BIFURCATION LOCI OF FINITE TYPE FAMILIES OF MEROMORPHIC MAPS

Adventourous **B**erkeley **C**omplex **D**ynamics

MSRI, Berkeley

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(Based on joined work with **Matthieu Astorg** and **Anna Miriam Benini**)

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$J-$STABILITY

Definition (J-stability)

f_{λ_0} is $J-$**stable** iff there exists a nbhd U of λ_0 and a **holomorphic motion** of $J(f_{\lambda_0})$ over U preserving the dynamics, i.e.

$$\begin{array}{rccc} H: & U \times J_{\lambda_0} & \longrightarrow & J_\lambda \\ & (\lambda, z) & \longmapsto & H_\lambda(z) \end{array}$$

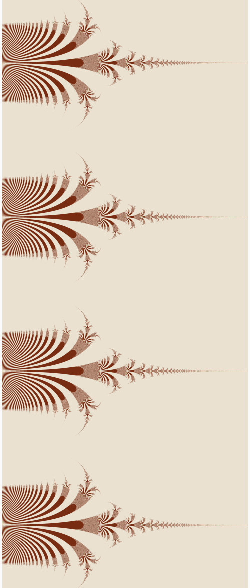
such that

- $z \mapsto H_\lambda(z)$ is injective for any $\lambda \in U$;
- $\lambda \mapsto H_\lambda(z)$ is holomorphic for any $z \in J_{\lambda_0}$;
- $H_{\lambda_0} = Id$
- $H_\lambda \circ f_{\lambda_0} = f_\lambda \circ H_\lambda$.

$\lambda-$Lemma [MSS'83]

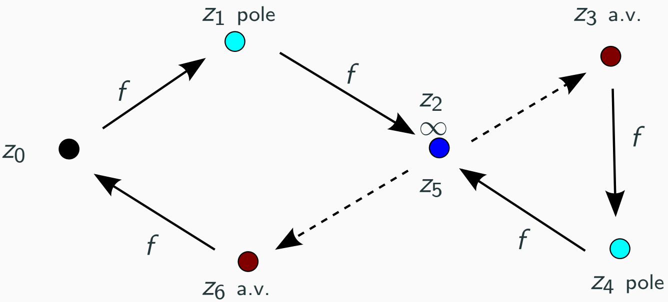
Any holomorphic motion H of a set $E \in \mathbb{C}$ estends to a holomorphic motion $\widehat{H}$ of $\overline{E}$, such that $\widehat{H}$ is jointly continuous and $\widehat{H}_\lambda(z)$ is quasiconformal.

