BROWN
THEORETICAL PHYSICS CENTER

How SUSY & Topology Led From Chern-Simons Theory To Solving A Forty Year-Old Mathematical Puzzle

S. James Gates, Jr.

Brown Theoretical Physics Center Director, Ford Foundation Professor of Physics,
Affiliate Mathematics Professor, and
Watson Institute for International & Public Affairs
Faculty Fellow

BTPC Director Office, Rm 110, Barus Hall, 340 Brook St., Providence, RI 02912
t: 401-863-6452 e: sylvester_gates@brown.edu w: https://sites.brown,edu/sjgates

Outline

1. Main Result – The Ectoplasmic Conjecture (EC)

2. Developmental Arc Description

3. Four Epiphany Moments (EM's)
   EM-1: No Extra Dimensions In SUSY QCD LEEA
   & SUSY Homotopy Operator
   EM-2: EC Appearance & Integration Theory
   EM-3: Superfield Chern-Simons Theory (CST),
   Minimal Homotopy & CST
   EM-5: EC & Adynkras

THE MAIN RESULT:

The Ectoplasmic Conjecture

The Ectoplasmic Conjecture

$$(\,x,\,\theta\,) = \mathcal{A}_d \,/\, SO(1, \mathrm{D}-1)$$

Given N supercharges in a 1D system, the minimum number d_{min} of bosons and equal number of fermions required to realize the N supercharges in a linear manner is given by

$$\mathrm{d}_{\min}(N) = \begin{cases} 2^{\frac{N-1}{2}}\,, & N \equiv 1,\,7 \quad \mathrm{mod}(8) \\ 2^{\frac{N}{2}}\,, & N \equiv 2,\,4,\,6 \quad \mathrm{mod}(8) \\ 2^{\frac{N+1}{2}}\,, & N \equiv 3,\,5 \quad \mathrm{mod}(8) \\ 2^{\frac{N-2}{2}}\,, & N \equiv 8 \quad \mathrm{mod}(8) \end{cases}$$

(where we exclude the case of $N = 0$ i.e. no supersymmetry)

Each higher dimensional superspace with D bosonic dimensions, (for purposes of counting) is equivalent to some value of d, which is the number of real components of θ. This is shown in a few cases below(where $d = \mathcal{F}(\mathrm{D})$).

d	D
4	4
8	5
16	10
32	11

Table 1: Relation Between D (Number of Spacetime) Dimensions and d For Some Superspaces

CONVENTIONS

$(-,+,+,\ldots,+)$ in every dimension and the corresponding Lorentz group is SO(1, D - 1)

The function $\mathcal{F}(\mathrm{D})$ for all values of D is given by applying Bott Periodicity to this table

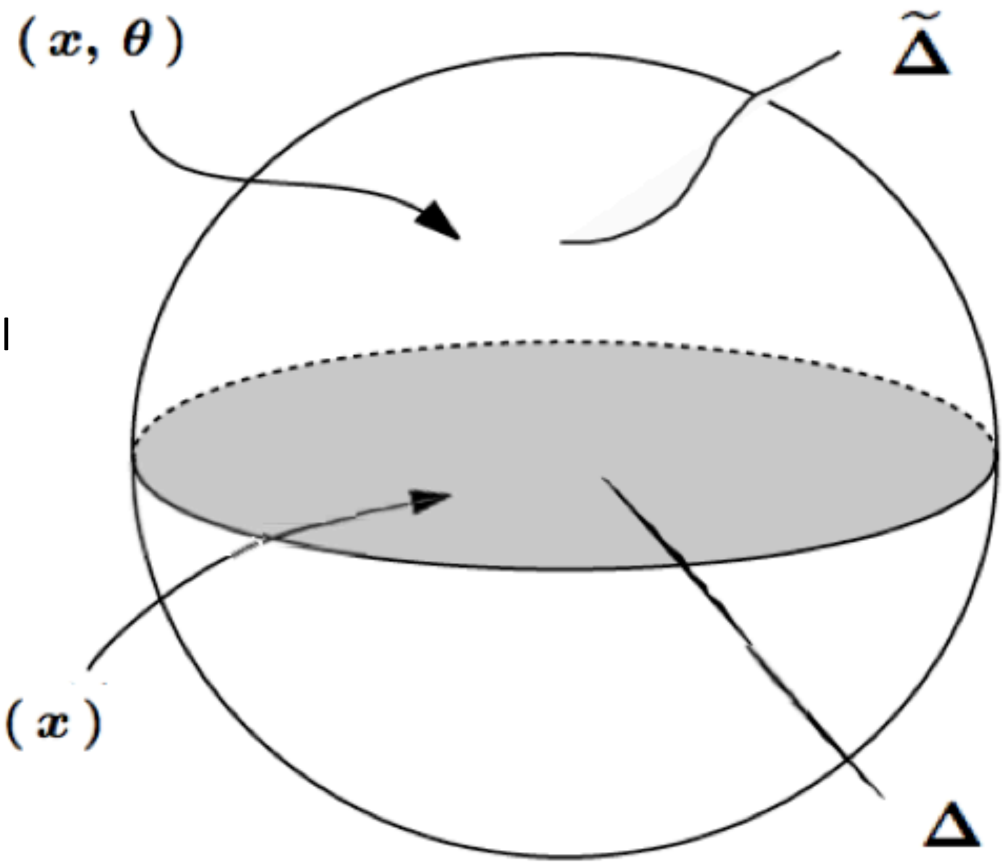
Spacetime Dimension	Lorentz Group	Type of Spinors	d
11	SO(1,10)	Majorana	32
10	SO(1,9)	Majorana-Weyl	16
9	SO(1,8)	Pseudo-Majorana	16
8	SO(1,7)	Pseudo-Majorana	16
7	SO(1,6)	SU(2)-Majorana	16
6	SO(1,5)	SU(2)-Majorana-Weyl	8
5	SO(1,4)	SU(2)-Majorana	8
4	SO(1,3)	Majorana/Weyl	4
3	SO(1,2)	Majorana	2
2	SO(1,1)	Majorana-Weyl	1

Summary of types of spinors in various dimensions

BROWN
THEORETICAL PHYSICS CENTER

A Representation of Superspace

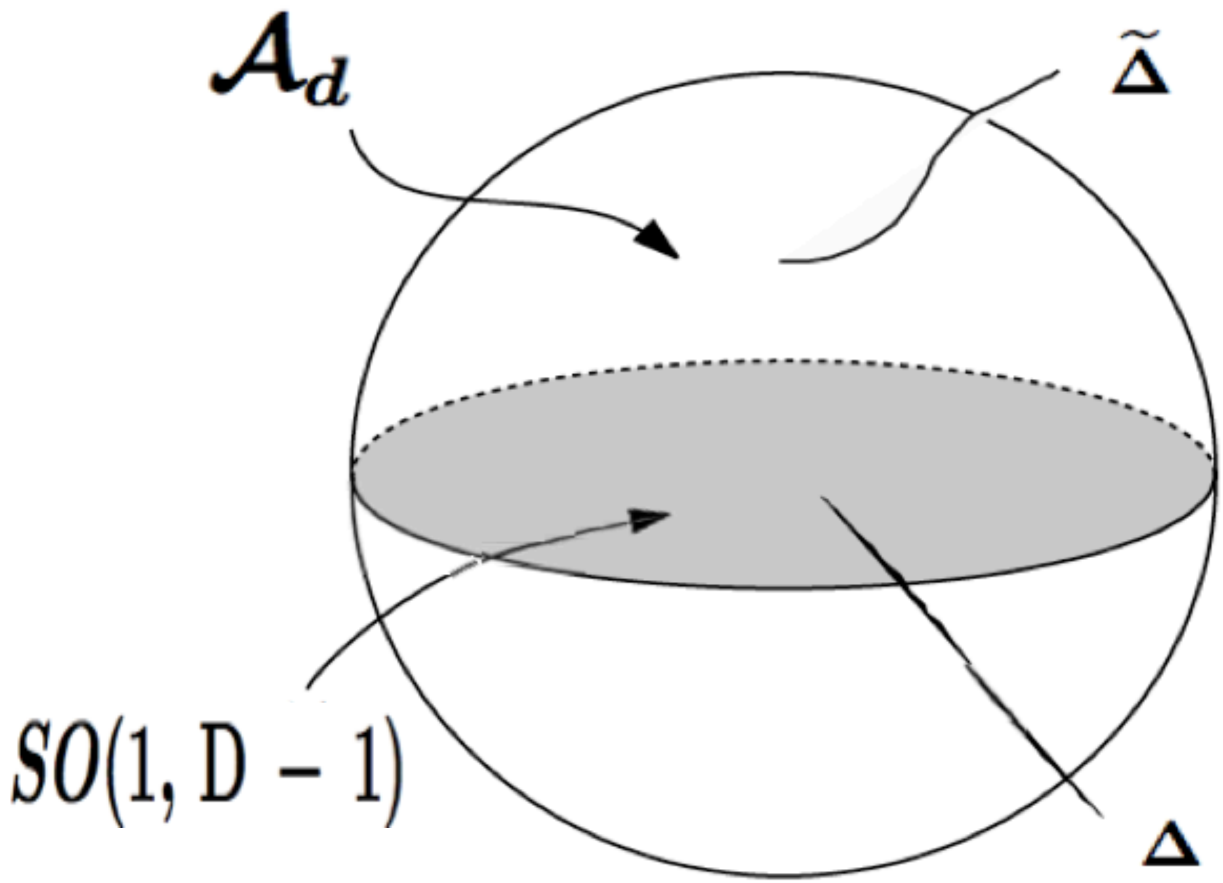
Ectoplasm:
Topological
'closeness'



The volume of the sphere represents the entirety of superspace and the equatorial plane represents the bosonic sub – space.

SJG
1997

A Representation of Superspace



The volume of the sphere represents the entirety of superspace and the equatorial plane represents the bosonic sub – space.

The Developmental Arc

From Start Until Now

Supersymmetry and Geometry in $D < 4$ Nonlinear σ Models

Gary Atkinson , Utpal Chattopadhyay , S.James Gates, Jr.

Published in: *Annals Phys.* 168 (1986) 387

Issues Addressed:

(a.) Lagrangian Reconstruction From Equations of Motion
(M. M. Vainberg, 1964; E. P. Hamilton & B. E. Goodwin, 1970

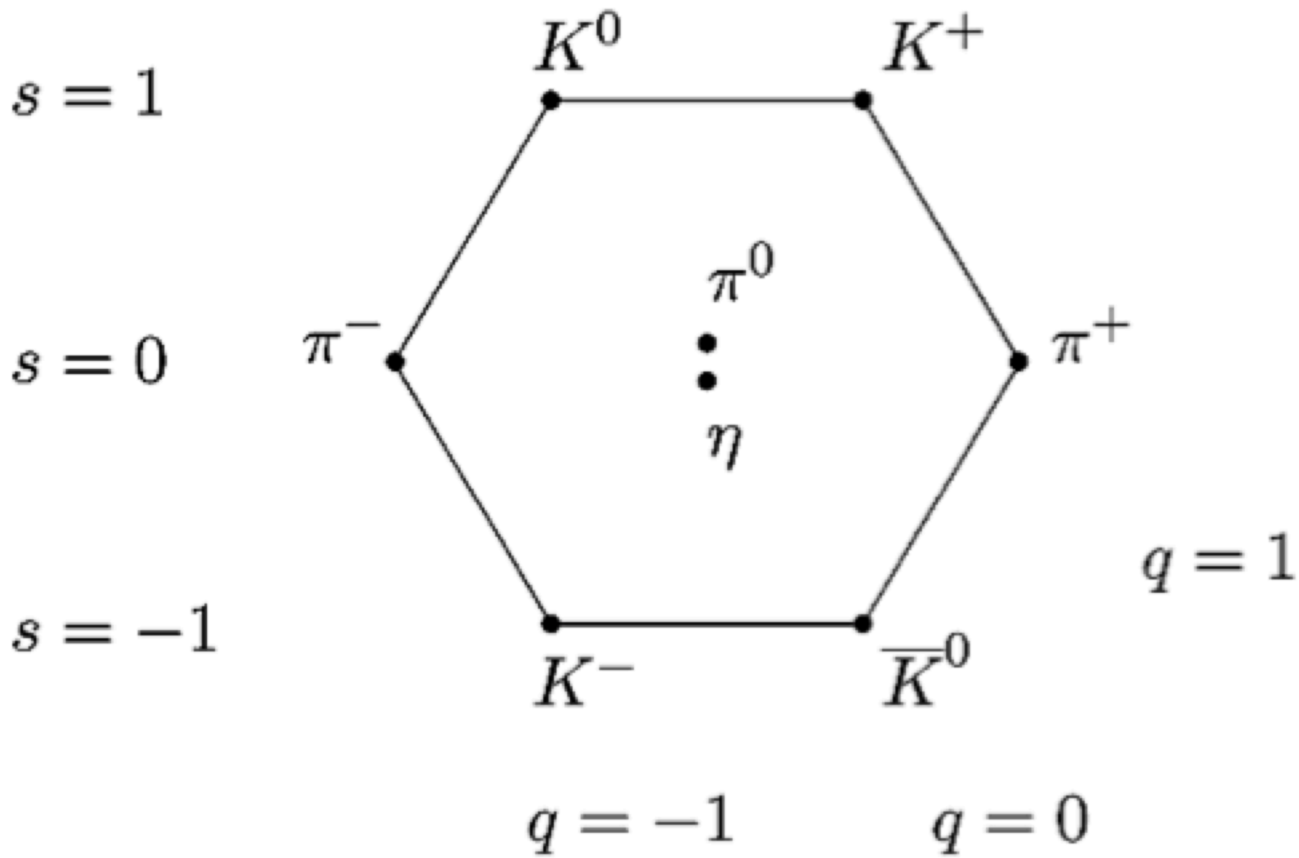
(b.) Novikov-Witten Proposal Extended To SUSY Theories

Conclusions Reached:

(a.) SUSY homotopy operators found for D < 4 Models

(b.) Interpretation of Novikov-Witten Picture Similar
to Interpretation of Berry Phases

Elementary Particles as Elements in the Weight Space of SU(3)



Map from $\mathcal{M}_4 : (t, \vec{x}) \to SU(3)$

$$U \;=\; exp\left[i\, f_\pi^{-1}\, 2\sqrt{2}\, \mathrm{M}\right] \quad , \quad \mathrm{M} \;=\; \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta^0 & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta^0 & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta^0 \end{pmatrix}$$

introduce of the derivations

$$d_\xi \;\equiv\; d\xi\,\frac{\partial}{\partial \xi} \quad , \quad d \;\equiv\; dt\,\frac{\partial}{\partial t} \;+\; dx\,\frac{\partial}{\partial x} \;+\; dy\,\frac{\partial}{\partial y} \;+\; dz\,\frac{\partial}{\partial z} \quad ,$$

$$\mathcal{A}_{NLM} \;=\; \frac{1}{16}\, f_\pi^2 \int d^4x \;\mathrm{Tr}\left[\left(dU^{-1}\right) \wedge \left({}^*d\,{}^*U\right)\right]$$

$$\left[\frac{\delta \mathcal{A}_{NSM}}{\delta \phi_k(x)}\right] \;\to\; d \wedge \left(U^{-1}\,{}^*d^*\,U\right) \;=\; 0$$

Witten's Observation: An extra symmetry not seen in experiments

$$\mathcal{B} : U \;\longleftrightarrow\; U^{-1} \quad , \quad \mathcal{P} : d^4x \;\to\; -\; d^4x$$

First consider an action $\mathcal{A}[\phi]$ dependent on fields $\phi_k(x)$ is given & yields Equations of Motion (EoM)

$$\left[\frac{\delta \mathcal{A}}{\delta \phi_k(x)}\right] = 0 \quad ,$$

where $\delta \mathcal{A}/\phi_k(x)$ is variational derivative of $\mathcal{A}[\phi]$,

$$\delta \mathcal{A}[\phi] = \int dx \left[\frac{\delta \mathcal{A}}{\delta \phi_k(x)}\right] \delta \phi_k(x)$$

Next only consider $\left[\frac{\delta \mathcal{A}}{\delta \phi_k(x)}\right]$. Is it possible to 'reconstruct' $\mathcal{A}[\phi]$?

