Spectrum of the Laplacian on the Basilica group and holomorphic dynamics (joint work with Eric Bedford, Rostislav Grigorchuk and Mikhail Lyubich)

Nguyen-Bac Dang

Université Paris-Saclay

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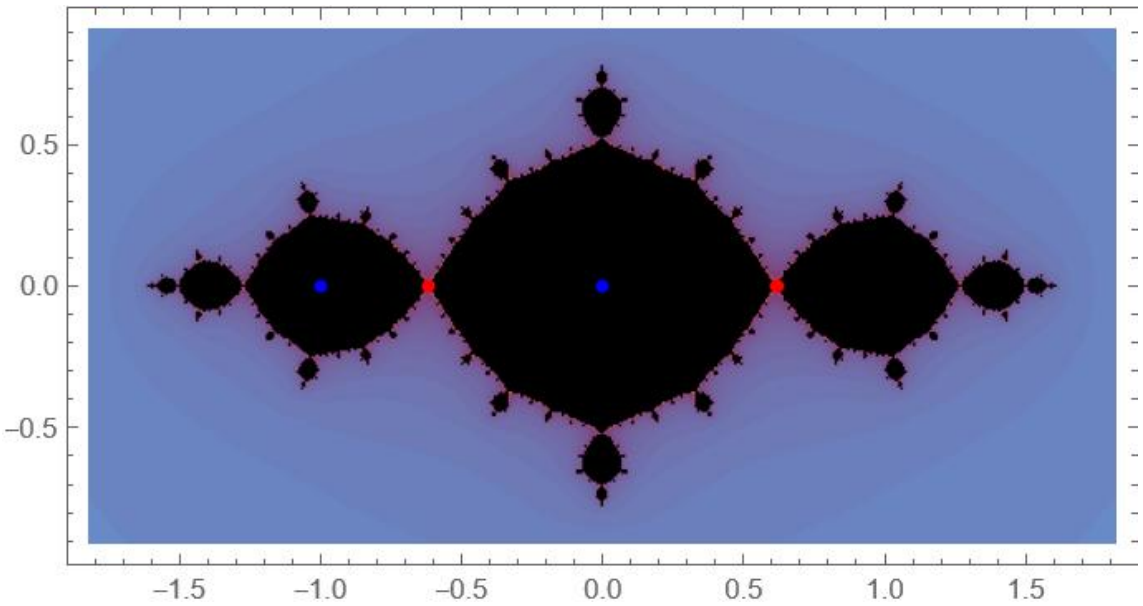
1. Basilica fractal and its iterated monodromy group
2. Spectrum of the Laplacian
3. Schur renormalization

