Elections and Representation

Michael A. Jones, *AMS | Mathematical Reviews*

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And the winner is <*insert winner here*>.

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OK, you just voted between two candidates:

Elections and Representation.

And the winner is <*insert winner here*>.

Since we are already talking about elections, let's continue ...

A brief overview

Mathematically define an election procedure

Consider alternative ways to determine allocation outcomes

Classical voting theory/social choice theory

Axiomatic characterizations: determine election procedures that satisfy desirable properties

Focus on one method in the context of increasing diversity

Give historical and recent account of this method

Mathematically, what is an election?

- There are n candidates, C_i for $i = 1$ to n.
- Each voter v has preferences over the candidates, resulting in a linear ordering, e.g., $C_1 \succ C_2 \succ C_3 \succ \cdots \succ C_n$.
- A collection of voters' preferences is a profile, P.
- An election procedure or social welfare function f maps the set of all profiles $\mathcal{P}$ to a partial order over the candidates. (If we are concerned only with one winner, then we take the top-ranked candidate(s).)

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When choosing between Elections (E) and Representation (R), selecting E (R) means $E \succ R$ ($R \succ E$). The profile of voters is:

k	ℓ
E	R
R	E

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k	ℓ
E	R
R	E

If $k > \ell$, then the outcome would be E; if $\ell > k$, then the outcome would be R. This is majority rule.

Deciding between two alternatives–majority rule

(Desirable) Axioms

1. (Resoluteness) Each set of preferences results in a unique winner.
2. (Anonymity) Voters are treated equally.
3. (Neutrality) Candidates are treated equally. (If every voter's preferences are reversed, then the group preference is reversed.)
4. (Positive responsiveness) Gaining support does not hurt a candidate. (If a voter changes its vote from the losing candidate to the winning candidate and all other votes remain fixed, then the winning candidate still wins.)

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Theorem (May, 1952)

For an odd number of voters, an election procedure satisfies Axioms 1-4 if and only if it is simple majority rule.

May, Kenneth O. A set of independent necessary and sufficient conditions for simple majority decisions. *Econometrica* 20 (1952), 680–684.

Elections among more than two alternatives

(Desirable) Axioms

1. (Pareto efficiency) If all voters prefer a to b, then the outcome of the social welfare function must rank a above b.

2. (Independence of Irrelevant Alternatives) The ranking of a and b by the social welfare function only depends on each voter's relative ordering of these two outcomes.

3. (Non-dictatorship) No one voter determines the social welfare ordering.

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There does not exist a social welfare function over three or more outcomes that satisfies Pareto efficiency, independence of irrelevant alternatives and non-dictatorship.

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There are variations of Arrow's theorem with other axioms.
This is known as Arrow's impossibility theorem.

Arrow, Kenneth J. Social choice and individual values. Cowles Commision Monograph No. 12. John Wiley & Sons, Inc., New York, N.Y. 1951.

Manipulation

Social choice theory studies the aggregation of preferences when each voter's preference is known. What if preferences are unknown?

Can an election procedure induce the truthful revelation of voter's preferences? (This is what *mechanism design* is about.)

A procedure is manipulable if there exists a profile P for which the election outcome is better for a voter if the voter submits insincere preferences as opposed to sincere preferences.

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Theorem (Gibbard, 1973; Satterthwaite, 1975)

The only election procedure for three or more candidates that is non-manipulable is a dictatorship.

Gibbard, Allan. Manipulation of voting schemes: a general result. Econometrica 41 (1973), 587–601.

Satterthwaite, Mark Allen. Strategy-proofness and Arrow's conditions: Existence and correspondence theorems for voting procedures and social welfare functions. J. Econom. Theory 10 (1975), no. 2, 187–217.