Mathematician M. M. Vainberg proposed a solution (1964):

(1.)

New field variables

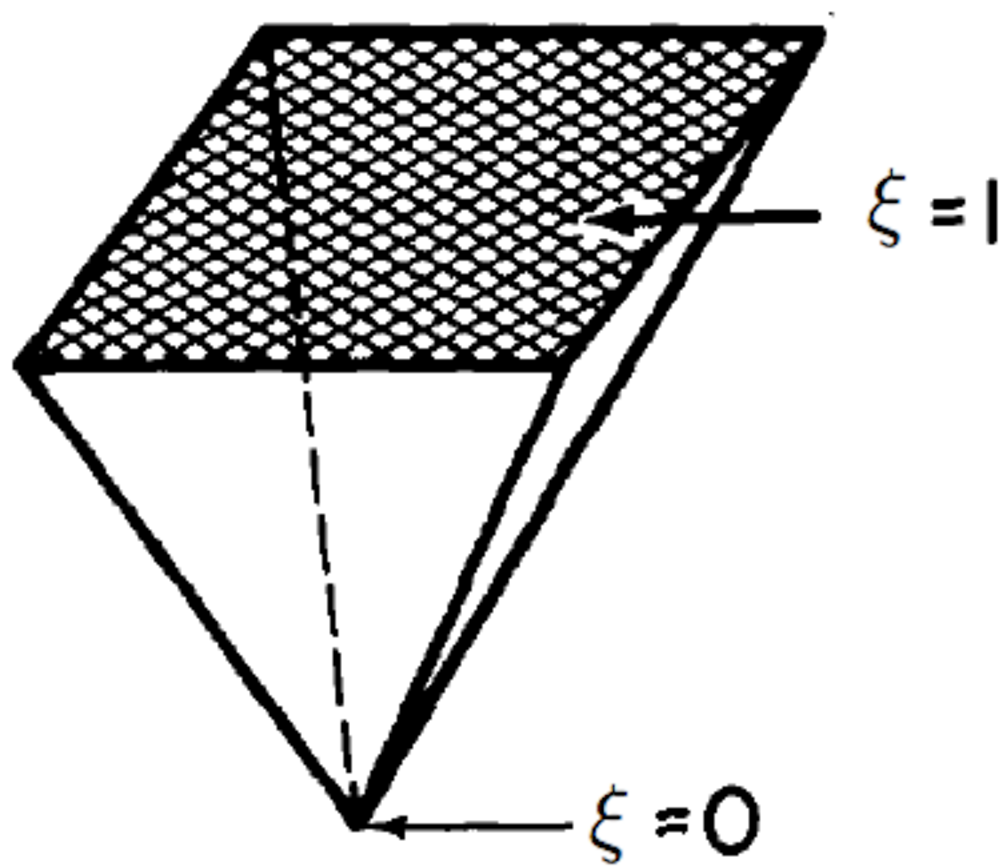
$\widehat{\phi}_k(x:\xi)$: $\quad 0 \leq \xi \leq 1 \quad \widehat{\phi}_k(x;\xi = 0) = 0, \quad \widehat{\phi}_k(x:\xi = 1) = \phi_k(x) \quad ,$

(2.)

Candidate Action

$$\mathcal{A}[\widehat{\phi}] = \int dx \int_0^1 d\xi \left[\frac{\delta \mathcal{A}}{\delta \phi_k(x)}\right]\Big|_{\phi=\phi(\tau)} \frac{d}{d\xi} \widehat{\phi}_k(x:\xi)$$

Elementary Particles as Elements in the Weight Space of SU(3)



Witten's Modification

$$\left[\frac{\delta \mathcal{A}}{\delta \phi_k(x)}\right] \rightarrow d \wedge (U^{-1}\, {}^*d^*U) \,+\, \lambda (U^{-1}\,dU)\wedge(U^{-1}\,dU)\wedge(U^{-1}\,dU)\wedge(U^{-1}\,dU)$$

$$\left[\frac{\delta \mathcal{A}}{\delta \phi_k(x)}\right]\Big|_{\phi=\phi(\tau)} \rightarrow d\wedge(\widehat{U}^{-1}\,{}^*d^*\widehat{U}) + \lambda(\widehat{U}^{-1}\,d\widehat{U})\wedge(\widehat{U}^{-1}\,d\widehat{U})\wedge(\widehat{U}^{-1}\,d\widehat{U})\wedge(\widehat{U}^{-1}\,d\widehat{U})$$

$$\frac{d}{d\xi}\,\widehat{\phi}_k(x:\xi) \rightarrow (\widehat{U}^{-1}d_\xi\widehat{U}) \qquad\qquad \mathrm{M} \rightarrow \xi\,\mathrm{M}$$

Witten's Modification of Vainberg's Candidate Action

$$\mathcal{A}[\widehat{\phi}] = \int dx \int_0^1 d\xi \left[\frac{\delta \mathcal{A}}{\delta \phi_k(x)}\right]\Big|_{\phi=\phi(\tau)} \frac{d}{d\xi}\,\widehat{\phi}_k(x:\xi)$$

leads to

$$\mathcal{A}_{NLM-W} = \frac{1}{16} f_\pi^2 \Big\{ \int d^4x \;\mathrm{Tr}\left[(dU^{-1})\wedge({}^*d^*U)\right] +$$

$$\lambda \int_0^1 d\xi\;\mathrm{Tr}\left[(\widehat{U}^{-1}\,d\widehat{U})\wedge(\widehat{U}^{-1}\,d\widehat{U})\wedge(\widehat{U}^{-1}\,d\widehat{U})\wedge(\widehat{U}^{-1}\,d\widehat{U})\,(\widehat{U}^{-1}d_\xi\widehat{U})\right] \Big\}$$

An identity of interest

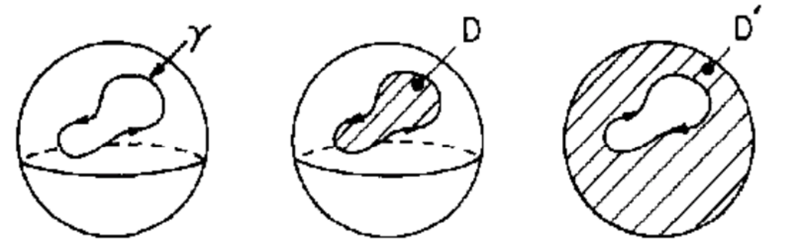
$$d_\xi \left\{\tfrac{1}{2}\,\mathrm{Tr}\left[\left(d\widehat{U}\right)\wedge\left({}^*d^*\widehat{U}^{-1}\right)\right]\right\} - d\wedge\left\{\mathrm{Tr}\left[\left(d_\xi\widehat{U}\right)\left({}^*d^*\widehat{U}^{-1}\right)\right]\right\} =$$

$$\mathrm{Tr}\left[\left(\widehat{U}^{-1}d_\xi\widehat{U}\right)d\wedge\left(\widehat{U}^{-1*}d^*\widehat{U}\right)\right] \quad .$$

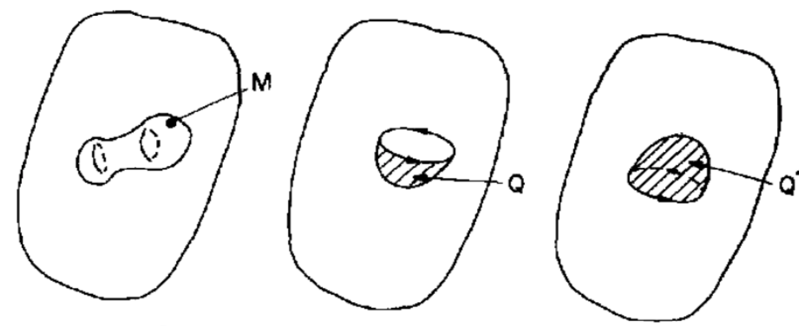
$$\exp\left(i\alpha\int_\gamma A_i dx^i\right) = \exp\left(i\alpha\int_D F_{ij}d\Sigma^{ij}\right)$$

$$\exp\left(i\alpha\int_\gamma A_i dx^i\right) = \exp\left(-i\alpha\int_{D'} F_{ij}d\Sigma^{ij}\right)$$

$$1 = \exp\left(i\alpha\int_{D+D'} F_{ij}d\Sigma^{ij}\right)$$



$$\int_{S_0^5} \omega_{ijklm} d\Sigma^{ijklm} = 2\pi$$



Novikov (1981) "Multivalued Functions and Functionals. An Analogue of the Morse Theory"

Witten (1983) "Global Aspects of Current Algebra"

The identity of interest does not require d_ξ and d t
to describe the tangent space of manifold.

(Echoes of
Berry phases)

BROWN
THEORETICAL PHYSICS CENTER

Remarks on the N=2 supersymmetric Chern-Simons theories

S.James Gates, Jr. Hitoshi Nishino

Published in: *Phys.Lett.B* 281 (1992) 72-80

Chern-Simons theories with supersymmetries in three-dimensions

Hitoshi Nishino S.James Gates, Jr.

Published in: *Int.J.Mod.Phys.A* 8 (1993) 3371-3422

Issues Addressed:

(a.) Superfield Lagrangian construction of 3D SUSY Chern-Simons Theory Plus Matter Couplings

Conclusions Reached:

(a.) Construction of supersymmetric Chern-Simons Theory with Extended SUSY and matter

Super connections
(Kahler geo. analogy)

$$\nabla_\alpha \equiv e^V D_\alpha e^{-V}, \quad \bar{\nabla}_\alpha \equiv e^{-V} \bar{D}_\alpha e^V,$$

$$\nabla_a \equiv -i \cdot \tfrac{1}{2} (\gamma_a)^{\alpha\beta} [\nabla_\alpha, \bar{\nabla}_\beta\} .$$

Super Yang-Mills
Field Strengths

$$[\nabla_\alpha, \nabla_\beta\} = 0 ,$$

$$[\nabla_\alpha, \bar{\nabla}_\beta\} = i (\gamma^c)_{\alpha\beta} \nabla_c + C_{\alpha\beta} S ,$$

$$[\nabla_\alpha, \nabla_b\} = (\gamma_b)_{\alpha\beta} \bar{W}^\beta , \quad [\nabla_a, \nabla_b\} = i F_{ab} .$$

Super 3-form

$$X^{(YM)}_{ABC} = \tfrac{1}{2} (A_{[A}{}^I F_{BC)}{}^I - \tfrac{1}{3} f^{IJK} A_{[A}{}^I A_B{}^J A_{C)}{}^K) .$$

Super 4-form

$$\tfrac{1}{6} D_{[A} X^{(YM)}_{BCD)} - \tfrac{1}{4} T_{[AB|}{}^E X^{(YM)}_{E|CD)} = \tfrac{1}{4} F_{[AB}{}^I F_{CD)}{}^I .$$

Irreducible
Decomposition of
Super 3-form
(Bianchi Identity
Implications

$$\nabla_\alpha S^I = -i \bar{W}_\alpha{}^I , \quad \nabla_\alpha \bar{W}_\beta{}^I = 0 ,$$

$$\nabla_\alpha F_{bc}{}^I = i (\gamma_{[b|})_{\alpha\beta} \nabla_{|c]} \bar{W}^\beta ,$$

$$\nabla_\alpha W_\beta{}^I = -\tfrac{1}{2} (\gamma^a)_{\alpha\beta} (\nabla_a S^I - i \cdot \tfrac{1}{2} \epsilon_a{}^{bc} F_{bc}{}^I)$$

$$+ i C_{\alpha\beta} D^I .$$

Vainberg Extension of Kahler-like potential of super-connections

$$\hat{V}(\theta, \bar{\theta}, x; \xi = 1) = V(\theta, \bar{\theta}, x) ,$$
$$\hat{V}(\theta, \bar{\theta}, x; \xi = 0) = 0 .$$

Vainberg Extension of Super-connections.

$$\hat{A}_\xi \equiv e^{\hat{V}} \partial_\xi e^{-\hat{V}}, \quad \bar{\hat{A}}_\xi \equiv e^{-\hat{V}} \partial_\xi e^{\hat{V}} .$$

Differential Equations between superconnections and Vainberg extensions.