Basilica Julia set

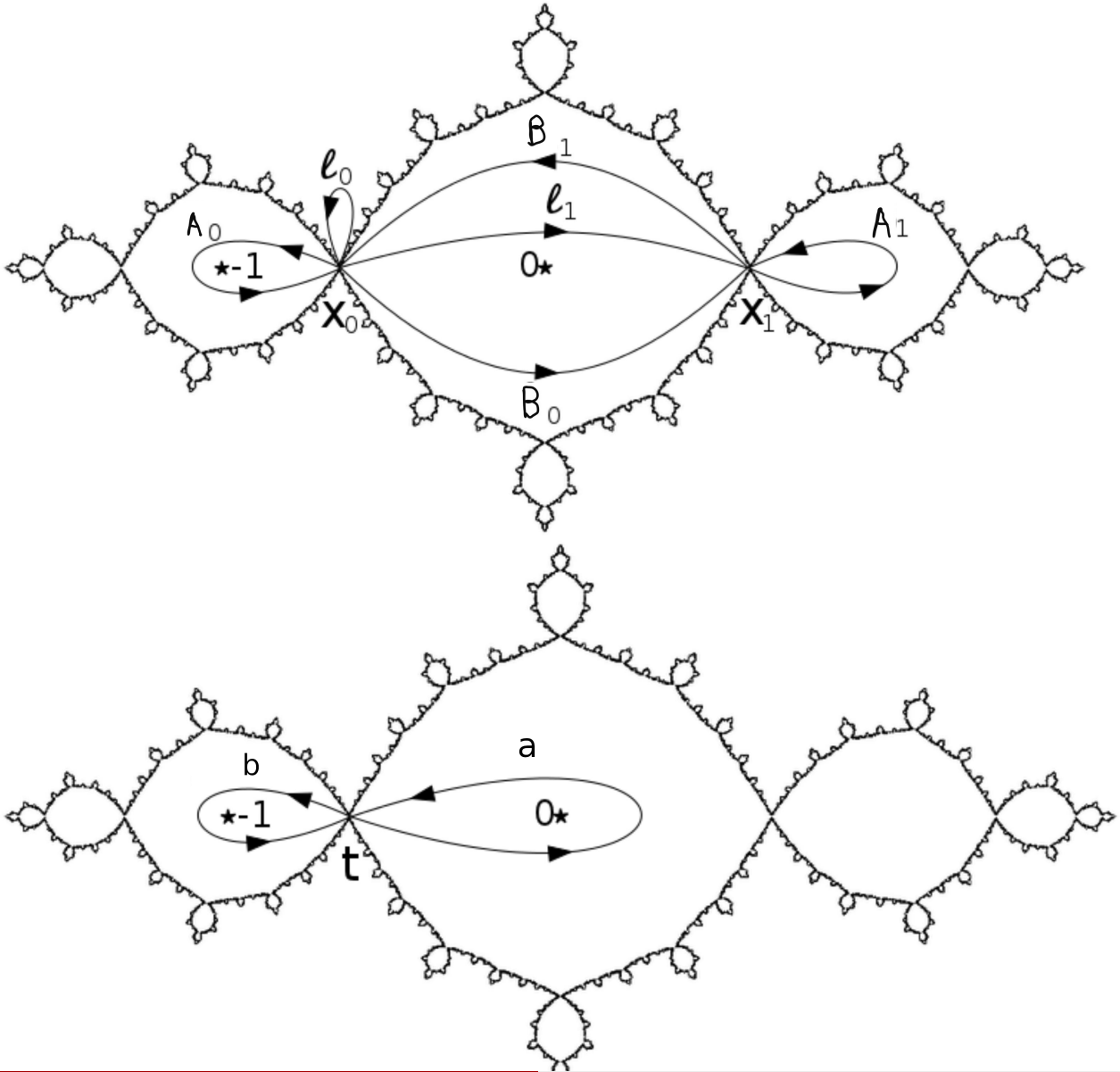
1. $f(z) = z^2 - 1$ (PCF map)
2. $f : \mathbb{C} \setminus \{0, 1, -1\} \to \mathbb{C} \setminus \{0, -1\}$ covering
3. $t = \dfrac{1 - \sqrt{5}}{2}$, $f(\pm t) = t$