Methods for three or more alternatives (plurality)

Plurality. The method most used in the U.S. in which each voter selects one candidate, presumably, their most preferred. The candidate with the most votes is elected.

ranking		**points**	**Example**		
C_1		1	$\frac{1}{3}+\varepsilon$	$\frac{1}{3}-\varepsilon$	$\frac{1}{3}$
C_2	$\Rightarrow$	0	C_1	C_2	C_3
$\vdots$		$\vdots$	C_2	C_3	C_2
C_n		0	C_3	C_1	C_1

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Concerns

Elected candidate may be disliked by a majority.

Leads to a two-party system.

Methods for three or more alternatives (approval voting)

Approval Voting. Each voter approves of a subset of candidates. Candidate with most approval votes wins.

ranking		**points**?		**points**?		**points**?	
C_1		1		1		1	
C_2		0		1		1	
C_3	$\Rightarrow$	0	$\Rightarrow$	0	$\Rightarrow$	1	etc.
$\vdots$		$\vdots$		$\vdots$		$\vdots$	
C_n		0		0		0	

Concerns/Usage

- Knowing preferences is not enough to determine approval votes.
- Approval votes do not measure intensity.
- Used by the MAA and AMS to elect committee members.
- Fargo, ND used approval voting to elect officials in June 2020.

Brams, Steven J.; Fishburn, Peter C. Approval voting. Second edition. Springer, New York, 2007.

Methods for three or more alternatives (Borda count)

Borda count. Each voter's linear preferences generates a voting vector. Candidate with the largest sum of points wins.

ranking	**points**
C_1	$n-1$
C_2	$n-2$
$\vdots$	$\vdots$
C_{n-1}	1
C_n	0

Concerns/Benefits

- If there are a lot of candidates, requires voters to rank all of them.
- Uses all the information about the preferences.
- Susceptible to strategic voting; "My scheme is only intended for honest men."
- Minimizes likelihood of manipulation for three candidates and for certain paradoxes for all n.

Saari, Donald G. Susceptibility to manipulation. Public Choice, Vol. 64, No. 1 (Jan., 1990), 21-41.

Methods for 3+ alternatives (ranked choice voting)

Ranked choice voting.

– If a candidate wins a majority of first-place votes, he or she wins the election.

– If no candidate wins a majority, eliminate the candidate with the fewest first-place votes.

– Transfer votes to second-place candidates.

– Repeat until a candidate has a majority of the first-place votes from the remaining voters; that candidate wins.

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Note: All four methods (and most others for three or more candidates) reduce to majority rule when there are two candidates and an odd number of voters. To elect k candidates, elected candidates must receive more than $\frac{1}{k+1}$ of the first-place votes.

All extend majority rule? None are perfect? Which to use?

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All extend majority rule? None are perfect? Which to use?

We will spend the rest of our time on elections on ranked choice voting, discussing its historical and current day use.

Ranked choice voting, history

- Ranked-choice voting (RCV) was invented in the 1850s in Europe as a PR system to be used in multi-winner elections.
- In the 1870s, it was adapted to the single-winner (or instant runoff) form by William Ware, an MIT professor.
- In 1915, Ashtabula, OH was the first place in U.S. to use RCV to elect its city council.

> *. . . proponents of democratization sought to use proportional electoral systems to restore power to ordinary people from . . . corrupt party bosses and corporate monopolists.*

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- Repeals in 1940s/50s. By 1962, Cambridge, MA was the only city using RCV system (city council and school board).

> *Today, proportional representation is promoted to facilitate more accurate representation of racial, ethnic, and gender groups in policy-making bodies.*

- We will look at the use of RCV in Oakland, CA & Eastpointe, MI.

Barber, Kathleen. Proportional representation and election reform in Ohio. Ohio State University Press, 1995.