$$[\hat{\nabla}_\alpha, \hat{A}_\xi\} = i\partial_\xi \hat{A}_\alpha , \quad [\bar{\hat{\nabla}}_\alpha, \bar{\hat{A}}_\xi\} = -i\partial_\xi \bar{\hat{A}}_\alpha ,$$
$$\partial_\xi \hat{S} = \tfrac{1}{2} C^{\alpha\beta}\left([\hat{\nabla}_\alpha, [\bar{\hat{\nabla}}_\beta, \bar{\hat{A}}_\xi\}\} - [\bar{\hat{\nabla}}_\alpha, [\hat{\nabla}_\beta, \hat{A}_\xi\}\}\right)$$

$$\mathscr{L}_{\text{Anyon}} = \mathscr{L}_{\text{SM}} + \mathscr{L}_{\text{CS}} + \mathscr{L}_{\text{Pot}} + \mathscr{L}_{\text{h}D}$$

$$I_{\text{CS}} = -\text{tr} \int \mathrm{d}^3x\, \mathrm{d}^4\theta \int_0^1 d\xi\, [\hat{A}_\xi(\hat{S} + \mathrm{i}\tfrac{1}{3}[\hat{A}^\alpha, \hat{\bar{A}}_\alpha\}) + \text{h.c.}]$$

$$I_{\text{SM}} = \int \mathrm{d}^3x\, \mathrm{d}^4\theta\, \bar{\Phi}\, \mathrm{e}^{-V}\, \Phi\,,$$

$$I_{\text{Pot}} = \int \mathrm{d}^3x [\mathrm{d}^2\theta\, (\tfrac{1}{2}M\Phi^2 + \tfrac{1}{3}\lambda\Phi^3 + \tfrac{1}{4}\gamma\Phi^4) + \text{h.c.}]\,.$$

$$I_{\text{h}D} = \int \mathrm{d}^3x\, \mathrm{d}^4\theta\, \text{h}^I V^I$$

$$\mathcal{L}_{\rm CS} = \tfrac{1}{2} m \epsilon^{\mu\nu\rho} (F_{\mu\nu}{}^I A_\rho{}^I - \tfrac{1}{3} f^{IJK} A_\mu{}^I A_\nu{}^J A_\rho{}^K)$$
$$- 2m\bar{\lambda}^I \lambda^I - 2mS^I D^I .$$

$$\mathcal{L}_{{\rm h}D} = \int d^3x\, d^4\theta\, {\rm h}^I D^I$$

$$\mathcal{L}_{\rm SM} = 2|\mathscr{D}_\mu A_i|^2 + i\bar{\chi}^i \not{\mathscr{D}} \chi_i + 2|F_i|^2$$
$$+ i(T_I)_i{}^j (\bar{\chi}^i \chi_j) S^I + 2i(T_I)_j{}^i D^I A_i A^{*j}$$
$$+ [2i(T_I)_j{}^i (\bar{\lambda}^I \chi_i) A^{*j} + 2i(T_I)_i{}^j (\lambda^I \bar{\chi}^i) A_j]$$
$$+ \{T_I, T_J\}_i{}^j S^I S^J A_j A^{*i} ,$$

$$\mathcal{L}_{\rm Pot} = [F_i W^i + \tfrac{1}{4} (\chi_i \chi_j) W^{ij}] + {\rm c.c.} ,$$
$$W^i \equiv \frac{\partial W(A)}{\partial A_i} , \quad W^{ij} \equiv \frac{\partial^2 W(A)}{\partial A_i\, \partial A_j} .$$

$$(\bar{A} T_I A) \equiv (T_I)_i{}^j A_j A^{*i} ,$$
$$(\bar{\chi} T_I \chi) \equiv (T_I)_i{}^j \bar{\chi}^i \chi_j , \quad \text{etc.}$$

$$\mathscr{L}_{\text{Anyon}} = \mathscr{L}_{\text{SM}} + \mathscr{L}_{\text{CS}} + \mathscr{L}_{\text{Pot}} + \mathscr{L}_{\text{h}D}$$

$$\begin{aligned}
\mathscr{L}_{\text{Anyon}} &= 2|\mathscr{D}_\mu A_i|^2 + \mathrm{i}\bar{\chi}^i \not{\mathscr{D}} \chi_i - \tfrac{1}{2}|W^i|^2 \\
&+ [\tfrac{1}{4}(\chi_i \chi_j) W^{ij} + \text{c.c.}] \\
&+ \tfrac{1}{2} m \epsilon^{\mu\nu\rho} (F_{\mu\nu}{}^I A_\rho{}^I - \tfrac{1}{3} f^{IJK} A_\mu{}^I A_\nu{}^J A_\rho{}^K) \\
&- \frac{1}{m} [(\bar{A} T_I A) - \tfrac{1}{2}\mathrm{i}\xi_I](\bar{\chi} T_I \chi) - \frac{2}{m}(\bar{\chi} T_I A)(\bar{A} T_I \chi) \\
&- \frac{1}{m^2}(\bar{A}\{T_I, T_J\}A)[(\bar{A} T_I A) - \tfrac{1}{2}\mathrm{i}\mathrm{h}_I] \\
&\times [(\bar{A} T_J A) - \tfrac{1}{2}\mathrm{i}\mathrm{h}_J] ,
\end{aligned}$$

$\mathscr{L}_{\text{Anyon}}$	Conformal Limit	$\mathscr{L}_{\text{ABJM}}$
1993		2008

Ectoplasm has no topology: The Prelude

S.James Gates, Jr.

Contribution to: SQS'97, 46-57 • e-Print: hep-th/9709104 [hep-th]

Component actions from curved superspace: Normal coordinates and ectoplasm

S.James Gates, Jr. Marcus T. Grisaru Marcia E. Knutt-Wehlau Warren Siegel

Published in: *Phys.Lett.B* 421 (1998) 203-210 • e-Print: hep-th/9711151 [hep-th]

Ectoplasm has no topology

S.James Gates, Jr.

Published in: *Nucl.Phys.B* 541 (1999) 615-650 • e-Print: hep-th/9809056 [hep-th]

$\tilde{\Delta}$ The topological 'closeness' of indices calculated in the superspace volume and the bosonic submanifold leads to SUSY measures in the latter. Δ

Issues Addressed

(a.) Exploration of the Ectoplasmic Conjecture to construct measures for superspace in the presence of curvature of the bosonic supermanifold

Conclusions Reached:

(a.) Principles and Construction of measures (supergravity projection operators) enunciated and examples given of technique.

BROWN
THEORETICAL PHYSICS CENTER

Holomorphy, minimal homotopy and the 4-D, N=1 supersymmetric Bardeen-Gross-Jackiw anomaly

S.James Gates, Jr. Marcus T. Grisaru Silvia Penati

Published in: *Phys.Lett.B* 481 (2000) 397-407 • e-Print: hep-th/0002045 [hep-th]

Supersymmetric gauge anomaly with general homotopic paths

S.James Gates, Jr. Marcus T. Grisaru Marcia E. Knutt-Wehlau Silvia Penati Hiroshi Suzuki

Published in: *Nucl.Phys.B* 596 (2001) 315-347 • e-Print: hep-th/0009192 [hep-th]

The Superspace WZNW action for 4-D, N=1 supersymmetric QCD

S.James Gates, Jr. Marcus T. Grisaru Marcia E. Knutt-Wehlau Silvia Penati

Published in: *Phys.Lett.B* 503 (2001) 349-354 • e-Print: hep-ph/0012301 [hep-ph]

Substantial computational advantages occur in supersymmetrical theories by use of the 'minimal homotopy operator.'

$$\widehat{U}(\xi) \equiv \mathbf{I} + \xi\,(\,U - \mathbf{I}\,)$$

$$\widehat{U}(\xi = 0) \equiv \mathbf{I}$$

$$\widehat{U}(\xi = 1) \equiv U$$

This is important as in some SUSY models, the Kahler-like potential for gauge superconnections takes the from of an exponential of a superfield valued over a Lie algebra.

3D, N = 2 SUSY Chern-Simons Theory possesses this feature and the minimal homotopy operator confers substantial computational efficiency in the study of such theories via supergraph computations.

The 4,294,967,296 Problem

Inspired by the understanding of how the topology of continuous manifold and that of the space fields and superfields are related suggested that supersymmetry and its representation theory might be connected to the topology of graphs and polytopes.

D	d	2^d	n_B	n_F
4	4	16	8	8
5	8	256	128	128
10	16	65,536	32,768	32,768
11	32	4,294,967,296	2,147,483,648	2,147,483,648

Number of independent components in *unconstrained* scalar superfields in D dimensional spacetime

4D Minimal Off-Shell Supermultiplet	d	2^{d-1}	$n_{B(min)}$	$n_{F(min)}$
$\mathcal{N} = 1$ Chiral	4	8	4	4
$\mathcal{N} = 2$ Vector	8	128	8	8
$\mathcal{N} = 4$ SG	16	32,768	128	128

Number of independent components in *maximally constrained* superfields

The Salam-Strathdee 4D, N = 1 Real Scalar Superfield

$$
\begin{aligned}
\mathcal{V}(x^{\underline{a}}, \theta^\alpha) \; &= v^{(0)}(x^{\underline{a}}) \; + \; \theta^\alpha \, v^{(1)}_\alpha(x^{\underline{a}}) \; + \; \theta^\alpha \theta^\beta \, v^{(2)}_{\alpha\beta}(x^{\underline{a}}) \\
&\quad + \; \theta^\alpha \theta^\beta \theta^\gamma \, v^{(3)}_{\alpha\beta\gamma}(x^{\underline{a}}) \; + \; \theta^\alpha \theta^\beta \theta^\gamma \theta^\delta \, v^{(4)}_{\alpha\beta\gamma\delta}(x^{\underline{a}})
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{V}(x^{\underline{a}}, \theta^\alpha) \; &= f(x^{\underline{a}}) \; + \; \theta^\alpha \, \psi_\alpha(x^{\underline{a}}) \; + \; \theta^\alpha \theta^\beta C_{\alpha\beta} \, g(x^{\underline{a}}) \\
&\quad + \; \theta^\alpha \theta^\beta i(\gamma^5)_{\alpha\beta} \, h(x^{\underline{a}}) \; + \; \theta^\alpha \theta^\beta i(\gamma^5 \gamma^{\underline{b}})_{\alpha\beta} \, v_{\underline{b}}(x^{\underline{a}}) \\
&\quad + \; \theta^\alpha \theta^\beta \theta^\gamma C_{\alpha\beta} C_{\gamma\delta} \, \chi^\delta(x^{\underline{a}}) \; + \; \theta^\alpha \theta^\beta \theta^\gamma \theta^\delta C_{\alpha\beta} C_{\gamma\delta} \, N(x^{\underline{a}})
\end{aligned}
$$

$$\mathcal{V}(x^{\underline{a}}, \theta^{\alpha}) \; = v^{(0)}(x^{\underline{a}}) \; + \; \theta^{\alpha} \, v^{(1)}_{\alpha}(x^{\underline{a}})$$

$$+ \; \theta^{\alpha}\theta^{\beta} \Big[\, C_{\alpha\beta} \, v^{(2)}_{1}(x^{\underline{a}}) \; + \; i(\gamma^{5})_{\alpha\beta} \, v^{(2)}_{2}(x^{\underline{a}}) \; + \; i(\gamma^{5}\gamma^{\underline{b}})_{\alpha\beta} \, v^{(2)}_{\underline{b}}(x^{\underline{a}}) \, \Big]$$

$$+ \; \theta^{\alpha}\theta^{\beta}\theta^{\gamma} C_{\alpha\beta} C_{\gamma\delta} \, v^{(3)\delta}(x^{\underline{a}}) \; + \; \theta^{\alpha}\theta^{\beta}\theta^{\gamma}\theta^{\delta} C_{\alpha\beta} C_{\gamma\delta} \, v^{(4)}(x^{\underline{a}})$$

Level	Adinkra nodes	Component fields	Irrep(s) in $\mathfrak{so}(4)$
0	$\mathcal{V}\vert$	$f(x^{\underline{a}})$	{1}
1	$\mathrm{D}_{\alpha}\mathcal{V}\vert$	$\psi_{\alpha}(x^{\underline{a}})$	{4}
2	$\mathrm{D}_{[\alpha}\mathrm{D}_{\beta]}\mathcal{V}\vert$	$g(x^{\underline{a}})$, $h(x^{\underline{a}})$, $v_{\underline{b}}(x^{\underline{a}})$	{1}, {1}, {4}
3	$\mathrm{D}_{[\alpha}\mathrm{D}_{\beta}\mathrm{D}_{\gamma]}\mathcal{V}\vert$	$\chi^{\delta}(x^{\underline{a}})$	{4}
4	$\mathrm{D}_{[\alpha}\mathrm{D}_{\beta}\mathrm{D}_{\gamma}\mathrm{D}_{\delta]}\mathcal{V}\vert$	$N(x^{\underline{a}})$	{1}

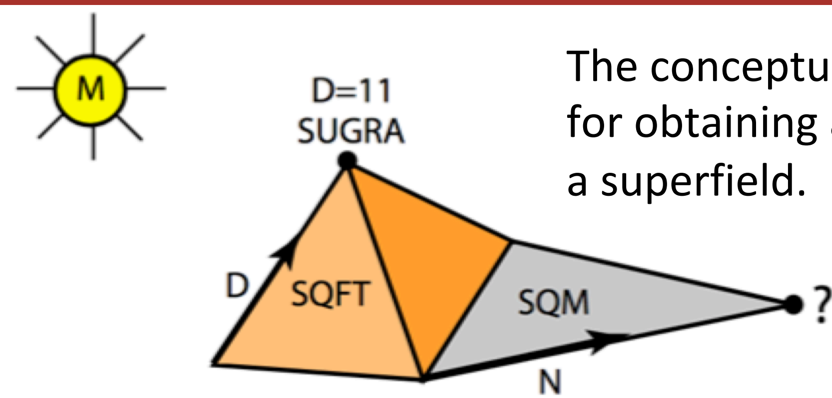
Explicit Relations between Adinkra Nodes, Component Fields, and Irreps

Q: What representations of SO(1,10) occur among the 4,294,967,296 degrees of freedom in the scalar superfield?

A: Until 2020, the answer was an unresolved puzzle.

BROWN
THEORETICAL PHYSICS CENTER

arXiv.org > hep-th > arXiv:hep-th/0408004



The conceptual background for obtaining an adinkra from a superfield.