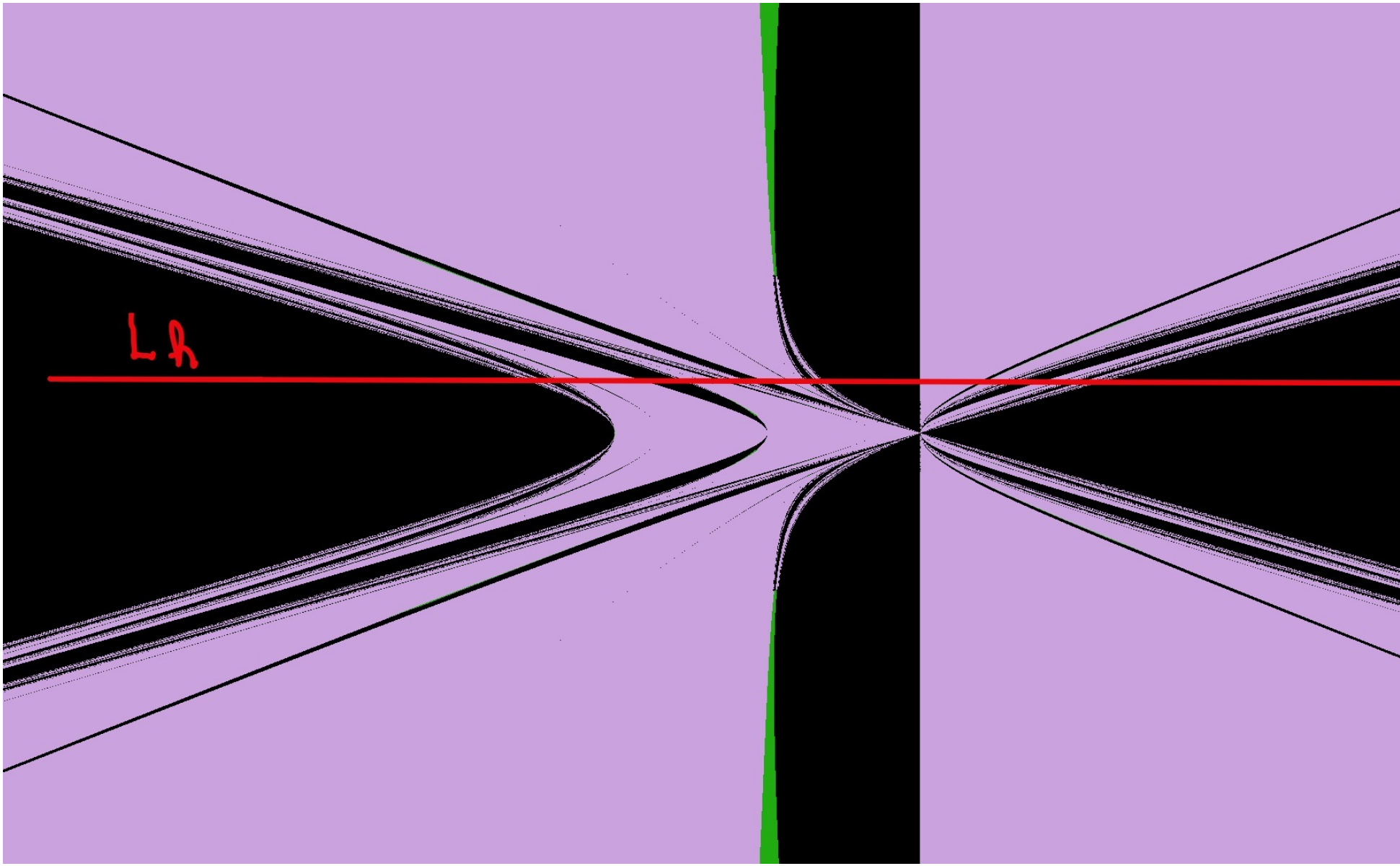
Iterated monodromy group

1. T_t: Hubbard tree of preimages of t
2. $\pi_1(\mathbb{C}\setminus\{0,-1\},t) =< a,b >$
3. $\rho_n : \pi_1(\mathbb{C}\setminus\{0,-1\},t) \to Aut(f^{-n}(t))$.
4. Basilica group $G = \pi_1(\mathbb{C}\setminus\{0,-1\},t)/\cap Ker\rho_n$.