FIXED POINTS DISAPPEARED TO INFINITY



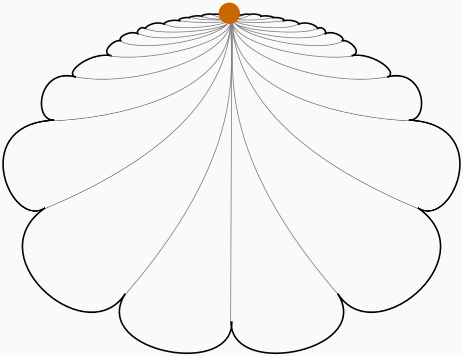
$$F_\lambda(z) = z + e^{-z}$$

VIRTUAL CYCLE OF PERIOD 6



$$\{z_0, z_1, \infty, z_3, z_4, \infty, z_6\}$$

ACCESSIBLE VIRTUAL CENTERS



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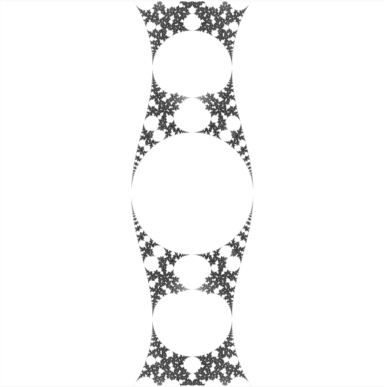
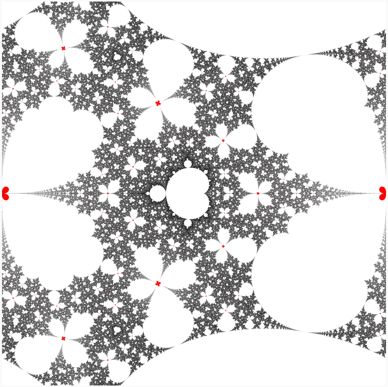
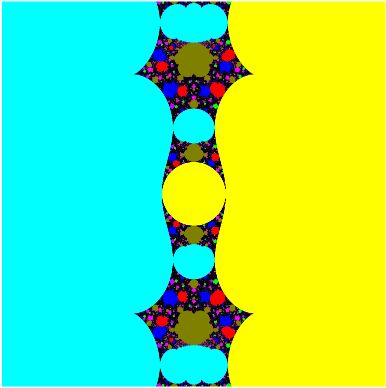




$T_\lambda(z) = \lambda \tan(z)$

$F_\lambda(z) = \pi \tan^2(z) + \lambda$



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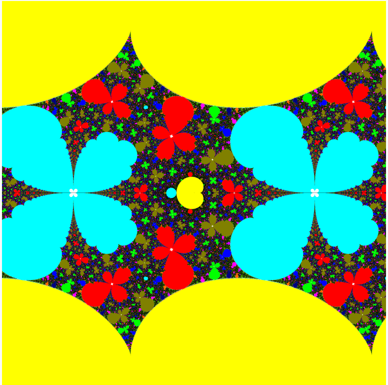

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$J-$stability Theorem

Theorem (Astorg-Benini-F)

$\{f_\lambda\}_{\lambda\in M}$ natural family of meromorphic maps of **finite type**. $U \subset M$ be a simply connected domain of parameters. Then, the following are equivalent.

(a) f_λ is ***J*-stable** for all $\lambda \in U$;

(b) Every singular value is **passive** on U;

(c) The **period** of attracting cycles is **bounded** in U;

(d) The number of **attracting cycles** is **constant** in U.

(e) U contains no non-persistent parabolic parameters.

Corollary

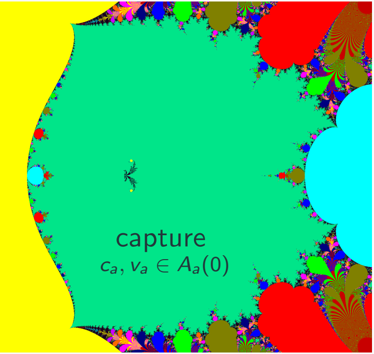
J-stable parameters form an open and dense set in M.

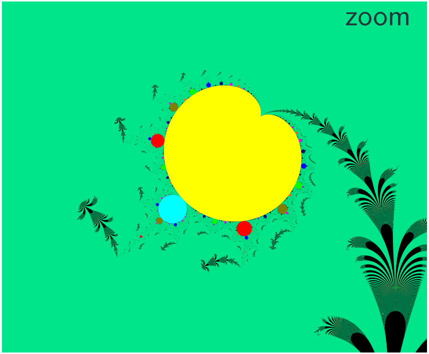
Thank you for your attention!

MSRI 1995

A (dynamically) natural family (1-D) [F-Keen'20]

$$f_a(z) = a\left(1 - \frac{ae^z}{(a+0.5)z+a}\right)$$





- attracting fixed point at $z = 0$, **constant multiplier** $\frac{1}{2}$
- persistent asymptotic value at ∞
- **Free asymptotic value** $v_a = a$ • **Free critical point** $c_a = \frac{1}{1+2a}$

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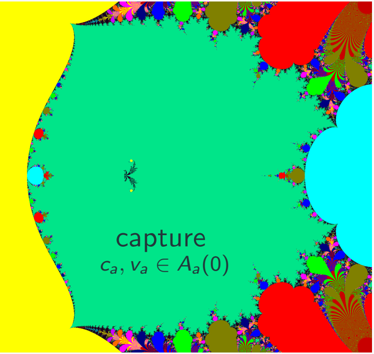