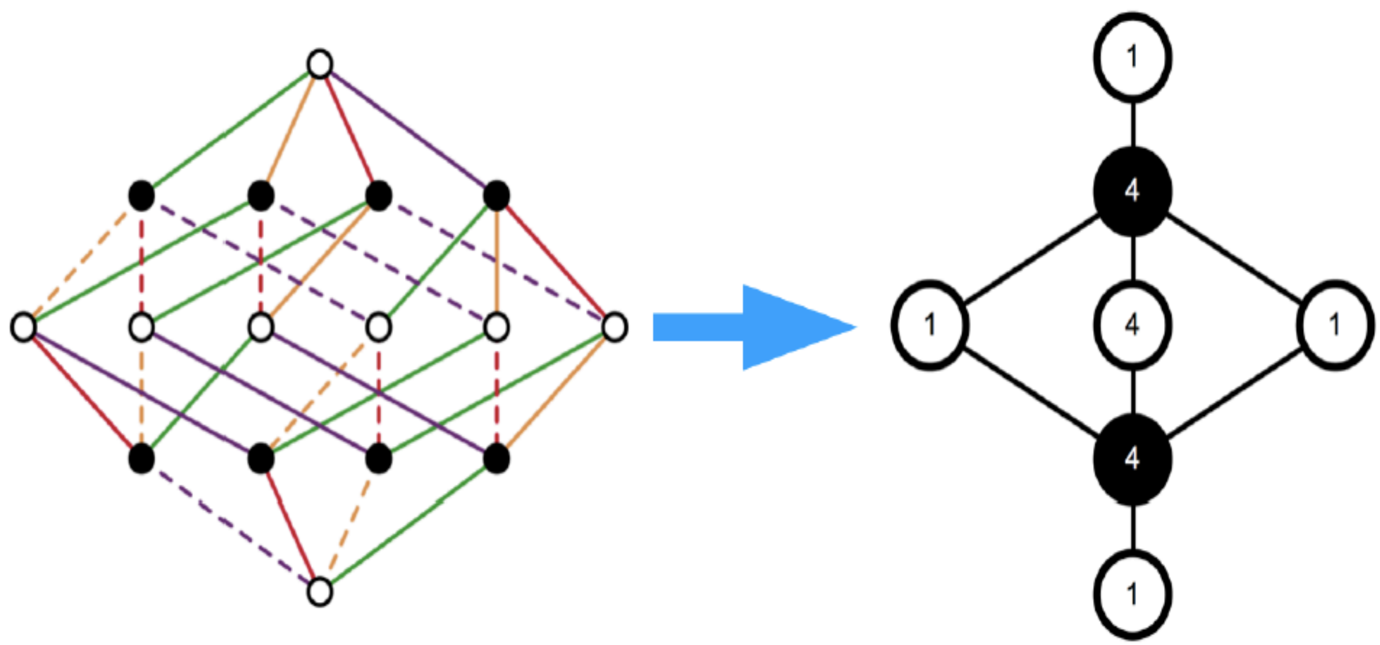
Each supersymmetric quantum field theory has a "shadow" in supersymmetric quantum mechanics obtained by dimensionally reducing all of the spatial dimensions in the field theory.

Adinkras: A Graphical Technology for Supersymmetric Representation Theory

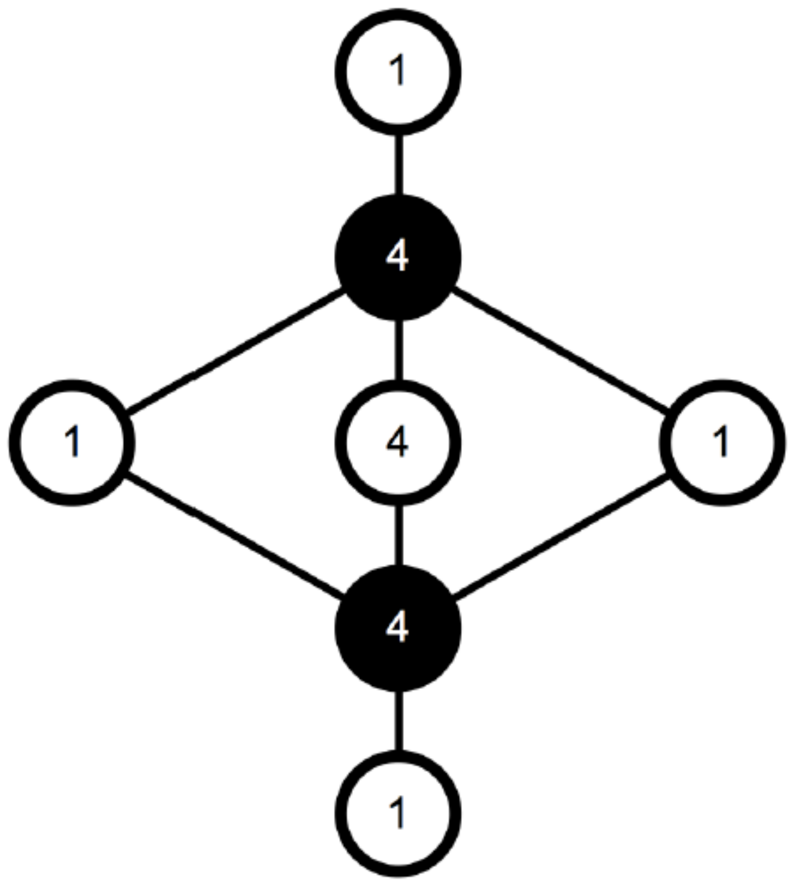
Michael Faux, S. J. Gates Jr

An adinkra with all nodes shown

An adinkra with some nodes collapsed with multiplicity shown



From 1D, $\mathcal{N} = 4$ Adinkra to 4D, $\mathcal{N} = 1$ Adinkra



Adinkra Diagram for 4D, $\mathcal{N} = 1$

Let $\mathcal{V}$ denote a scalar superfield in a Lorentz superspace of signature SO(1, D - 1), *then at each even level* n *of the superfield the equation*

$$\frac{d!}{n!(d-n)!} = \sum_{\mathcal{R}} b_{\{\mathcal{R}\}} \, d_{\{\mathcal{R}\}}$$

and at each odd level of the superfield the equation

$$\frac{d!}{n!(d-n)!} = \sum_{\mathcal{R}} b_{\{\mathcal{R}\}} \, d_{\{\mathcal{R}\}}$$

are both determined by the branching rules of the totally antisymmetric representations of $\mathcal{A}_{d-1}$ series of the Cartan classification of compact Lie algebras under the projection to its SO(1, D - 1) *subalgebra.*

The Salam-Strathdee 11D, N = 1 Real Scalar Superfield

$$
\begin{aligned}
\mathcal{V}(\theta, x) \; = \; & \varphi^{(0)}(x) \, + \, \theta^{\alpha}\, \varphi^{(1)}_{\alpha}(x) \, + \, \Theta^{(1)}\, \varphi^{(2)}(x) \, + \, \Theta^{(2)\, \underline{abc}}\, \varphi^{(2)}_{\underline{abc}}(x) \, + \, \Theta^{(3)\, \underline{abcd}}\, \varphi^{(2)}_{\underline{abcd}}(x) \\
& + \, \Theta^{(1)}\, \theta^{\alpha}\, \varphi^{(3)}_{\alpha}(x) \, + \, \Theta^{(2)\, \underline{abc}}\, \theta^{\alpha}\, \varphi^{(3)}_{\alpha\, \underline{abc}}(x) \, + \, \Theta^{(3)\, \underline{abcd}}\, \theta^{\alpha}\, \varphi^{(3)}_{\alpha\, \underline{abcd}}(x) \\
& + \, \Theta^{(1)}\, \Theta^{(1)}\, \varphi^{(4)}(x) \, + \, \Theta^{(1)}\, \Theta^{(2)\, \underline{abc}}\, \varphi^{(4)}_{\underline{abc}}(x) \, + \, \Theta^{(1)}\, \Theta^{(3)\, \underline{abcd}}\, \varphi^{(4)}_{\underline{abcd}}(x) \\
& + \, \Theta^{(2)\, \underline{abc}}\, \Theta^{(2)\, \underline{def}}\, \varphi^{(4)}_{\underline{abc}\, \underline{def}}(x) \, + \, \Theta^{(2)\, \underline{abc}}\, \Theta^{(3)\, \underline{defg}}\, \varphi^{(4)}_{\underline{abc}\, \underline{defg}}(x) \\
& + \, \Theta^{(3)\, \underline{abcd}}\, \Theta^{(3)\, \underline{efgh}}\, \varphi^{(4)}_{\underline{abcd}\, \underline{efgh}}(x) \, + \, \dots
\end{aligned}
$$

$$
\begin{array}{llll}
\{1\} & \Theta^{(1)} & = C_{\alpha\beta}\, \theta^{\alpha}\theta^{\beta} & , \\
\{165\} & \Theta^{(2)\, \underline{abc}} & = (\gamma^{\underline{abc}})_{\alpha\beta}\, \theta^{\alpha}\theta^{\beta} & , \\
\{330\} & \Theta^{(3)\, \underline{abcd}} & = (\gamma^{\underline{abcd}})_{\alpha\beta}\, \theta^{\alpha}\theta^{\beta} & .
\end{array}
$$

The First Sign Of Trouble

$$\{5,280\} \qquad [\,\Theta^{(3)\,\underline{abcd}}\,\theta_\alpha\,]_{IR} \quad ,$$

$$\{3,520\} \qquad [\,\Theta^{(3)\,\underline{abcd}}\,(\gamma_{\underline{d}})_{\alpha\beta}\,\theta^\beta\,]_{IR} \quad , \quad [\,\Theta^{(2)\,\underline{abc}}\,\theta_\alpha\,]_{IR} \quad ,$$

$$\{1,408\} \qquad [\,\Theta^{(3)\,\underline{abcd}}\,(\gamma_{\underline{cd}})_{\alpha\beta}\,\theta^\beta\,]_{IR} \quad , \quad [\,\Theta^{(2)\,\underline{abc}}\,(\gamma_{\underline{c}})_{\alpha\beta}\,\theta^\beta\,]_{IR} \quad ,$$

$$\{320\} \qquad [\,\Theta^{(3)\,\underline{abcd}}\,(\gamma_{\underline{bcd}})_{\alpha\beta}\,\theta^\beta\,]_{IR} \quad , \quad [\,\Theta^{(2)\,\underline{abc}}\,(\gamma_{\underline{bc}})_{\alpha\beta}\,\theta^\beta\,]_{IR} \quad ,$$

$$\{32\} \qquad \Theta^{(3)\,\underline{abcd}}\,(\gamma_{\underline{abcd}})_{\alpha\beta}\,\theta^\beta \quad , \quad \Theta^{(2)\,\underline{abc}}\,(\gamma_{\underline{abc}})_{\alpha\beta}\,\theta^\beta \quad , \quad \Theta^{(1)}\,\theta_\alpha \quad .$$

$$\{32\} \wedge \{32\} \wedge \{32\} = \frac{\{32\} \times \{31\} \times \{30\}}{3 \times 2} = \{4,960\} = \{32\} \oplus \{1,408\} \oplus \{3,520\}$$

The main message of this section of our work is that explicit θ-expansion of the eleven dimensional scalar superfield is considerably more complicated than in lower dimensions. One must contend with three separate problems:

(a.) there are multiple equivalent ways to express the required θ-monomials,

(b.) some apparently reasonable monimal combinations actually vanish,

(c.) the requirement of irreducibility of the θ-monomial expansion requires carefully constructed combinations.

The resolution of the first two of these problems relies of the derivation of Fierz identities. With regard to the final problem, the only methodology known to us is brute force establishment of their existences.

$$
\mathcal{V} = \left\{ \begin{array}{ll}
\text{Level} - 0 & \{1\} \quad , \\
\text{Level} - 1 & \{32\} \quad , \\
\text{Level} - 2 & \{32\} \wedge \{32\} \quad , \\
\text{Level} - 3 & \{32\} \wedge \{32\} \wedge \{32\} \\
\vdots & \vdots \\
\text{Level} - n & \underbrace{\{32\} \wedge \ \dots \ \wedge \{32\}}_{n \text{ times}} \\
\vdots & \vdots \\
\text{Level} - 32 & \{1\} \quad .
\end{array} \right\}
= \left\{ \begin{array}{ll}
\text{Level} - 0 & 1 \quad , \\
\text{Level} - 1 & 32 \quad , \\
\text{Level} - 2 & 496 \quad , \\
\text{Level} - 3 & 4960 \quad , \\
\vdots & \vdots \\
\text{Level} - n & \frac{32!}{n!(32-n)!} \quad , \\
\vdots & \vdots \\
\text{Level} - 32 & 1 \quad .
\end{array} \right.
$$

$$
\frac{32!}{n!(32-n)!} = \sum_{\mathcal{R}} b_{\{\mathcal{R}\}} \, d_{\{\mathcal{R}\}} \qquad \frac{32!}{n!(32-n)!} = \sum_{\mathcal{R}} b_{\{\mathcal{R}\}} \, d_{\{\mathcal{R}\}}
$$