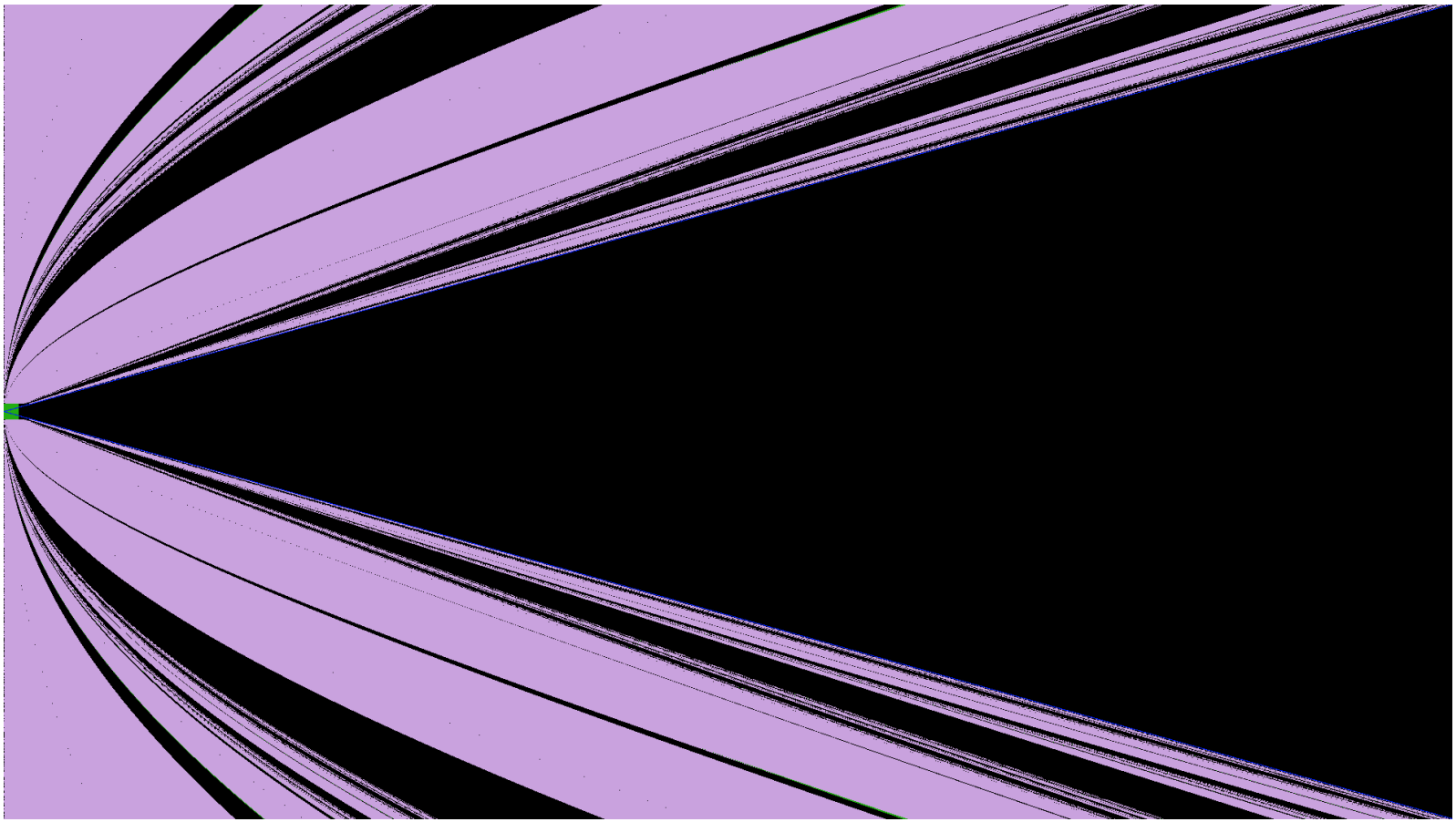
Level 1 monodromy action



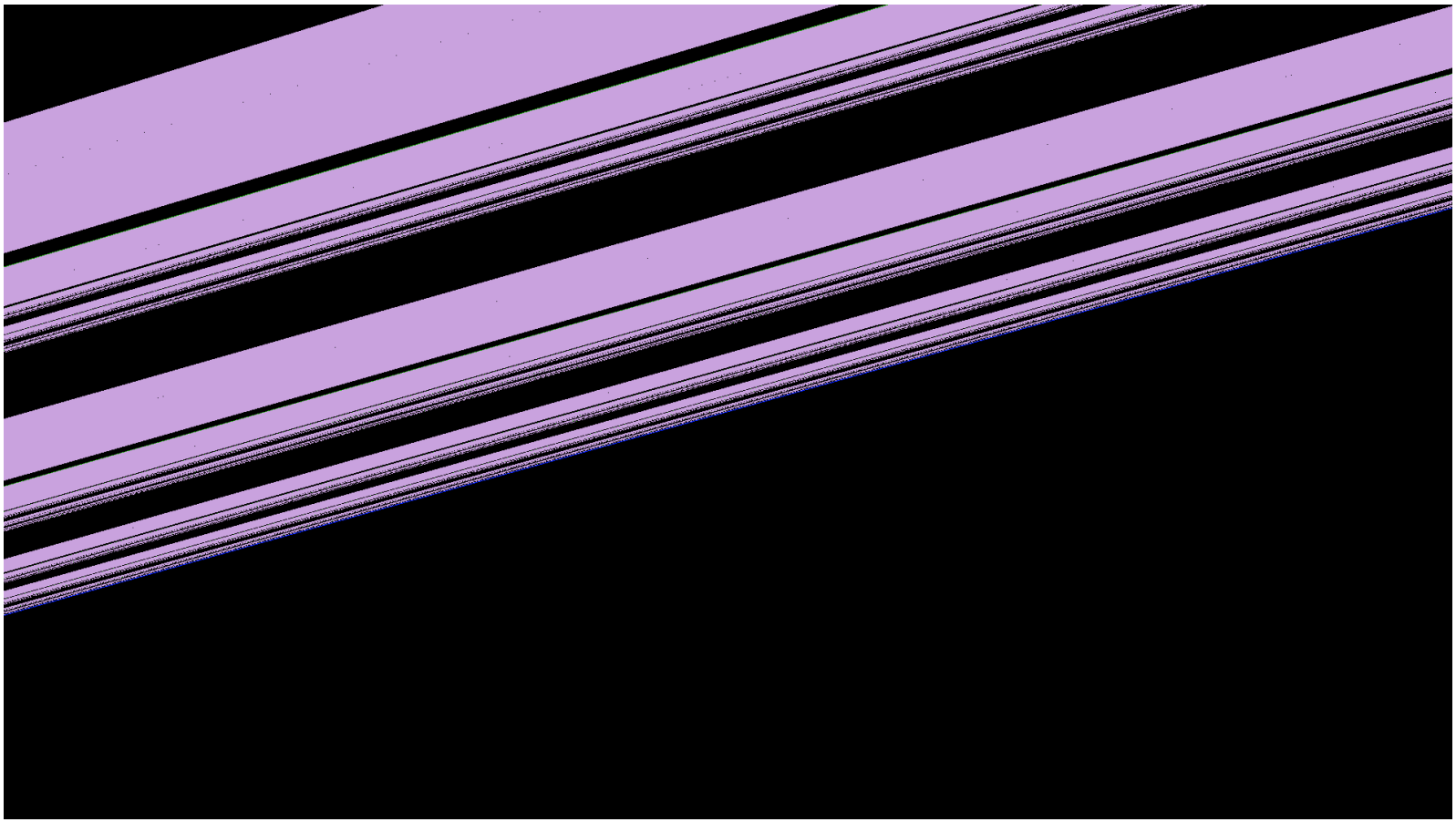