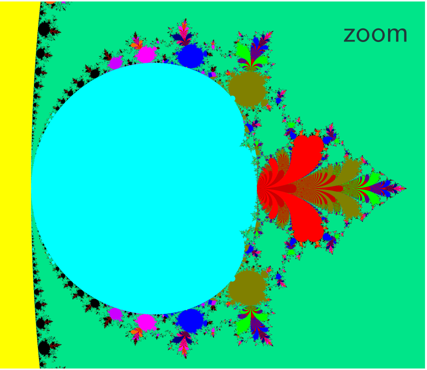

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Proof of activity

- Suppose $t \mapsto (\lambda(t), z(t))$ curve in P_n

s.t $\lambda(t) \underset{t\to\infty}{\longrightarrow} \lambda_0, \qquad z(t) \underset{t\to\infty}{\longrightarrow} \infty.$

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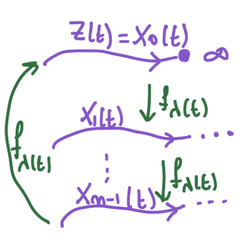

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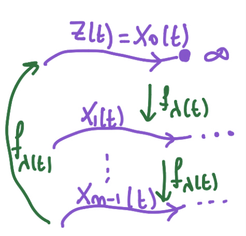

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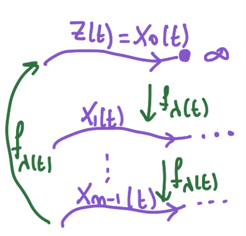

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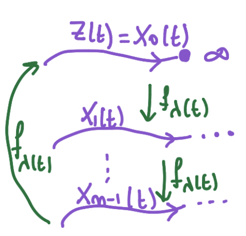
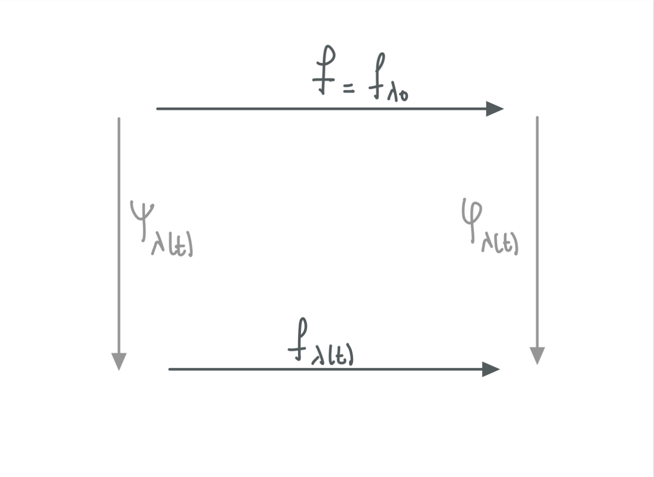


- Limit points eventually map to ∞.
- By discreteness of prepoles, $x_m(t)$ has a limit.
- Let $a_m = \lim_{t\to\infty} x_m(t) \in \widehat{\mathbb{C}}$.

Proof of activity

Assume $a_m = \infty$



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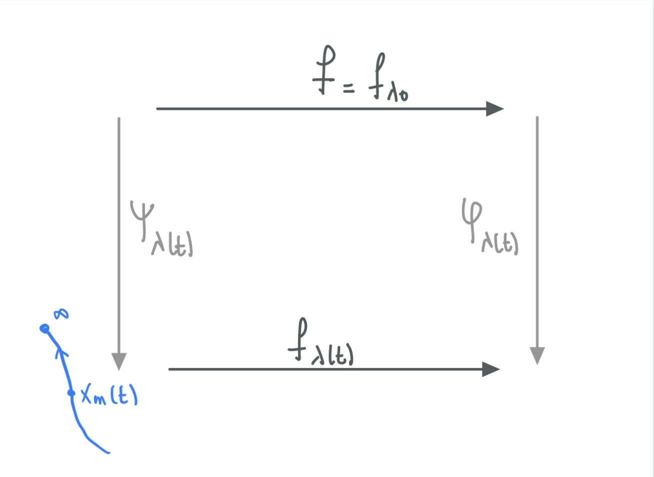

