $d \geqslant 2$, and it involves geometric ideas in order to get further improvement beyond just orthogonality. We'll explain a little more about it below. The first number theory problem we described involves a one-dimensional integral, but there are similar problems that lead to higher-dimensional integrals. Decoupling theory leads to an improvement in this higher-dimensional setting using geometric ideas that come out of restriction theory.

(continued on next page)

Harmonic Analysis (continued from previous page)

The Approach to Higher Dimensions

Higher dimensional integrals appear whenever we study a system of Diophantine equations instead of a single equation. Suppose that $\vec{P} = (P_1, \ldots, P_n)$ is an n-tuple of polynomials with integer coefficients. The number of solutions of the system $P_1(a_1, \ldots, a_r) = \cdots = P_n(a_1, \ldots, a_r) = 0$ with $1 \leqslant a_i \leqslant A$ is given by an n-dimensional version of the integral (∗):

$$\int_{[0,1]^n} \sum_{1 \leqslant a_i \leqslant A} e^{2\pi i \vec{P}(a_1, \ldots, a_r) \cdot \vec{x}} dx_1 \ldots dx_n \qquad (**)$$

In special cases, this expression can be written as the moment of a simpler trigonometric polynomial. For instance, Vinogradov proposed the following Diophantine system:

$$\begin{aligned} a_1 + \cdots + a_s &= b_1 + \cdots + b_s, \\ a_1^2 + \cdots + a_s^n &= b_1^2 + \cdots + b_s^2, \\ &\vdots \\ a_1^n + \cdots + a_s^n &= b_1^n + \cdots + b_s^n. \end{aligned}$$

The number of solutions to this system with $1 \leqslant a_i, b_i \leqslant A$ is given by a moment integral:

$$\int_{[0,1]^n} \left| \sum_{1 \leqslant a \leqslant A} e^{2\pi i M(a) \cdot x} \right|^{2s} dx, \qquad (\ddagger)$$

where $M(a) = (a, a^2, \ldots, a^n)$. In other words, the number of solutions is the $2s$-moment of the trigonometric sum $f(x)$:

$$f(x) = \sum_{1 \leqslant a \leqslant A} e^{2\pi i M(a) \cdot x}.$$

The case $n = 2$ was understood in Vinogradov's time. Only recently, Trevor Wooley proved nearly sharp estimates for $n = 3$. Even more recently, decoupling has led to a good understanding of this problem for all n. Bourgain, Demeter, and I used the decoupling approach to give nearly sharp estimates for the number of solutions to the Vinogradov system by estimating the moment integral (‡).

The First Three Stages of Decoupling

The decoupling approach considers many different pieces of the trigonometric sum $f(x)$. For any interval $I \subset [1, A]$, we let

$$f_I(x) = \sum_{a \in I} e^{2\pi i M(a) \cdot x}.$$

In the first stage, we consider intervals I of some small length L_1, much smaller than A. One can prove some rough bounds for f_I using orthogonality as above. At the second stage, we consider larger intervals J of length L_2, much bigger than L_1 but still much smaller than A. We express $f_J = \sum_{I \subset J} f_I$, and we study how the different functions f_I interact with each other. At this stage, some geometric reasoning comes into play which gives an improvement over just orthogonality.

If the different functions f_I were supported in disjoint regions, then we would immediately get an estimate much stronger than orthogonality:

$$\int |f_J|^{2s} \leqslant \sum_{I \subset J} \int |f_I|^{2s}.$$

A bit more generally, if the functions f_I are concentrated in regions that overlap only a little, one still gets good estimates.

Restriction theory provides tools to help understand when the functions f_I are concentrated in regions that don't overlap much. For each I, there is a tiling of space, $\mathbb{T}_I$, and $|f_I|$ is roughly constant on each tile of the tiling. The tiles of $\mathbb{T}_I$ are highly eccentric rectangles. For different intervals I of the same length, the tilings $\mathbb{T}_I$ are congruent to each other, but they are oriented at different angles. In particular, if I_1 and I_2 are intervals that are not too close together, then a single tile of $\mathbb{T}_{I_1}$ and a single tile of $\mathbb{T}_{I_2}$ do not intersect each other very much.