Spectrum and dynamics



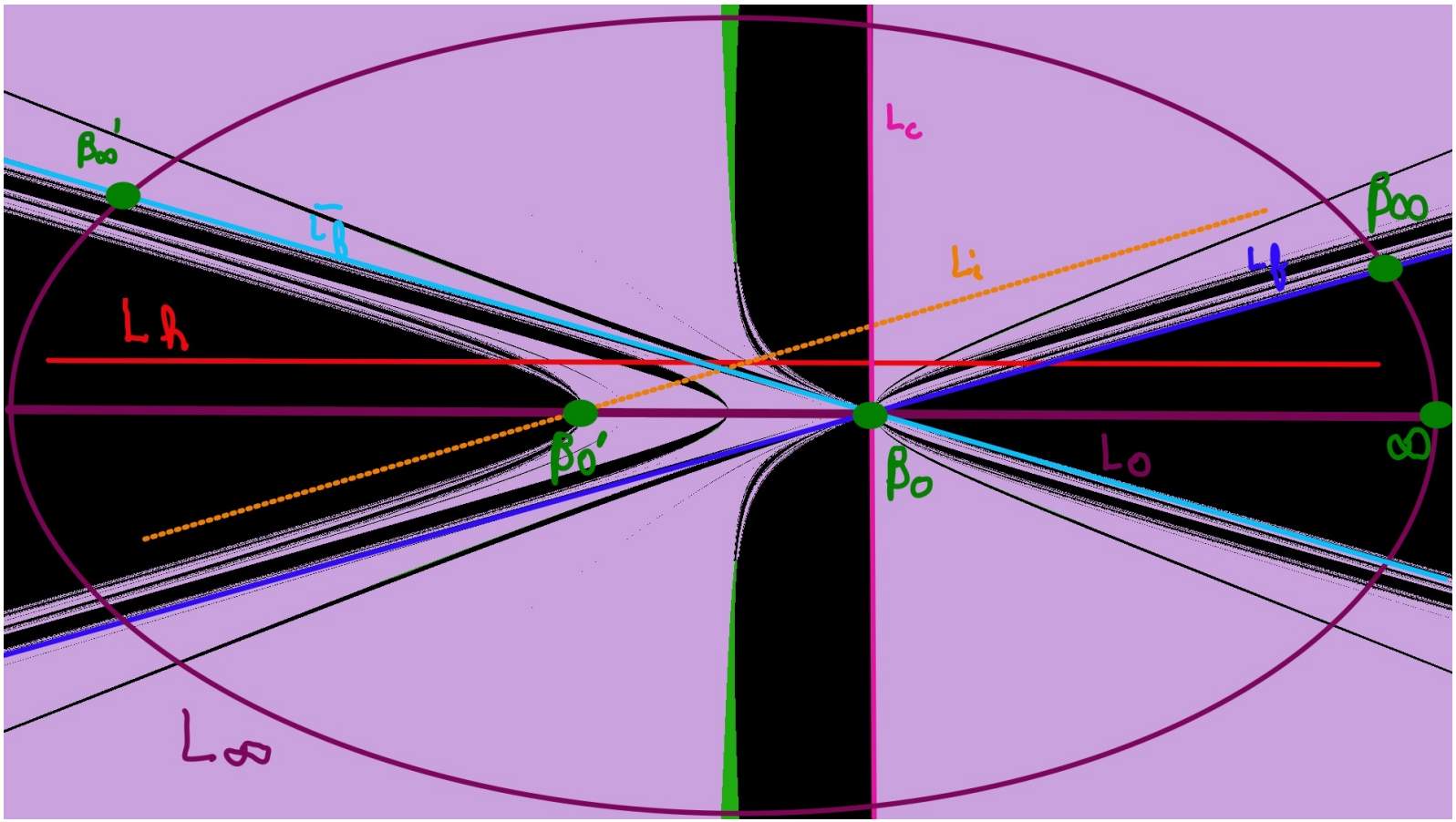