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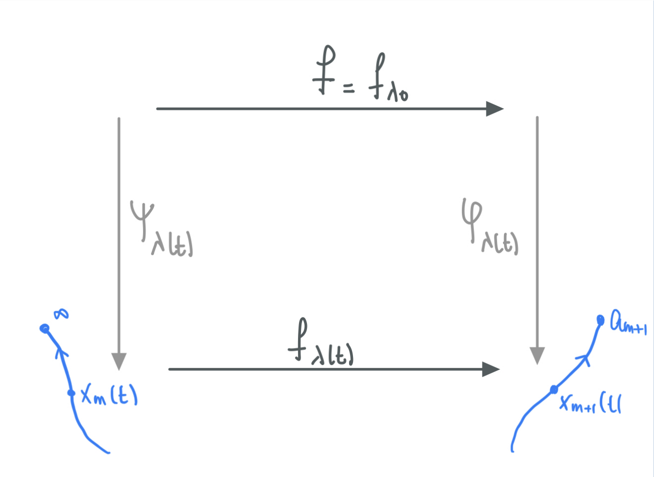

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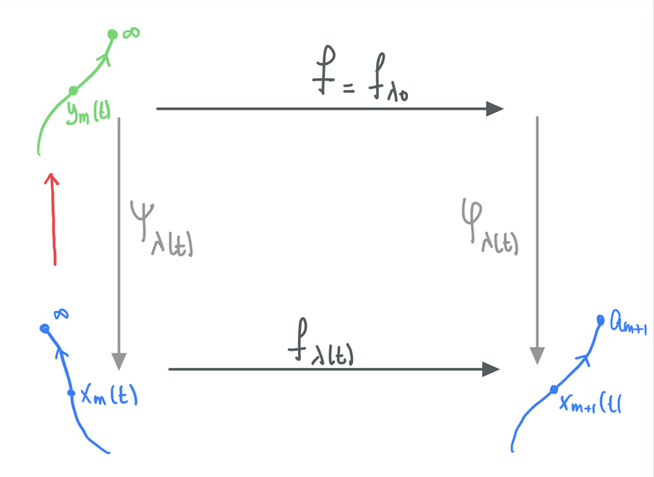

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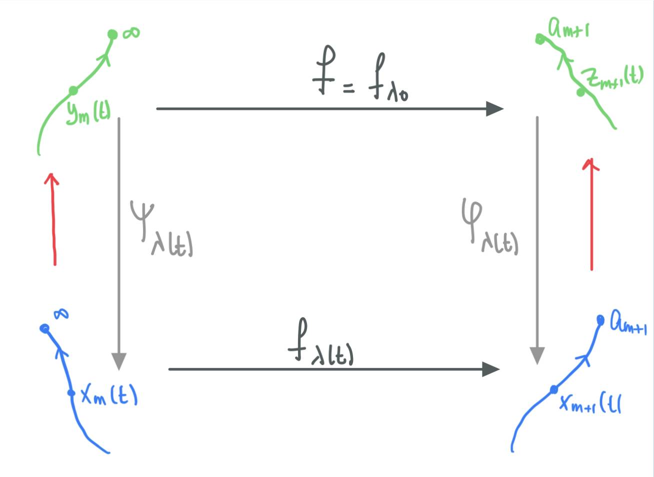

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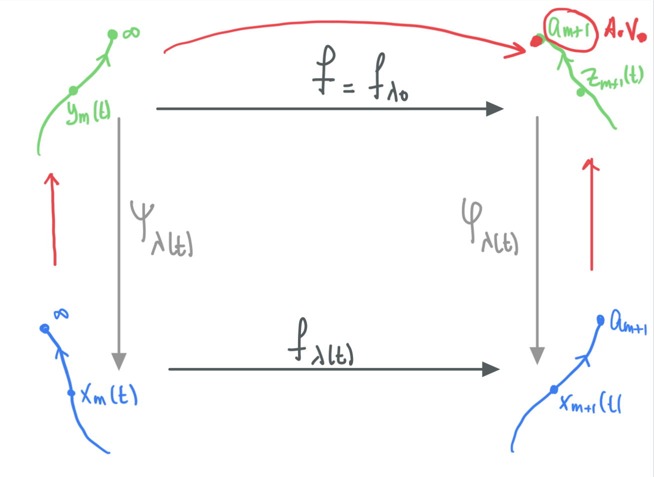

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