Now the previous analysis by orthogonality would be sharp in a situation where the mass of f_I is concentrated into a small number of tiles. If the mass of f_I is spread among many tiles of $\mathbb{T}_I$, then we get a better estimate at the first stage. But if the mass of f_I is concentrated in just a few tiles for each I, then these tiles do not intersect very much, and the geometry gives us a better estimate at the second stage. Either way, this gives enough leverage to improve on the estimate that comes just from orthogonality.

Next one goes to the third stage, considering intervals K of length L_3, much bigger than L_2 but still much smaller than A. We can write $f_K = \sum_{J \subset K} f_J$. If the mass of each function f_J is concentrated into a small number of tiles of $\mathbb{T}_J$, then the same geometry argument as above gives an improvement over the basic orthogonality estimate. But decoupling uses more than this. Bourgain and Demeter consider both the geometry of the functions f_J, $J \subset K$, and the geometry of the functions f_I, $I \subset K$. Combining geometric information from the first stage and the second stage, they are able to get an even better estimate at the third stage.

Surprises, Open Questions

The decoupling approach considers many different scales of intervals. At the s-th stage, the argument combines geometric information from all of the previous $s - 1$ stages. For the argument to work well, it is necessary to use a large number of different stages, and the novelty of decoupling is the way that it combines information from these many different scales. The proof is intricate, but it only uses orthogonality, the shapes of the tiles in the tiling, and this new way of combining information from many scales.

It came as a big surprise to me, and I think to many others, that such strong estimates could be proven using only these ingredients. It is not yet clear how far one can push these methods and which problems one can solve using just these tools.

Named Postdocs — Spring 2017

Berlekamp

Maria Nastasescu, a member of the Analytic Number Theory program, is the Spring 2017 Berlekamp Endowed Postdoctoral Scholar. Maria studies analytic and algebraic aspects of automorphic forms and especially non-vanishing theorems for L-functions of different types. During her graduate studies, she proved, for instance, that elliptic curves over the rationals that have semistable reduction at a prime p are characterized, up to isogeny and quadratic twist, by their adjoint p-adic L-function. This algebraic result is obtained by an analytic statement concerning the determination of automorphic forms on GL(3) from a sparse set of special values of twisted L-functions, which improves significantly earlier work of Luo and Ramakrishnan. The Berlekamp Postdoctoral Fellowship was established in 2014 by a group of Elwyn Berlekamp's friends, colleagues, and former students whose lives he has touched in many ways. He is well known for his algorithms in coding theory and has made important contributions to game theory. He is also known for his love of mathematical puzzles.


Maria Nastasescu

Huneke

Max Engelstein is the Huneke Postdoc this spring in the Harmonic Analysis program. Max obtained his Ph.D. at the University of Chicago in June 2016, under the direction of Carlos Kenig. He took up a C.L.E. Moore Postdoctoral Instructorship at MIT, beginning Fall 2016, working with David Jerison. Max works on problems at the interface between PDE and geometric measure theory, often using techniques of harmonic analysis. Most of his work to date concerns free boundary problems, in which information about the regularity of some solution to a PDE (or to some variational problem) may be used to glean information about the regularity of the free boundary. Max has already obtained some striking results in this area, concerning both 1-phase and 2-phase problems, and in both the elliptic and parabolic settings. The Huneke postdoctoral fellowship is funded by a generous endowment from Professor Craig Huneke, who is internationally recognized for his work in commutative algebra and algebraic geometry.


Max Engelstein

Strauch

Marina Iliopoulou is this spring's Strauch Postdoctoral Fellow as part of the Harmonic Analysis program. Marina was born in Greece and finished her Bachelor degree in Mathematics at the University of Athens in 2009, earning multiple academic awards along the way. She completed her Ph.D. at the University of Edinburgh in 2013. After several years as Research Fellow at the University of Birmingham (UK), she joined UC Berkeley in Fall 2016 as a Morrey Visiting Assistant Professor. Marina's research has focused on the exciting interface between Euclidean harmonic analysis and extremal combinatorics. She and collaborators are working to develop combinatorial and algebraic methods suited to the analysis of inequalities connected to the Fourier transform and to Schrödinger equations. She has also investigated incidence counting problems and their connection with Fourier analysis. The Strauch Fellowship is funded by a generous annual gift from Roger Strauch, Chairman of The Roda Group. He is a member of the Engineering Dean's College Advisory Boards of UC Berkeley and Cornell University, and is also currently the chair of MSRI's Board of Trustees.