BRANCHING RULES & PLETHYSM INDUCED TRANSPARENCY FOR THE 11D, N = 1 SCALAR SUPERFIELD: RESULTS

- Level-0: $\{1\}$
- Level-1: $\{32\}$
- Level-2: $\{1\} \oplus \{165\} \oplus \{330\}$
- Level-3: $\{32\} \oplus \{1,408\} \oplus \{3,520\}$
- Level-4: $\{1\} \oplus \{165\} \oplus \{330\} \oplus \{1,144\} \oplus \{4,290\} \oplus \{5,005\} \oplus \{7,865\} \oplus \{17,160\}$
- Level-5: $\{32\} \oplus \{1,408\} \oplus \{3,520\} \oplus \{4,224\} \oplus \{10,240\} \oplus \{24,960\} \oplus \{28,512\} \oplus \{36,960\} \oplus \{91,520\}$
- Level-6: $\{1\} \oplus \{165\} \oplus \{330\} \oplus \{1,144\} \oplus \{4,290\} \oplus \{5,005\} \oplus \{7,128\} \oplus \{7,865\} \oplus \{15,400\} \oplus (2)\{17,160\} \oplus \{28,314\} \oplus \{33,033\} \oplus \{37,752\} \oplus \{70,070\} \oplus \{78,650\} \oplus \{117,975\} \oplus \{175,175\} \oplus \{289,575\}$
- Level-7: $\{32\} \oplus \{1,408\} \oplus \{3,520\} \oplus \{4,224\} \oplus \{7,040\} \oplus \{10,240\} \oplus \{24,960\} \oplus (2)\{28,512\} \oplus \{36,960\} \oplus \{45,056\} \oplus \{45,760\} \oplus (2)\{91,520\} \oplus \{134,784\} \oplus \{137,280\} \oplus \{147,840\} \oplus \{160,160\} \oplus \{219,648\} \oplus \{264,000\} \oplus \{274,560\} \oplus \{573,440\} \oplus \{1,034,880\}$
- Level-8: $\{1\} \oplus \{165\} \oplus \{330\} \oplus \{935\} \oplus \{1,144\} \oplus \{4,290\} \oplus \{5,005\} \oplus \{7,128\} \oplus \{7,293\} \oplus (2)\{7,865\} \oplus (2)\{15,400\} \oplus (2)\{17,160\} \oplus \{22,275\} \oplus \{23,595\} \oplus \{23,595'\} \oplus \{28,314\} \oplus \{28,798\} \oplus \{33,033\} \oplus \{37,752\} \oplus \{57,915\} \oplus \{58,344\} \oplus \{70,070\} \oplus \{72,930\} \oplus (2)\{78,650\} \oplus \{85,085\} \oplus \{112,200\} \oplus (2)\{117,975\} \oplus (2)\{175,175\} \oplus \{178,750\} \oplus \{188,760\} \oplus \{255,255\} \oplus \{268,125\} \oplus (2)\{289,575\} \oplus \{333,234\} \oplus \{382,239\} \oplus \{503,965\} \oplus \{802,230\} \oplus \{868,725\} \oplus \{875,160\} \oplus \{984,555\} \oplus \{1,274,130\} \oplus \{1,519,375\}$

- Level-9: $\{32\} \oplus \{1,408\} \oplus \{3,520\} \oplus \{4,224\} \oplus (2)\{7,040\} \oplus \{10,240\} \oplus \{22,880\} \oplus \{24,960\} \oplus (3)\{28,512\} \oplus \{36,960\} \oplus (2)\{45,056\} \oplus (2)\{45,760\} \oplus (3)\{91,520\} \oplus \{128,128\} \oplus (2)\{134,784\} \oplus \{137,280\} \oplus (2)\{147,840\} \oplus \{157,696\} \oplus (2)\{160,160\} \oplus \{183,040\} \oplus (3)\{219,648\} \oplus \{251,680\} \oplus (2)\{264,000\} \oplus \{274,560\} \oplus \{292,864\} \oplus \{480,480\} \oplus \{570,240\} \oplus (2)\{573,440\} \oplus \{798,720\} \oplus \{896,896\} \oplus \{901,120\} \oplus (3)\{1,034,880\} \oplus \{1,351,680\} \oplus \{1,921,920\} \oplus \{1,936,000\} \oplus \{2,114,112\} \oplus \{2,168,320\} \oplus \{2,288,000\} \oplus \{4,212,000\}$

- Level-10: $\{1\} \oplus \{165\} \oplus \{330\} \oplus \{935\} \oplus \{1,144\} \oplus \{4,290\} \oplus \{5,005\} \oplus (2)\{7,128\} \oplus \{7,293\} \oplus (2)\{7,865\} \oplus (3)\{15,400\} \oplus (3)\{17,160\} \oplus \{22,275\} \oplus \{23,595\} \oplus \{23,595'\} \oplus \{26,520\} \oplus \{28,314\} \oplus \{28,798\} \oplus (2)\{33,033\} \oplus (2)\{37,752\} \oplus \{57,915\} \oplus (2)\{58,344\} \oplus (2)\{70,070\} \oplus \{72,930\} \oplus (3)\{78,650\} \oplus \{81,510\} \oplus (2)\{85,085\} \oplus \{112,200\} \oplus (3)\{117,975\} \oplus \{137,445\} \oplus (3)\{175,175\} \oplus (2)\{178,750\} \oplus \{181,545\} \oplus \{182,182\} \oplus (2)\{188,760\} \oplus \{255,255\} \oplus \{268,125\}$

  $\oplus (4)\{289,575\} \oplus (2)\{333,234\} \oplus (2)\{382,239\} \oplus \{386,750\} \oplus \{448,305\} \oplus (3)\{503,965\} \oplus \{525,525\} \oplus \{616,616\} \oplus \{650,650\} \oplus \{715,715\} \oplus (2)\{802,230\} \oplus (2)\{868,725\} \oplus (2)\{875,160\} \oplus (2)\{984,555\} \oplus \{1,002,001\} \oplus \{1,100,385\} \oplus (2)\{1,274,130\} \oplus (2)\{1,310,309\} \oplus \{1,412,840\} \oplus (2)\{1,519,375\} \oplus \{1,673,672\} \oplus \{1,786,785\} \oplus \{2,571,250\} \oplus \{3,128,697\} \oplus \{3,641,274\} \oplus \{3,792,360\} \oplus \{4,506,040\} \oplus \{5,214,495\} \oplus \{7,900,750\}$

- Level-11: $\{32\} \oplus \{1,408\} \oplus \{3,520\} \oplus (2)\{4,224\} \oplus (2)\{7,040\} \oplus (2)\{10,240\} \oplus \{22,880\} \oplus (2)\{24,960\} \oplus (4)\{28,512\} \oplus (2)\{36,960\} \oplus (2)\{45,056\} \oplus (3)\{45,760\} \oplus (4)\{91,520\} \oplus \{128,128\} \oplus (4)\{134,784\} \oplus (2)\{137,280\} \oplus (3)\{147,840\} \oplus (2)\{157,696\} \oplus (3)\{160,160\} \oplus (2)\{183,040\} \oplus (4)\{219,648\} \oplus \{251,680\} \oplus (2)\{264,000\} \oplus (2)\{274,560\} \oplus (2)\{292,864\} \oplus \{457,600\} \oplus (3)\{480,480\} \oplus (2)\{570,240\} \oplus (4)\{573,440\} \oplus \{672,672\} \oplus (2)\{798,720\} \oplus (2)\{896,896\} \oplus (2)\{901,120\} \oplus (5)\{1,034,880\} \oplus \{1,140,480\} \oplus \{1,351,680\} \oplus \{1,425,600\} \oplus \{1,757,184\} \oplus (2)\{1,921,920\} \oplus \{1,936,000\} \oplus \{2,013,440\} \oplus \{2,038,400\} \oplus (3)\{2,114,112\} \oplus (2)\{2,168,320\} \oplus (3)\{2,288,000\} \oplus \{2,358,720\} \oplus \{3,706,560\} \oplus (3)\{4,212,000\} \oplus \{5,857,280\} \oplus \{5,930,496\} \oplus \{6,040,320\} \oplus \{7,208,960\} \oplus \{8,781,696\} \oplus \{9,123,840\} \oplus \{11,714,560\}$

- Level-12: $\{1\} \oplus \{165\} \oplus \{330\} \oplus \{935\} \oplus (2)\{1,144\} \oplus (2)\{4,290\} \oplus (2)\{5,005\} \oplus (2)\{7,128\} \oplus \{7,150\} \oplus \{7,293\} \oplus (3)\{7,865\} \oplus (3)\{15,400\} \oplus (4)\{17,160\} \oplus (2)\{22,275\} \oplus \{23,595\} \oplus \{23,595'\} \oplus \{26,520\} \oplus (2)\{28,314\} \oplus (2)\{28,798\} \oplus (3)\{33,033\} \oplus (2)\{37,752\} \oplus \{47,190\} \oplus (2)\{57,915\} \oplus (2)\{58,344\} \oplus (2)\{70,070\} \oplus \{72,930\} \oplus (4)\{78,650\} \oplus \{81,510\} \oplus (3)\{85,085\} \oplus \{91,960\} \oplus \{112,200\} \oplus (5)\{117,975\} \oplus (2)\{137,445\} \oplus (4)\{175,175\} \oplus (3)\{178,750\} \oplus \{181,545\} \oplus \{182,182\} \oplus (2)\{188,760\} \oplus \{235,950\} \oplus \{251,680'\} \oplus (3)\{255,255\} \oplus \{266,266\} \oplus (2)\{268,125\} \oplus (5)\{289,575\} \oplus (3)\{333,234\} \oplus (3)\{382,239\} \oplus \{386,750\} \oplus (2)\{448,305\} \oplus (5)\{503,965\} \oplus (2)\{525,525\} \oplus \{616,616\} \oplus \{650,650\} \oplus \{715,715\} \oplus \{722,358\} \oplus (4)\{802,230\} \oplus \{862,125\} \oplus (4)\{868,725\} \oplus (3)\{875,160\} \oplus \{948,090\} \oplus (3)\{984,555\} \oplus \{1,002,001\} \oplus (2)\{1,100,385\} \oplus \{1,115,400\} \oplus \{1,123,122\} \oplus \{1,245,090\} \oplus (3)\{1,274,130\} \oplus (3)\{1,310,309\} \oplus \{1,412,840\} \oplus (3)\{1,519,375\} \oplus (3)\{1,673,672\} \oplus \{1,718,496\} \oplus (2)\{1,786,785\} \oplus \{2,147,145\} \oplus \{2,450,250\} \oplus \{2,571,250\} \oplus \{2,743,125\} \oplus (3)\{3,128,697\} \oplus (2)\{3,641,274\} \oplus (2)\{3,792,360\} \oplus \{3,993,990\} \oplus (2)\{4,506,040\} \oplus \{4,708,704\} \oplus (3)\{5,214,495\} \oplus \{5,651,360\} \oplus \{5,834,400\} \oplus \{6,276,270\} \oplus \{7,468,032\} \oplus \{7,487,480\} \oplus (2)\{7,900,750\} \oplus \{11,981,970\} \oplus \{14,889,875\} \oplus \{20,084,064\}$

- Level-13: $\{32\}\oplus(2)\{1,408\}\oplus(2)\{3,520\}\oplus(2)\{4,224\}\oplus(2)\{7,040\}\oplus(3)\{10,240\}\oplus\{22,880\}\oplus(3)\{24,960\}\oplus(5)\{28,512\}\oplus(3)\{36,960\}\oplus(3)\{45,056\}\oplus(4)\{45,760\}\oplus(5)\{91,520\}\oplus(2)\{128,128\}\oplus(5)\{134,784\}\oplus(3)\{137,280\}\oplus(4)\{147,840\}\oplus(3)\{157,696\}\oplus(4)\{160,160\}\oplus(3)\{183,040\}\oplus(5)\{219,648\}\oplus\{251,680\}\oplus(2)\{264,000\}\oplus(3)\{274,560\}\oplus(2)\{292,864\}\oplus\{302,016\}\oplus(2)\{457,600\}\oplus(4)\{480,480\}\oplus(3)\{570,240\}\oplus(6)\{573,440\}\oplus(2)\{672,672\}\oplus(3)\{798,720\}\oplus(4)\{896,896\}\oplus(3)\{901,120\}\oplus(7)\{1,034,880\}\oplus(2)\{1,140,480\}\oplus\{1,171,456\}\oplus\{1,351,680\}\oplus(2)\{1,425,600\}\oplus(2)\{1,757,184\}\oplus(2)\{1,921,920\}\oplus(2)\{1,936,000\}\oplus(2)\{2,013,440\}\oplus(2)\{2,038,400\}\oplus(4)\{2,114,112\}\oplus(3)\{2,168,320\}\oplus(5)\{2,288,000\}\oplus\{2,342,912\}\oplus(2)\{2,358,720\}\oplus\{2,402,400\}\oplus(2)\{3,706,560\}\oplus\{3,706,560'\}\oplus(2)\{3,794,560\}\oplus(5)\{4,212,000\}\oplus\{5,720,000\}\oplus(2)\{5,857,280\}\oplus\{5,930,496\}\oplus(2)\{6,040,320\}\oplus\{6,864,000\}\oplus(2)\{7,208,960\}\oplus(2)\{8,781,696\}\oplus(2)\{9,123,840\}\oplus\{10,570,560'\}\oplus(2)\{11,714,560\}\oplus\{11,927,552\}\oplus\{12,390,400\}\oplus\{13,246,464\}\oplus\{13,453,440\}\oplus\{33,554,432\}$

- Level-14: $\{1\} \oplus (2)\{165\} \oplus (2)\{330\} \oplus \{935\} \oplus (2)\{1,144\} \oplus \{1,430\} \oplus \{3,003\} \oplus (2)\{4,290\} \oplus (2)\{5,005\} \oplus (3)\{7,128\} \oplus \{7,150\} \oplus \{7,293\} \oplus (3)\{7,865\} \oplus \{11,583\} \oplus (4)\{15,400\} \oplus (5)\{17,160\} \oplus (2)\{22,275\} \oplus (2)\{23,595\} \oplus \{235,95'\} \oplus (2)\{26,520\} \oplus (2)\{28,314\} \oplus (2)\{28,798\} \oplus (3)\{33,033\} \oplus (3)\{37,752\} \oplus \{47,190\} \oplus (2)\{57,915\} \oplus (3)\{58,344\} \oplus (3)\{70,070\} \oplus \{72,930\} \oplus (5)\{78,650\} \oplus (2)\{81,510\} \oplus (3)\{85,085\} \oplus \{91,960\} \oplus \{112,200\} \oplus (6)\{117,975\} \oplus (2)\{137,445\} \oplus \{162,162\} \oplus (5)\{175,175\} \oplus (4)\{178,750\} \oplus (2)\{181,545\} \oplus (2)\{182,182\} \oplus (2)\{188,760\} \oplus \{218,295\} \oplus \{235,950\} \oplus \{2516,80'\} \oplus (3)\{255,255\} \oplus \{266,266\} \oplus (2)\{268,125\} \oplus (7)\{289,575\} \oplus (4)\{333,234\} \oplus (4)\{382,239\} \oplus (2)\{386,750\} \oplus (2)\{448,305\} \oplus \{490,490\} \oplus (6)\{503,965\} \oplus (3)\{525,525\} \oplus \{526,240\} \oplus \{616,616\} \oplus (2)\{650,650\} \oplus \{715,715\} \oplus \{722,358\} \oplus (5)\{802,230\} \oplus \{825,825\} \oplus \{862,125\} \oplus (5)\{868,725\} \oplus (3)\{875,160\} \oplus (2)\{948,090\} \oplus (4)\{984,555\} \oplus \{1,002,001\} \oplus (3)\{1,100,385\} \oplus \{1,115,400\} \oplus \{1,123,122\} \oplus \{1,190,112\} \oplus \{1,191,190\} \oplus \{1,245,090\} \oplus (4)\{1,274,130\} \oplus (5)\{1,310,309\} \oplus (2)\{1,412,840\} \oplus (4)\{1,519,375\} \oplus \{1,533,675\} \oplus (4)\{1,673,672\} \oplus \{1,718,496\} \oplus (3)\{1,786,785\} \oplus \{2,147,145\} \oplus \{2,450,250\} \oplus (2)\{2,571,250\} \oplus (2)\{2,743,125\} \oplus \{3,083,080\} \oplus (4)\{3,128,697\} \oplus \{3,586,440\} \oplus (3)\{3,641,274\} \oplus (2)\{3,792,360\} \oplus \{3,993,990\} \oplus \{4,332,042\} \oplus (4)\{4,506,040\} \oplus (2)\{4,708,704\} \oplus \{4,781,920\} \oplus (5)\{5,214,495\} \oplus \{52,144,95'\} \oplus \{5,651,360\} \oplus \{5,834,400\} \oplus (2)\{6,276,270\} \oplus \{7,468,032\} \oplus (2)\{7,487,480\} \oplus \{7,865,000\} \oplus (3)\{7,900,750\} \oplus \{9,845,550\} \oplus \{10,830,105\} \oplus (2)\{11,981,970\} \oplus \{12,972,960\} \oplus \{14,889,875\} \oplus \{17,606,160\} \oplus \{18,718,700\} \oplus (2)\{20,084,064\} \oplus \{31,082,480\}$