Ranked choice voting: Oakland, CA

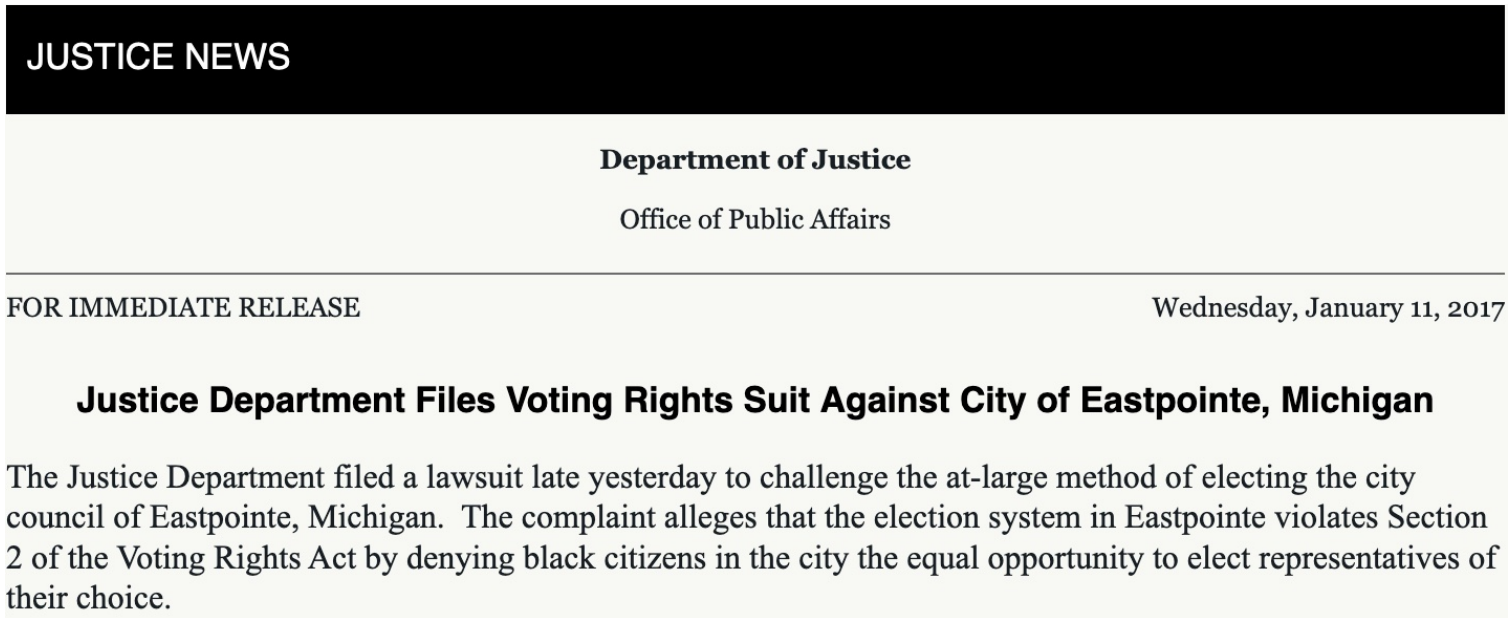
2010 Oakland Mayoral Election

Candidate	Round 1	Round 2	Round 3	Round 4	Round 5	Round 6	Round 7	Round 8	Round 9	Round 10
Jean Quan	29,266	29,299	29,391	29,514	29,645	30,500	30,884	31,655	35,033	**53,897**
Don Perata	**40,342**	**40,374**	**40,455**	**40,606**	**40,728**	**40,814**	**41,364**	**42,188**	**45,465**	51,872
Rebecca Kaplan	25,813	25,831	25,890	26,026	26,117	26,496	26,831	27,475	32,719	
Joe Tuman	14,347	14,357	14,471	14,552	14,780	14,949	15,202	15,462		
Marcie Hodge	2,994	2,999	3,033	3,155	3,200	3,250	3,625			
Terence Candell	2,315	2,316	2,386	2,497	2,613	2,680				
Don MacLeay	1,630	1,636	1,677	1,719	1,852					
Greg Harland	966	968	1,059	1,087						
Larry Lionel "LL" Young Jr.	933	939	976							
Arnold Fields	733	738								
Write-in	268									
Continuing votes	119,607	119,457	119,338	119,156	118,935	118,689	117,906	116,780	113,217	105,769
Exhausted ballots	0	149	262	435	376	893	1,655	2,766	6,284	13,667

- RCV has been suggested as a way to reduce political divide by candidates courting more voters.