Marina Iliopoulou

Viterbi

Julia Brandes is the Viterbi Postdoctoral Fellow in this spring's Harmonic Analysis program. Julia's research concerns Diophantine equations and in particular the counting of the number of solutions of certain systems of polynomial equations with special structure. In her Ph.D. thesis, and later, she studied the linear subspaces contained in a hypersurface, exploiting the geometric nature of this condition to go beyond earlier estimates. In recent work, partly joint with S. Parsell, she has considered systems of diagonal equations in much greater generality than had been done before, obtaining in some cases the optimal result expected from general heuristics. The Viterbi Endowed Postdoctoral Scholarship is funded by a generous endowment from Dr. Andrew Viterbi, well known as the co-inventor of Code Division Multiple Access based digital cellular technology and the Viterbi decoding algorithm, used in many digital communication systems.


Julia Brandes

***Naser Talebizadeh Sardari**, this semester's first Uhlenbeck Postdoctoral Fellow, is profiled on page 3.*

A Kind of Silence

Ed Baker
November 1, 2016

Something divided by nothing
Becomes everything;
But only in the limit
A limit that's never reached.
Expanding without end
Not as cause and effect;
As the air, swept along imperceptibly
By the butterfly's shimmering wings.
What is this asymptote beyond our reach?
The end of time? God?
That nothing in everything,
Empty space, whiteness, chaos —
Silence.

That something pervades the emptiness;
The emptiness as emptiness is.
Art moulds it through a form
A formula
A beginning, an end, a middle;
An audience,
Reduced to its bare essentials
Its bareness
Naked, vulnerable and contained.
Like a thought, it floats
Out of the air
We hear an invigorated freshness,
A disenchanted emptiness,
A teased and tormented aloofness;
We hear it — that confounding
Fiddle-about silence.
As a mirror
As a page of whiteness
Suggestively gleaming back at us
The point of view has shifted
Beauty, if there beauty be,
Can only be — well, beheld;
The beauty of the moment.
When we put that conch shell to our ear
The sound of the eternal
Is surely what we hear
Eternal emptiness.
She sells seashells, for profit
This was another point of objection
As each tick-tock ticks by
We sense the freedom —
The ultimate anarchy of silence.

Can you bear it any longer?
The allure of ice cream,
Or a martini
And yet it moves along
Everything is tickety-boo.
Perhaps the coda has begun,
We decide which path
Best takes us to the end;
A grand crescendo to a blustery climax,
A quiet fade into the stillness of the night.
That moment
When past and future coalesce
Into a predetermined point,
Invented and revised;
The silence never stops.

Call for Proposals

All proposals can be submitted to the Director or Deputy Director or any member of the Scientific Advisory Committee with a copy to proposals@msri.org. For detailed information, please see the website msri.org.

Thematic Programs

The Scientific Advisory Committee (SAC) of the Institute meets in January, May, and November each year to consider letters of intent, pre-proposals, and proposals for programs. The deadlines to submit proposals of any kind for review by the SAC are **March 15**, **October 15**, and **December 15**. Successful proposals are usually developed from the pre-proposal in a collaborative process between the proposers, the Directorate, and the SAC, and may be considered at more than one meeting of the SAC before selection. For complete details, see tinyurl.com/msri-progprop.

Hot Topics Workshops

Each year MSRI runs a week-long workshop on some area of intense mathematical activity chosen the previous fall. Proposals should be received by **March 15**, **October 15**, or **December 15** for review at the upcoming SAC meeting. See tinyurl.com/msri-htw.

Summer Graduate Schools

Every summer MSRI organizes several 2-week long summer graduate workshops, most of which are held at MSRI. Proposals must be submitted by **March 15**, **October 15** or **December 15** for review at the upcoming SAC meeting. See tinyurl.com/msri-sgs.

Puzzles Column

Elwyn Berlekamp and Joe P. Buhler

Modified from © Sergey Ilin | Dreamstime.com

0. How many meanings can you ascribe to the following word sequence?

a pretty light brown doll house

1. An arbitrary square is removed from a 2^n by 2^n chessboard. Prove that the remaining squares can be exactly tiled by right trominoes. (A right tromino is the 3-square tile obtained by removing one square from a 2 by 2 chessboard.)

Comment: This year's ITA (Information Theory and Applications) conference near UC San Diego included a special session in memory of Solomon W. Golomb, which had a "puzzle" component. The first five problems here (0–4) came up in that session, and problems 0 and 1 are due to Sol.