Novikov-Shubin invariant



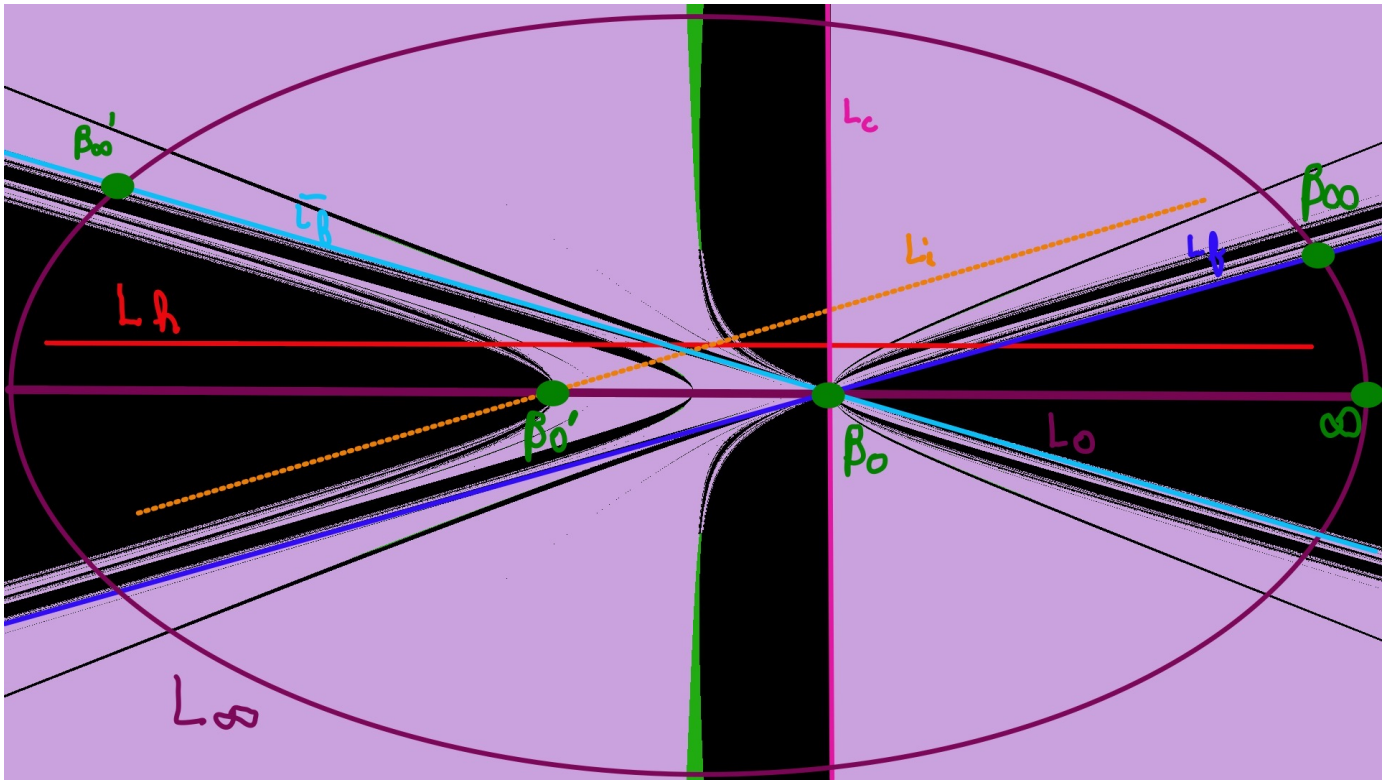
Novikov-Shubin invariant (zoom in)