$\Rightarrow$ Quan won because she actively encouraged supporters of other candidates to rank her in second place.

Ranked choice voting: Eastpointe, MI



JUSTICE NEWS

Department of Justice

Office of Public Affairs

FOR IMMEDIATE RELEASE — Wednesday, January 11, 2017

Justice Department Files Voting Rights Suit Against City of Eastpointe, Michigan

The Justice Department filed a lawsuit late yesterday to challenge the at-large method of electing the city council of Eastpointe, Michigan. The complaint alleges that the election system in Eastpointe violates Section 2 of the Voting Rights Act by denying black citizens in the city the equal opportunity to elect representatives of their choice.

> *Section 2 of the Voting Rights Act of 1965 prohibits voting practices or procedures that discriminate on the basis of race, color, or membership in one of the language minority groups . . .*

- Previously, winner-take-all elections "diluted the voting power of its African-American population, allowing the white majority to elect their candidates of choice to the council nearly every time."
- June 5, 2019: Under the agreement, the city council members will continue to be elected on a citywide basis, but under the new ranked choice voting system . . .

Ranked choice voting: Eastpointe, MI

• Importantly, Eastpointe was required to "conduct a robust voter education program".
• Since adopting RCV, Eastpointe has seen a higher number of African-American and female candidates running for and winning office.

Eastpointe 2020 City Council election

Contest	**Eastpointe City Council - November 3, 2020**
Jurisdiction	Eastpointe
Office	City Council - Partial Term
Date	11/3/2020
Winner(s)	**Sylvia L. Moore**
Threshold	**5802**

Rounds	Round 1	Round 2	Round 3	Round 4
Eliminated	Tonia Gladney		Rob Baker	
Elected		Sylvia L. Moore		
Sylvia L. Moore	5177	7525	7525	11602
Rob Baker	4756	5651	5651	0
Tonia Gladney	3989	0	0	0
Inactive ballots	4171	4917	4917	6491

• On November 2, 2021 the City of Eastpointe will elect two city council members using RCV.

A mathematician's role ...

As mathematicians and mathematics educators, we can use our training to

- analyze claims about election procedures,
- educate voters about new election procedures so that the procedures have a chance to work,
- evaluate election data to see whether procedures are having desired impact, and
- use real data in the classroom to have younger generations think about the application of mathematical ideas in politics.

Representation

Classic apportionment

In the U.S., apportionment methods have been used to determine the number of representatives each state receives in the House (and, ultimately the number of electoral votes).

The Apportionment Problem

- Let h the number of representatives in the House.
- State i has population p_i and $P = \sum_{i=i}^{n} p_i$ is the total population.
- Each state i should receive $q_i = \frac{p_i}{P} h$ of the representatives.
- Because the q_i are rarely integers and there are no fractional representatives, the apportionment problem is to round the q_i to a_i so that $\sum_{i=1}^{n} a_i = h$.

Balinski, Michel L.; Young, H. Peyton. Fair representation: Meeting the ideal of one man, one vote. Brookings Institution Press. Washington, D.C. 2001.

Brief history of apportionment in the U.S.

A natural solution to the problem was due to Alexander Hamilton: round the quota down; give another representative to the states with the largest fractional remainders to fill the house. Recall that $q_i = \frac{p_i}{P} h$.

Example. Let $h = 10$.

State	Population	Quota	Floor	Apportionment
1	502	6.526	6	6
2	377	4.901	4	5
3	121	1.5730	1	2
T	1000	13	11	13

Striking down Hamilton's method was the first veto by Washington. Hamilton to Jefferson: Vermont ($3 \to 2$) and Virginia ($18 \to 19$).