- Level-15: (2){32} ⊕ {320} ⊕ (2){1, 408} ⊕ {1, 760} ⊕ (3){3, 520} ⊕ (2){4, 224} ⊕ {5, 280} ⊕ (3){7, 040} ⊕ (3){10, 240} ⊕ (2){22, 880} ⊕ (3){24, 960} ⊕ (6){28, 512} ⊕ (3){36, 960} ⊕ (4){45, 056} ⊕ (4){45, 760} ⊕ {64, 064} ⊕ (6){91, 520} ⊕ (3){128, 128} ⊕ (6){134, 784} ⊕ (3){137, 280} ⊕ (4){147, 840} ⊕ (3){157, 696} ⊕ (5){160, 160} ⊕ {160, 160′} ⊕ (3){183, 040} ⊕ (6){219, 648} ⊕ {251, 680} ⊕ (3){264, 000} ⊕ (3){274, 560} ⊕ (3){292, 864} ⊕ {302, 016} ⊕ {366, 080} ⊕ (2){457, 600} ⊕ (5){480, 480} ⊕ (3){570, 240} ⊕ (7){573, 440} ⊕ (2){672, 672} ⊕ (4){798, 720} ⊕ (5){896, 896} ⊕ (4){901, 120} ⊕ (8){1, 034, 880} ⊕ (3){1, 140, 480} ⊕ {1, 171, 456} ⊕ {1, 208, 064} ⊕ (2){1, 351, 680} ⊕ (3){1, 425, 600} ⊕ (2){1, 757, 184} ⊕ (2){1, 921, 920} ⊕ (3){1, 936, 000} ⊕ (3){2, 013, 440} ⊕ (2){2, 038, 400} ⊕ (5){2, 114, 112} ⊕ (3){2, 168, 320} ⊕ (6){2, 288, 000} ⊕ {2, 342, 912} ⊕ (3){2, 358, 720} ⊕ (2){2, 402, 400} ⊕ {2, 446, 080} ⊕ (3){3, 706, 560} ⊕ (2){3, 706, 560′} ⊕ (3){3, 794, 560} ⊕ {4, 026, 880} ⊕ (6){4, 212, 000} ⊕ (2){5, 720, 000} ⊕ (2){5, 857, 280} ⊕ {5, 930, 496} ⊕ (3){6, 040, 320} ⊕ {6, 307, 840} ⊕ {6, 864, 000} ⊕ (3){7, 208, 960} ⊕ (3){8, 781, 696} ⊕ (3){9, 123, 840} ⊕ {10, 570, 560} ⊕ {10, 570, 560′} ⊕ (2){11, 714, 560} ⊕ {11, 927, 552} ⊕ (2){12, 390, 400} ⊕ (2){13, 246, 464} ⊕ (2){13, 453, 440} ⊕ {15, 375, 360} ⊕ {30, 201, 600} ⊕ {33, 116, 160} ⊕ {33, 554, 432}

- Level-16: $(2)\{1\} \oplus \{11\} \oplus \{65\} \oplus (2)\{165\} \oplus \{275\} \oplus (2)\{330\} \oplus \{462\} \oplus (2)\{935\} \oplus (2)\{1,144\} \oplus \{1,430\} \oplus \{2,717\} \oplus \{3,003\} \oplus (3)\{4,290\} \oplus (2)\{5,005\} \oplus \{7,007\} \oplus (3)\{7,128\} \oplus \{7,150\} \oplus \{7,293\} \oplus (4)\{7,865\} \oplus \{11,583\} \oplus (4)\{15,400\} \oplus \{16,445\} \oplus (5)\{17,160\} \oplus (3)\{22,275\} \oplus (3)\{23,595\} \oplus (2)\{23,595'\} \oplus (2)\{26,520\} \oplus (2)\{28,314\} \oplus (2)\{28,798\} \oplus (3)\{33,033\} \oplus \{35,750\} \oplus (3)\{37,752\} \oplus \{47,190\} \oplus (3)\{57,915\} \oplus (3)\{58,344\} \oplus (3)\{70,070\} \oplus \{72,930\} \oplus (5)\{78,650\} \oplus (2)\{81,510\} \oplus (4)\{85,085\} \oplus \{91,960\} \oplus (2)\{112,200\} \oplus (6)\{117,975\} \oplus (2)\{137,445\} \oplus \{162,162\} \oplus (5)\{175,175\} \oplus (5)\{178,750\} \oplus (2)\{181,545\} \oplus (2)\{182,182\} \oplus (3)\{188,760\} \oplus \{218,295\} \oplus \{235,950\} \oplus \{251,680'\} \oplus (4)\{255,255\} \oplus (2)\{266,266\} \oplus (3)\{268,125\} \oplus (7)\{289,575\} \oplus (4)\{333,234\} \oplus (4)\{382,239\} \oplus (2)\{386,750\} \oplus (2)\{448,305\} \oplus \{490,490\} \oplus (6)\{503,965\} \oplus (3)\{525,525\} \oplus \{526,240\} \oplus \{616,616\} \oplus \{628,320\} \oplus (2)\{650,650\} \oplus \{674,817\} \oplus \{715,715\} \oplus (2)\{722,358\} \oplus (6)\{802,230\} \oplus \{825,825\} \oplus (2)\{862,125\} \oplus (6)\{868,725\} \oplus (4)\{875,160\} \oplus (2)\{948,090\} \oplus (4)\{984,555\} \oplus \{1,002,001\} \oplus (3)\{1,100,385\} \oplus (2)\{1,115,400\} \oplus (2)\{1,123,122\} \oplus \{1,190,112\} \oplus \{1,191,190\} \oplus \{1,245,090\} \oplus (4)\{1,274,130\} \oplus (5)\{1,310,309\} \oplus (2)\{1,412,840\} \oplus (5)\{1,519,375\} \oplus \{1,533,675\} \oplus (4)\{1,673,672\} \oplus (2)\{1,718,496\} \oplus \{1,758,120\} \oplus (3)\{1,786,785\} \oplus \{2,147,145\} \oplus (2)\{2,450,250\} \oplus (2)\{2,571,250\} \oplus \{2,598,960\} \oplus (3)\{2,743,125\} \oplus \{2,858,856\} \oplus \{3,056,625\} \oplus \{3,083,080\} \oplus (4)\{3,128,697\} \oplus \{3,586,440\} \oplus (3)\{3,641,274\} \oplus (2)\{3,792,360\} \oplus \{3,993,990\} \oplus \{4,332,042\} \oplus (4)\{4,506,040\} \oplus (2)\{4,708,704\} \oplus \{4,781,920\} \oplus (6)\{5,214,495\} \oplus (2)\{5,214,495'\} \oplus (2)\{5,651,360\} \oplus \{5,834,400\} \oplus (2)\{6,276,270\} \oplus \{7,468,032\} \oplus (3)\{7,487,480\} \oplus (2)\{7,865,000\} \oplus (3)\{7,900,750\} \oplus \{8,893,500\} \oplus \{9,845,550\} \oplus \{10,696,400'\} \oplus \{10,830,105\} \oplus (2)\{11,981,970\} \oplus \{12,972,960\} \oplus \{14,889,875\} \oplus \{17,606,160\} \oplus \{18,718,700\} \oplus (3)\{20,084,064\} \oplus \{30,604,288\} \oplus \{31,082,480\}$

- Level-16: $(2)\{1\} \oplus \{11\} \oplus \{65\} \oplus (2)\{165\} \oplus \{275\} \oplus (2)\{330\} \oplus \{462\} \oplus (2)\{935\} \oplus (2)\{1,144\} \oplus \{1,430\} \oplus \{2,717\} \oplus \{3,003\} \oplus (3)\{4,290\} \oplus (2)\{5,005\} \oplus \{7,007\} \oplus (3)\{7,128\} \oplus \{7,150\} \oplus \{7,293\} \oplus (4)\{7,865\} \oplus \{11,583\} \oplus (4)\{15,400\} \oplus \{16,445\} \oplus (5)\{17,160\} \oplus (3)\{22,275\} \oplus (3)\{23,595\} \oplus (2)\{23,595'\} \oplus (2)\{26,520\} \oplus (2)\{28,314\} \oplus (2)\{28,798\} \oplus (3)\{33,033\} \oplus \{35,750\} \oplus (3)\{37,752\} \oplus \{47,190\} \oplus (3)\{57,915\} \oplus (3)\{58,344\} \oplus (3)\{70,070\} \oplus \{72,930\} \oplus (5)\{78,650\} \oplus (2)\{81,510\} \oplus (4)\{85,085\} \oplus \{91,960\} \oplus (2)\{112,200\} \oplus (6)\{117,975\} \oplus (2)\{137,445\} \oplus \{162,162\} \oplus (5)\{175,175\} \oplus (5)\{178,750\} \oplus (2)\{181,545\} \oplus (2)\{182,182\} \oplus (3)\{188,760\} \oplus \{218,295\} \oplus \{235,950\} \oplus \{251,680'\} \oplus (4)\{255,255\} \oplus (2)\{266,266\} \oplus (3)\{268,125\} \oplus (7)\{289,575\} \oplus (4)\{333,234\} \oplus (4)\{382,239\} \oplus (2)\{386,750\} \oplus (2)\{448,305\} \oplus \{490,490\} \oplus (6)\{503,965\} \oplus (3)\{525,525\} \oplus \{526,240\} \oplus \{616,616\} \oplus \{628,320\} \oplus (2)\{650,650\} \oplus \{674,817\} \oplus \{715,715\} \oplus (2)\{722,358\} \oplus (6)\{802,230\} \oplus \{825,825\} \oplus (2)\{862,125\} \oplus (6)\{868,725\} \oplus (4)\{875,160\} \oplus (2)\{948,090\} \oplus (4)\{984,555\} \oplus \{1,002,001\} \oplus (3)\{1,100,385\} \oplus (2)\{1,115,400\} \oplus (2)\{1,123,122\} \oplus \{1,190,112\} \oplus \{1,191,190\} \oplus \{1,245,090\} \oplus (4)\{1,274,130\} \oplus (5)\{1,310,309\} \oplus (2)\{1,412,840\} \oplus (5)\{1,519,375\} \oplus \{1,533,675\} \oplus (4)\{1,673,672\} \oplus (2)\{1,718,496\} \oplus \{1,758,120\} \oplus (3)\{1,786,785\} \oplus \{2,147,145\} \oplus (2)\{2,450,250\} \oplus (2)\{2,571,250\} \oplus \{2,598,960\} \oplus (3)\{2,743,125\} \oplus \{2,858,856\} \oplus \{3,056,625\} \oplus \{3,083,080\} \oplus (4)\{3,128,697\} \oplus \{3,586,440\} \oplus (3)\{3,641,274\} \oplus (2)\{3,792,360\} \oplus \{3,993,990\} \oplus \{4,332,042\} \oplus (4)\{4,506,040\} \oplus (2)\{4,708,704\} \oplus \{4,781,920\} \oplus (6)\{5,214,495\} \oplus (2)\{5,214,495'\} \oplus (2)\{5,651,360\} \oplus \{5,834,400\} \oplus (2)\{6,276,270\} \oplus \{7,468,032\} \oplus (3)\{7,487,480\} \oplus (2)\{7,865,000\} \oplus (3)\{7,900,750\} \oplus \{8,893,500\} \oplus \{9,845,550\} \oplus \{10,696,400'\} \oplus \{10,830,105\} \oplus (2)\{11,981,970\} \oplus \{12,972,960\} \oplus \{14,889,875\} \oplus \{17,606,160\} \oplus \{18,718,700\} \oplus (3)\{20,084,064\} \oplus \{30,604,288\} \oplus \{31,082,480\}$