2. The number 545 has the interesting property that if you erase any of the digits and write in a new digit in that place (the new digit may be a leading zero or the same as the old digit), then you never get a multiple of 11. Find two more such integers.

Comment: We heard this from Richard Stong, and it was originally due to Mike Boshernitzan.

3. Given a triangle T with vertices A, B, C, construct external 30/30/120 triangles on the sides of T with the 120° angles being at the new vertices X, Y, Z. Prove that XYZ is an equilateral triangle.

4. A group of n people plan to paint the outside of a fence surrounding a large circular field using the following curious process. Each painter takes a bucket of paint to a random point on the circumference and, on a signal, paints towards their furthest neighbor, stopping when they reach a painted surface. What is the expected fraction of the fence that will be left unpainted at the end of this process?

Comment: We heard this from Paul Cuff, who remembers hearing it many years ago at one of Tom Cover's seminars at Stanford. The reader may enjoy figuring out the fraction of unpainted space if the painters paint towards their *nearest* neighbor, and deciding beforehand by pure thought whether this should be more or less than one-half.

5. The king has invited N quarrelsome knights to dinner. Many pairs of them have deep-seated mutual hostilities that are so bad that if seated next to each other bloodshed will likely ensue. Can you always find a safe seating arrangement around a circular table if each knight is hostile to less than (or at most?) half of the other knights? An arrangement of the N knights is of course safe if no pair of adjacent knights are hostile.

Comment: We heard this problem from Zvezda Stankova.

Call for Membership

MSRI invites membership applications for the 2018–2019 academic year in these positions:

Research Professors	by October 1, 2017
Research Members	by December 1, 2017
Postdoctoral Fellows	by December 1, 2017

In the academic year 2018–2019, the research programs are:

Hamiltonian Systems, from Topology to Applications through Analysis
Aug 13–Dec 14, 2018
Organized by Rafael de la Llave, Albert Fathi (Lead), Vadim Kaloshin, Robert Littlejohn, Philip Morrison, Tere M. Seara, Serge Tabachnikov, Amie Wilkinson

Derived Algebraic Geometry
Jan 22–May 24, 2019
Organized by Julie Bergner, Bhargav Bhatt (Lead), Dennis Gaitsgory, David Nadler, Nikita Rozenblyum, Peter Scholze, Bertrand Toën, Gabriele Vezzosi

Birational Geometry and Moduli Spaces
Jan 22–May 24, 2019
Organized by Antonella Grassi, Christopher Hacon (Lead), Sándor Kovács, Mircea Mustață, Martin Olsson

MSRI uses **MathJobs** to process applications for its positions. Interested candidates must apply online at mathjobs.org after August 1, 2017. For more information about any of the programs, please see msri.org/scientific/programs.

Clay Senior Scholarships

The Clay Mathematics Institute (www.claymath.org) has announced the 2017–2018 recipients of its Senior Scholar awards. The awards provide support for established mathematicians to play a leading role in a topical program at an institute or university away from their home institution. Here are the Clay Senior Scholars who will work at MSRI in 2017–2018:

Geometric Functional Analysis and Applications (Fall 2017)
William B. Johnson, Texas A&M University

Geometric and Topological Combinatorics (Fall 2017)
Francisco Santos, Universidad de Cantabria

Group Representation Theory and Applications (Spring 2018)
Gunter Malle, Universität Kaiserslautern

Enumerative Geometry Beyond Numbers (Spring 2018)
Hiraku Nakajima, Kyoto University

2016 Annual Report

We gratefully acknowledge the supporters of MSRI whose generosity allows us to fulfill MSRI's mission to advance and communicate the fundamental knowledge in mathematics and the mathematical sciences; to develop human capital for the growth and use of such knowledge; and to cultivate in the larger society awareness and appreciation of the beauty, power, and importance of mathematical ideas and ways of understanding the world.

This report acknowledges grants and gifts received from January 1 – December 31, 2016. In preparation of this report, we have tried to avoid errors and omissions. If any are found, please accept our apologies, and report them to development@msri.org. If your name was not listed as you prefer, please let us know so we can correct our records. If your gift was received after December 31, 2016, your name will appear in the 2017 Annual Report. For more information on our giving program, please visit www.msri.org.

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17 Gauss Way, Berkeley CA 94720-5070
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