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Hamilton to Jefferson: Vermont ($3 \to 2$) and Virginia ($18 \to 19$).

Small	– – –	Divisor Methods	– – –	Large
Adams	Dean	Hill-Huntington	Webster	Jefferson
down	harmonic	geometric	arithmetic	up

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After every census, the H-H method is used to apportionment the representatives.

A new use, presidential primaries

The United States (US) Presidential Primaries are a series of state elections held every four years to determine the Democratic and Republican Parties' candidates for President.

For a state's presidential primary, apportionment methods are being used to determine the number of delegates a candidate receives based on the number of votes the candidate received.

The Apportionment Problem

- Let D be the number of delegates to be awarded.
- Candidate i receives v_i votes and $V = \sum_{i=i}^{n} v_i$ is the total number of votes.
- Each candidate i should receive $q_i = \frac{v_i}{V} D$ of the delegates.
- Because the q_i are rarely integers and there are no fractional delegates, the apportionment problem is to round the q_i to d_i so that $\sum_{i=1}^{n} d_i = D$.

If a candidate wins a majority of the delegates at the end of the process, the candidate is endorsed as the party's nominee for President.

A modern history of presidential primaries in the U.S.

- The modern system by which the U.S. Republican and Democratic Parties select their presidential candidates dates from the late 1960s.
- Prior to this, decisions were were brokered among party officials and leaders who had influence over large blocs of state delegates.
- Protests for the 1968 nomination of Hubert Humphrey despite support for Robert Kennedy are credited with the changes.

$\Rightarrow$ First the Democrats, and then the Republicans, reformed the primary process to increase public input.

- In 2008, Republicans used plurality rule to award delegates while Democrats used Hamilton's method.

$\Rightarrow$ Consequence: McCain secured a majority of the delegates by early March. The Obama-Clinton race went into June.

- Republicans formed the Temporary Delegate Selection Committee to adjust the primary calendar and delegate apportionment.
- Primaries before April 1 had to use a proportional method; the method was not dictated.

Jewitt, Caitlin E. 2019. The Primary Rules: Parties, Voters, and Presidential Nominations. The University of Michigan Press.

Example

With no guidance, the Republicans required their state parties to solve an apportionment problem.
How would you do? (. . . we'll make this multiple choice)

Table: A sample election with $n = 6$ and $D = 101$. The bold numbers indicate when quota is broken.

i	v_i	q_i	1	2	3	4	5	6
1	29,130	25.473	**27**	26	25	26	**28**	26
2	20,000	17.489	17	18	18	18	17	18
3	17,720	15.495	15	15	16	16	15	16
4	16,750	14.647	15	15	15	14	14	15
5	16,550	14.472	14	14	14	14	14	15
6	15,350	13.423	13	13	13	13	13	**11**
Total	115,500	101	101	101	101	101	101	101

Zoom poll. Vote for which you think is the fairest among allocations 1 through 6?

Apportionment methods used in the presidential primaries

The state parties came up with methods based on rounding down, nearest-integer rounding, and rounding up. They are: Nearest-integer Extremes; Nearest-integer Sequential; Hamilton; Large; Iterated Lower Quota; Sequential Upper Quota.

Table 1
A sample election with $n = 6$ and $D = 101$. The bold numbers indicate when quota is broken.

i	v_i	q_i	$[q_i]$	$\lfloor q_i \rfloor$	$\lceil q_i \rceil$	NIE	NIS	HAM	LAR	ILQ	SUQ
1	29,130	25.473	25	25	26	**27**	26	25	26	**28**	26
2	20,000	17.489	17	17	18	17	18	18	18	17	18
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4	16,750	14.647	15	14	15	15	15	15	14	14	15
5	16,550	14.472	14	14	15	14	14	14	14	14	15
6	15,350	13.423	13	13	14	13	13	13	13	13	**11**
Total	115,500	101	99	98	104	101	101	101	101	101	101

Jones, Michael A.; McCune, David; Wilson, Jennifer M. New quota-based apportionment methods: The allocation of delegates in the Republican Presidential Primary. Math. Social Sciences 108 (2020) 122–137.