- Level-16: $(2)\{1\} \oplus \{11\} \oplus \{65\} \oplus (2)\{165\} \oplus \{275\} \oplus (2)\{330\} \oplus \{462\} \oplus (2)\{935\} \oplus (2)\{1,144\} \oplus \{1,430\} \oplus \{2,717\} \oplus \{3,003\} \oplus (3)\{4,290\} \oplus (2)\{5,005\} \oplus \{7,007\} \oplus (3)\{7,128\} \oplus \{7,150\} \oplus \{7,293\} \oplus (4)\{7,865\} \oplus \{11,583\} \oplus (4)\{15,400\} \oplus \{16,445\} \oplus (5)\{17,160\} \oplus (3)\{22,275\} \oplus (3)\{23,595\} \oplus (2)\{23,595'\} \oplus (2)\{26,520\} \oplus (2)\{28,314\} \oplus (2)\{28,798\} \oplus (3)\{33,033\} \oplus \{35,750\} \oplus (3)\{37,752\} \oplus \{47,190\} \oplus (3)\{57,915\} \oplus (3)\{58,344\} \oplus (3)\{70,070\} \oplus \{72,930\} \oplus (5)\{78,650\} \oplus (2)\{81,510\} \oplus (4)\{85,085\} \oplus \{91,960\} \oplus (2)\{112,200\} \oplus (6)\{117,975\} \oplus (2)\{137,445\} \oplus \{162,162\} \oplus (5)\{175,175\} \oplus (5)\{178,750\} \oplus (2)\{181,545\} \oplus (2)\{182,182\} \oplus (3)\{188,760\} \oplus \{218,295\} \oplus \{235,950\} \oplus \{251,680'\} \oplus (4)\{255,255\} \oplus (2)\{266,266\} \oplus (3)\{268,125\} \oplus (7)\{289,575\} \oplus (4)\{333,234\} \oplus (4)\{382,239\} \oplus (2)\{386,750\} \oplus (2)\{448,305\} \oplus \{490,490\} \oplus (6)\{503,965\} \oplus (3)\{525,525\} \oplus \{526,240\} \oplus \{616,616\} \oplus \{628,320\} \oplus (2)\{650,650\} \oplus \{674,817\} \oplus \{715,715\} \oplus (2)\{722,358\} \oplus (6)\{802,230\} \oplus \{825,825\} \oplus (2)\{862,125\} \oplus (6)\{868,725\} \oplus (4)\{875,160\} \oplus (2)\{948,090\} \oplus (4)\{984,555\} \oplus \{1,002,001\} \oplus (3)\{1,100,385\} \oplus (2)\{1,115,400\} \oplus (2)\{1,123,122\} \oplus \{1,190,112\} \oplus \{1,191,190\} \oplus \{1,245,090\} \oplus (4)\{1,274,130\} \oplus (5)\{1,310,309\} \oplus (2)\{1,412,840\} \oplus (5)\{1,519,375\} \oplus \{1,533,675\} \oplus (4)\{1,673,672\} \oplus (2)\{1,718,496\} \oplus \{1,758,120\} \oplus (3)\{1,786,785\} \oplus \{2,147,145\} \oplus (2)\{2,450,250\} \oplus (2)\{2,571,250\} \oplus \{2,598,960\} \oplus (3)\{2,743,125\} \oplus \{2,858,856\} \oplus \{3,056,625\} \oplus \{3,083,080\} \oplus (4)\{3,128,697\} \oplus \{3,586,440\} \oplus (3)\{3,641,274\} \oplus (2)\{3,792,360\} \oplus \{3,993,990\} \oplus \{4,332,042\} \oplus (4)\{4,506,040\} \oplus (2)\{4,708,704\} \oplus \{4,781,920\} \oplus (6)\{5,214,495\} \oplus (2)\{5,214,495'\} \oplus (2)\{5,651,360\} \oplus \{5,834,400\} \oplus (2)\{6,276,270\} \oplus \{7,468,032\} \oplus (3)\{7,487,480\} \oplus (2)\{7,865,000\} \oplus (3)\{7,900,750\} \oplus \{8,893,500\} \oplus \{9,845,550\} \oplus \{10,696,400'\} \oplus \{10,830,105\} \oplus (2)\{11,981,970\} \oplus \{12,972,960\} \oplus \{14,889,875\} \oplus \{17,606,160\} \oplus \{18,718,700\} \oplus (3)\{20,084,064\} \oplus \{30,604,288\} \oplus \{31,082,480\}$

- Level-16: $(2)\{1\} \oplus \{11\} \oplus \{65\} \oplus (2)\{165\} \oplus \{275\} \oplus (2)\{330\} \oplus \{462\} \oplus (2)\{935\} \oplus (2)\{1,144\} \oplus$
$\{1,430\} \oplus \{2,717\} \oplus \{3,003\} \oplus (3)\{4,290\} \oplus (2)\{5,005\} \oplus \{7,007\} \oplus (3)\{7,128\} \oplus \{7,150\} \oplus$
$\{7,293\} \oplus (4)\{7,865\} \oplus \{11,583\} \oplus (4)\{15,400\} \oplus \{16,445\} \oplus (5)\{17,160\} \oplus (3)\{22,275\} \oplus$
$(3)\{23,595\} \oplus (2)\{23,595'\} \oplus (2)\{26,520\} \oplus (2)\{28,314\} \oplus (2)\{28,798\} \oplus (3)\{33,033\} \oplus$
$\{35,750\} \oplus (3)\{37,752\} \oplus \{47,190\} \oplus (3)\{57,915\} \oplus (3)\{58,344\} \oplus (3)\{70,070\} \oplus \{72,930\} \oplus$
$(5)\{78,650\} \oplus (2)\{81,510\} \oplus (4)\{85,085\} \oplus \{91,960\} \oplus (2)\{112,200\} \oplus (6)\{117,975\} \oplus$
$(2)\{137,445\} \oplus \{162,162\} \oplus (5)\{175,175\} \oplus (5)\{178,750\} \oplus (2)\{181,545\} \oplus (2)\{182,182\} \oplus$
$(3)\{188,760\} \oplus \{218,295\} \oplus \{235,950\} \oplus \{251,680'\} \oplus (4)\{255,255\} \oplus (2)\{266,266\} \oplus$
$(3)\{268,125\} \oplus (7)\{289,575\} \oplus (4)\{333,234\} \oplus (4)\{382,239\} \oplus (2)\{386,750\} \oplus (2)\{448,305\} \oplus$
$\{490,490\} \oplus (6)\{503,965\} \oplus (3)\{525,525\} \oplus \{526,240\} \oplus \{616,616\} \oplus \{628,320\} \oplus$
$(2)\{650,650\} \oplus \{674,817\} \oplus \{715,715\} \oplus (2)\{722,358\} \oplus (6)\{802,230\} \oplus \{825,825\} \oplus$
$(2)\{862,125\} \oplus (6)\{868,725\} \oplus (4)\{875,160\} \oplus (2)\{948,090\} \oplus (4)\{984,555\} \oplus \{1,002,001\} \oplus$
$(3)\{1,100,385\} \oplus (2)\{1,115,400\} \oplus (2)\{1,123,122\} \oplus \{1,190,112\} \oplus \{1,191,190\} \oplus$
$\{1,245,090\} \oplus (4)\{1,274,130\} \oplus (5)\{1,310,309\} \oplus (2)\{1,412,840\} \oplus (5)\{1,519,375\} \oplus$
$\{1,533,675\} \oplus (4)\{1,673,672\} \oplus (2)\{1,718,496\} \oplus \{1,758,120\} \oplus (3)\{1,786,785\} \oplus$
$\{2,147,145\} \oplus (2)\{2,450,250\} \oplus (2)\{2,571,250\} \oplus \{2,598,960\} \oplus (3)\{2,743,125\} \oplus$
$\{2,858,856\} \oplus \{3,056,625\} \oplus \{3,083,080\} \oplus (4)\{3,128,697\} \oplus \{3,586,440\} \oplus (3)\{3,641,274\} \oplus$
$(2)\{3,792,360\} \oplus \{3,993,990\} \oplus \{4,332,042\} \oplus (4)\{4,506,040\} \oplus (2)\{4,708,704\} \oplus$
$\{4,781,920\} \oplus (6)\{5,214,495\} \oplus (2)\{5,214,495'\} \oplus (2)\{5,651,360\} \oplus \{5,834,400\} \oplus$
$(2)\{6,276,270\} \oplus \{7,468,032\} \oplus (3)\{7,487,480\} \oplus (2)\{7,865,000\} \oplus (3)\{7,900,750\} \oplus$
$\{8,893,500\} \oplus \{9,845,550\} \oplus \{10,696,400'\} \oplus \{10,830,105\} \oplus (2)\{11,981,970\} \oplus$
$\{12,972,960\} \oplus \{14,889,875\} \oplus \{17,606,160\} \oplus \{18,718,700\} \oplus (3)\{20,084,064\} \oplus$
$\{30,604,288\} \oplus \{31,082,480\}$

BROWN
THEORETICAL PHYSICS CENTER

- Level-16: $(2)\{1\} \oplus \{11\} \oplus \{65\} \oplus (2)\{165\} \oplus \{275\} \oplus (2)\{330\} \oplus \{462\} \oplus (2)\{935\} \oplus (2)\{1,144\} \oplus \{1,430\} \oplus \{2,717\} \oplus \{3,003\} \oplus (3)\{4,290\} \oplus (2)\{5,005\} \oplus \{7,007\} \oplus (3)\{7,128\} \oplus \{7,150\} \oplus \{7,293\} \oplus (4)\{7,865\} \oplus \{11,583\} \oplus (4)\{15,400\} \oplus \{16,445\} \oplus (5)\{17,160\} \oplus (3)\{22,275\} \oplus (3)\{23,595\} \oplus (2)\{23,595'\} \oplus (2)\{26,520\} \oplus (2)\{28,314\} \oplus (2)\{28,798\} \oplus (3)\{33,033\} \oplus \{35,750\} \oplus (3)\{37,752\} \oplus \{47,190\} \oplus (3)\{57,915\} \oplus (3)\{58,344\} \oplus (3)\{70,070\} \oplus \{72,930\} \oplus (5)\{78,650\} \oplus (2)\{81,510\} \oplus (4)\{85,085\} \oplus \{91,960\} \oplus (2)\{112,200\} \oplus (6)\{117,975\} \oplus (2)\{137,445\} \oplus \{162,162\} \oplus (5)\{175,175\} \oplus (5)\{178,750\} \oplus (2)\{181,545\} \oplus (2)\{182,182\} \oplus (3)\{188,760\} \oplus \{218,295\} \oplus \{235,950\} \oplus \{251,680'\} \oplus (4)\{255,255\} \oplus (2)\{266,266\} \oplus (3)\{268,125\} \oplus (7)\{289,575\} \oplus (4)\{333,234\} \oplus (4)\{382,239\} \oplus (2)\{386,750\} \oplus (2)\{448,305\} \oplus \{490,490\} \oplus (6)\{503,965\} \oplus (3)\{525,525\} \oplus \{526,240\} \oplus \{616,616\} \oplus \{628,320\} \oplus (2)\{650,650\} \oplus \{674,817\} \oplus \{715,715\} \oplus (2)\{722,358\} \oplus (6)\{802,230\} \oplus \{825,825\} \oplus (2)\{862,125\} \oplus (6)\{868,725\} \oplus (4)\{875,160\} \oplus (2)\{948,090\} \oplus (4)\{984,555\} \oplus \{1,002,001\} \oplus (3)\{1,100,385\} \oplus (2)\{1,115,400\} \oplus (2)\{1,123,122\} \oplus \{1,190,112\} \oplus \{1,191,190\} \oplus \{1,245,090\} \oplus (4)\{1,274,130\} \oplus (5)\{1,310,309\} \oplus (2)\{1,412,840\} \oplus (5)\{1,519,375\} \oplus \{1,533,675\} \oplus (4)\{1,673,672\} \oplus (2)\{1,718,496\} \oplus \{1,758,120\} \oplus (3)\{1,786,785\} \oplus \{2,147,145\} \oplus (2)\{2,450,250\} \oplus (2)\{2,571,250\} \oplus \{2,598,960\} \oplus (3)\{2,743,125\} \oplus \{2,858,856\} \oplus \{3,056,625\} \oplus \{3,083,080\} \oplus (4)\{3,128,697\} \oplus \{3,586,440\} \oplus (3)\{3,641,274\} \oplus (2)\{3,792,360\} \oplus \{3,993,990\} \oplus \{4,332,042\} \oplus (4)\{4,506,040\} \oplus (2)\{4,708,704\} \oplus \{4,781,920\} \oplus (6)\{5,214,495\} \oplus (2)\{5,214,495'\} \oplus (2)\{5,651,360\} \oplus \{5,834,400\} \oplus (2)\{6,276,270\} \oplus \{7,468,032\} \oplus (3)\{7,487,480\} \oplus (2)\{7,865,000\} \oplus (3)\{7,900,750\} \oplus \{8,893,500\} \oplus \{9,845,550\} \oplus \{10,696,400'\} \oplus \{10,830,105\} \oplus (2)\{11,981,970\} \oplus \{12,972,960\} \oplus \{14,889,875\} \oplus \{17,606,160\} \oplus \{18,718,700\} \oplus (3)\{20,084,064\} \oplus \{30,604,288\} \oplus \{31,082,480\}$

- Level-17: $(2)\{32\} \oplus \{320\} \oplus (2)\{1,408\} \oplus \{1,760\} \oplus (3)\{3,520\} \oplus (2)\{4,224\} \oplus \{5,280\} \oplus (3)\{7,040\} \oplus (3)\{10,240\} \oplus (2)\{22,880\} \oplus (3)\{24,960\} \oplus (6)\{28,512\} \oplus (3)\{36,960\} \oplus (4)\{45,056\} \oplus (4)\{45,760\} \oplus \{64,064\} \oplus (6)\{91,520\} \oplus (3)\{128,128\} \oplus (6)\{134,784\} \oplus (3)\{137,280\} \oplus (4)\{147,840\} \oplus (3)\{157,696\} \oplus (5)\{160,160\} \oplus \{160,160'\} \oplus (3)\{183,040\} \oplus (6)\{219,648\} \oplus \{251,680\} \oplus (3)\{264,000\} \oplus (3)\{274,560\} \oplus (3)\{292,864\} \oplus \{302,016\} \oplus \{366,080\} \oplus (2)\{457,600\} \oplus (5)\{480,480\} \oplus (3)\{570,240\} \oplus (7)\{573,440\} \oplus (2)\{672,672\} \oplus (4)\{798,720\} \oplus (5)\{896,896\} \oplus (4)\{901,120\} \oplus (8)\{1,034,880\} \oplus (3)\{1,140,480\} \oplus \{1,171,456\} \oplus \{1,208,064\} \oplus (2)\{1,351,680\} \oplus (3)\{1,425,600\} \oplus (2)\{1,757,184\} \oplus (2)\{1,921,920\} \oplus (3)\{1,936,000\} \oplus (3)\{2,013,440\} \oplus (2)\{2,038,400\} \oplus (5)\{2,114,112\} \oplus (3)\{2,168,320\} \oplus (6)\{2,288,000\} \oplus \{2,342,912\} \oplus (3)\{2,358,720\} \oplus (2)\{2,402,400\} \oplus \{2,446,080\} \oplus (3)\{3,706,560\} \oplus (2)\{3,706,560'\} \oplus (3)\{3,794,560\} \oplus \{4,026,880\} \oplus (6)\{4,212,000\} \oplus (2)\{5,720,000\} \oplus (2)\{5,857,280\} \oplus \{5,930,496\} \oplus (3)\{6,040,320\} \oplus \{6,307,840\} \oplus \{6,864,000\} \oplus (3)\{7,208,960\} \oplus (3)\{8,781,696\} \oplus (3)\{9,123,840\} \oplus \{10,570,560\} \oplus \{10,570,560'\} \oplus (2)\{11,714,560\} \oplus \{11,927,552\} \oplus (2)\{12,390,400\} \oplus (2)\{13,246,464\} \oplus (2)\{13,453,440\} \oplus \{15,375,360\} \oplus \{30,201,600\} \oplus \{33,116,160\} \oplus \{33,554,432\}$