Dynamical features of F



Dynamical features of F



$$\frac{1}{2^n}[\chi_n = 0] = \sum_{k=0}^{n-1} c_{n-1-k}\pi_* \frac{(\tilde{F}^k)^*}{2^k}[\tilde{L}_c] + \pi_* \frac{(\tilde{F}^n)^*[\tilde{L}_f]}{2^n} \qquad (1)$$

Asymptotic formulas

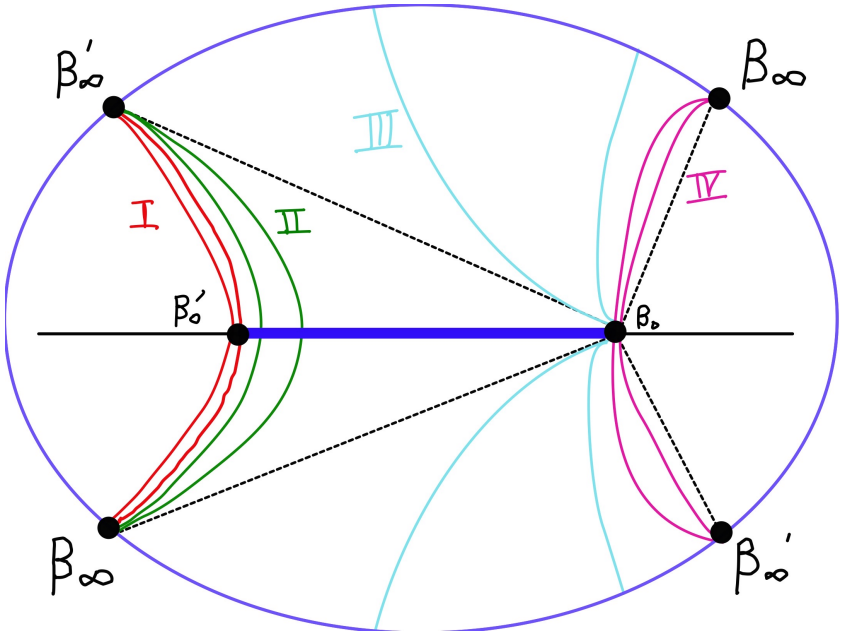
$$\frac{1}{2^n}[\chi_n = 0] = \sum_{k=0}^{n-1} c_{n-1-k}\pi_* \frac{(\tilde{F}^k)^*}{2^k}[\tilde{L}_c] + \pi_* \frac{(\tilde{F}^n)^*[\tilde{L}_f]}{2^n} \tag{2}$$

General term of the series: $\frac{(\tilde{F}^k)^*}{2^k}[\tilde{L}_c]$

$$T_B = \sum_{k=0}^{\infty} \alpha_k \pi_* \frac{(\tilde{F}^k)^*}{2^k}[\tilde{L}_c] \tag{3}$$

where $\alpha_k = \lim_n c_{n-1-k}$

Markov partition



$$\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$$