Bias in these new apportionment methods

Is bias bad or good? It depends on where the state is on the primary calendar.

- Early in the primary season, methods should favor "small" candidates to generate ideas and participation.
- Later in the primary season, methods should favor "large" candidates to consolidate support.

Table 2
The 2016 primary calendar and delegate apportionment for states and territories (AS: American Samoa; GU: Guam; NMI: Northern Marianas Islands; PR: Puerto Rico; VI: Virgin Islands).

Date	Method:	State(s)	Date	Method:	State(s)
Feb 1	HAM:	IA	Mar 10	DEL:	VI
Feb 9	NIE:	NH[d]	Mar 12	DEL:	GU
Feb 20	WIN:	SC	Mar 15	WIN:	FL, IL, MO, NMI, OH
Feb 23	UN:	NV		UN:	NC
Mar 1	HAM:	VA[a], AR[c]	Mar 22	DEL:	AS
	HNH:	MN		SUQ:	UT
	ILQ:	GA		WIN:	AZ
	NIE:	AL, VT[a]	Apr 1–3	NM:	ND
	NIS:	MA	Apr 5	WIN:	WI
	SUQ:	TN, TX	Apr 19	NIE:	NY
	UN:	AK, ND, OK	Apr 26	NIE:	CT[a], RI[b]
	NM:	CO, WY		WIN:	DE, MD, PA
Mar 5	HAM:	KY[a]	May 3	WIN:	IN
	SUQ:	KS, ME	May 10	WIN:	NE
	UC:	LA[d]		NM:	WV
Mar 6	UN:	PR	May 17	HAM:	OR
Mar 8	NIE:	MI[a]	May 24	HAM:	WA[d]
	SUQ:	HI	Jun 7	HAM:	NM[a]
	UN:	MS		WIN:	CA, MT, NJ, SD
	NM:	ID	Jun 24	HAM:	DC

In Practice

Democratic Delegate Selection Rules (Section 13, Part D); available online at www.democrats.org.

Step 1. Tabulate the percentage of the vote that each presidential preference (including uncommitted status) receives in the congressional district to three decimals.

Step 2. Retabulate the percentage of the vote to three decimals, received by each presidential preference, excluding the votes of presidential preferences whose percentage in Step 1 falls below 15%.

Step 3. Multiply the number of delegates to be allocated by the percentage received by each presidential preference.

Step 4. Delegates shall be allocated to each presidential preference based on the whole numbers that result from the multiplication in Step 3.

Step 5. Remaining delegates, if any, shall be awarded in order of the highest fractional remainders in Step 3.

Example

i	v_i	q_i	Hamilton	q_i^*	Hamilton*
1	6625	3.313	3	3.524	4
2	2775	1.388	2	1.476	1
3	599	0.300	0		

Other implications

- The example demonstrates that under Hamilton's method when votes are eliminated by a cutoff (the 15%), then it is possible that a candidate's delegate count decreases.

- This is called the Elimination Paradox. It can occur when a candidate drops out of a race, too.

Theorem

All of the delegate allocation methods used by the Democrats and Republicans in their state primaries (except for winner-take-all) are subject to the Elimination Paradox.

- Why is this bad? Think about open primaries and confidence in the election process.

- Divisor methods are immune from this paradox.

Jones, Michael A.; McCune, David; Wilson, Jennifer M. The elimination paradox: apportionment in the Democratic Party. Public Choice, 2019, vol. 178, issue 1, No 4, 53-65.

Concluding remarks

Similar to elections, other political processes are rarely stagnant.

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Thank you.