- Level-17: $(2)\{32\} \oplus \underline{\{320\}} \oplus (2)\{1,408\} \oplus \{1,760\} \oplus (3)\{3,520\} \oplus (2)\{4,224\} \oplus \{5,280\} \oplus (3)\{7,040\} \oplus (3)\{10,240\} \oplus (2)\{22,880\} \oplus (3)\{24,960\} \oplus (6)\{28,512\} \oplus (3)\{36,960\} \oplus (4)\{45,056\} \oplus (4)\{45,760\} \oplus \{64,064\} \oplus (6)\{91,520\} \oplus (3)\{128,128\} \oplus (6)\{134,784\} \oplus (3)\{137,280\} \oplus (4)\{147,840\} \oplus (3)\{157,696\} \oplus (5)\{160,160\} \oplus \{160,160'\} \oplus (3)\{183,040\} \oplus (6)\{219,648\} \oplus \{251,680\} \oplus (3)\{264,000\} \oplus (3)\{274,560\} \oplus (3)\{292,864\} \oplus \{302,016\} \oplus \{366,080\} \oplus (2)\{457,600\} \oplus (5)\{480,480\} \oplus (3)\{570,240\} \oplus (7)\{573,440\} \oplus (2)\{672,672\} \oplus (4)\{798,720\} \oplus (5)\{896,896\} \oplus (4)\{901,120\} \oplus (8)\{1,034,880\} \oplus (3)\{1,140,480\} \oplus \{1,171,456\} \oplus \{1,208,064\} \oplus (2)\{1,351,680\} \oplus (3)\{1,425,600\} \oplus (2)\{1,757,184\} \oplus (2)\{1,921,920\} \oplus (3)\{1,936,000\} \oplus (3)\{2,013,440\} \oplus (2)\{2,038,400\} \oplus (5)\{2,114,112\} \oplus (3)\{2,168,320\} \oplus (6)\{2,288,000\} \oplus \{2,342,912\} \oplus (3)\{2,358,720\} \oplus (2)\{2,402,400\} \oplus \{2,446,080\} \oplus (3)\{3,706,560\} \oplus (2)\{3,706,560'\} \oplus (3)\{3,794,560\} \oplus \{4,026,880\} \oplus (6)\{4,212,000\} \oplus (2)\{5,720,000\} \oplus (2)\{5,857,280\} \oplus \{5,930,496\} \oplus (3)\{6,040,320\} \oplus \{6,307,840\} \oplus \{6,864,000\} \oplus (3)\{7,208,960\} \oplus (3)\{8,781,696\} \oplus (3)\{9,123,840\} \oplus \{10,570,560\} \oplus \{10,570,560'\} \oplus (2)\{11,714,560\} \oplus \{11,927,552\} \oplus (2)\{12,390,400\} \oplus (2)\{13,246,464\} \oplus (2)\{13,453,440\} \oplus \{15,375,360\} \oplus \{30,201,600\} \oplus \{33,116,160\} \oplus \{33,554,432\}$

THE ADYNKRA GRAPH OF THE 11D, N = 1 SCALAR SUPERFIELD

The Component Fields of the Salam-Strathdee 11D, N = 1 Real Scalar Superfield

Level #	Component Field Count
0	1
1	1
2	3
3	3
4	8
5	9
6	19
7	23
8	49
9	55
10	99
11	106
12	173
13	171
14	247
15	225
16	296

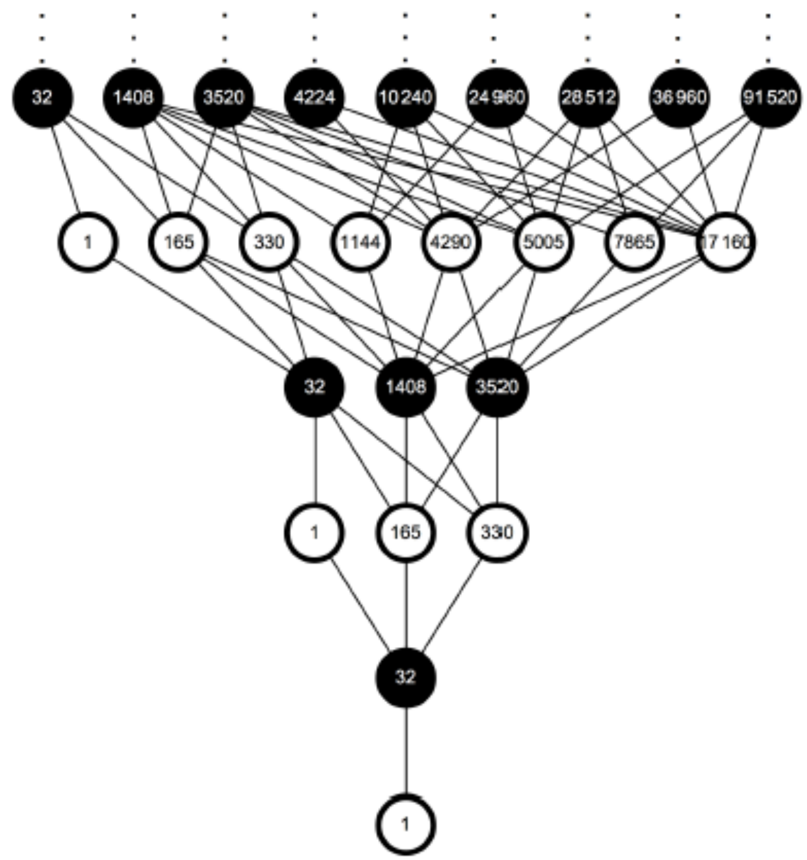
: Number of Independent Fields at Each Level

$$N_{Bosonic\ Fields} = 1,198$$

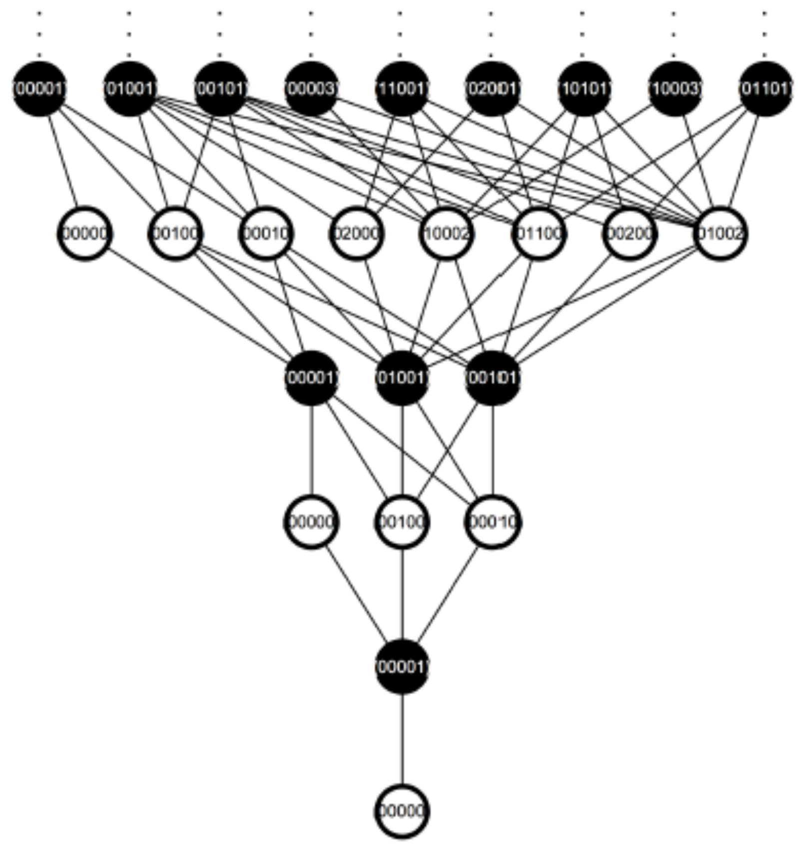
$$\dots, h_{\mu\nu}, A_{\mu\nu\rho}, \dots$$

$$N_{Fermionic\ Fields} = 1,186$$

$$\dots, \Psi_{\mu}{}^{\alpha}, \dots$$



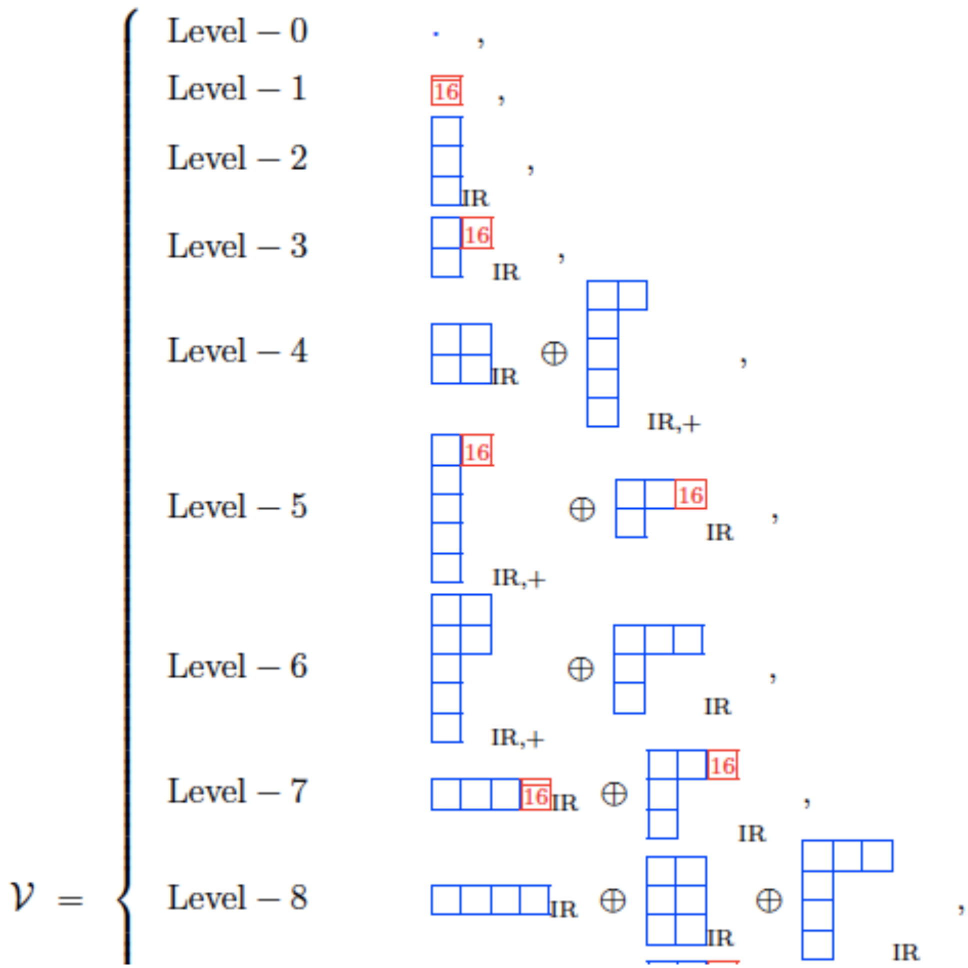
Adinkra Diagram for 11D, $\mathcal{N} = 1$ (using dimensions)



Adinkra Diagram for 11D, $\mathcal{N} = 1$ (using Dynkin labels)

VISIBLE INSIGHTS FROM THE 10D, N = 1 SCALAR SUPERFIELD

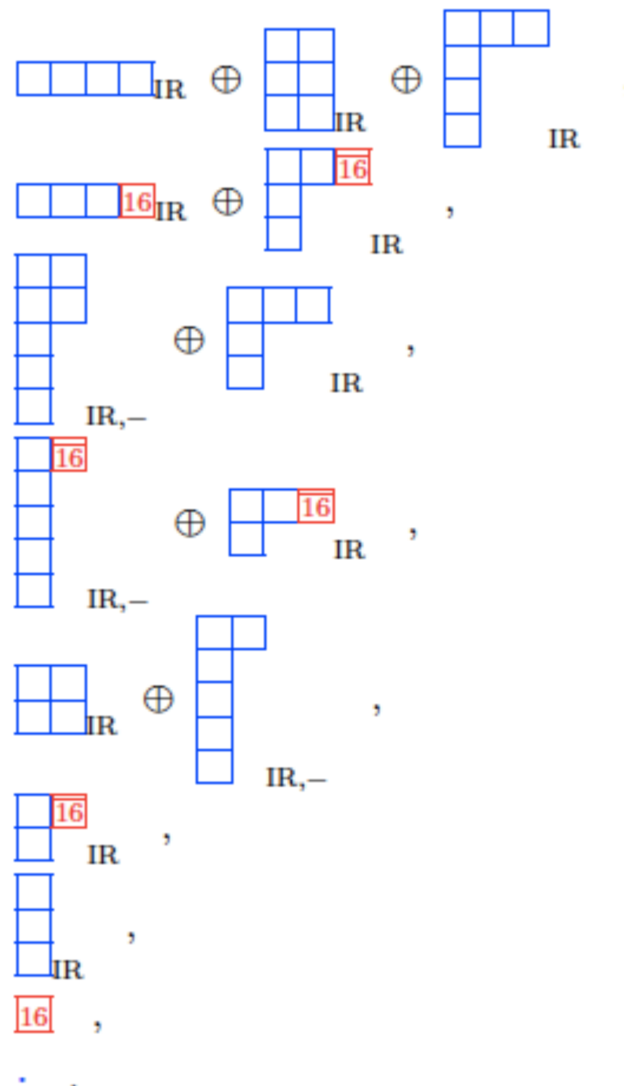
BROWN
THEORETICAL PHYSICS CENTER



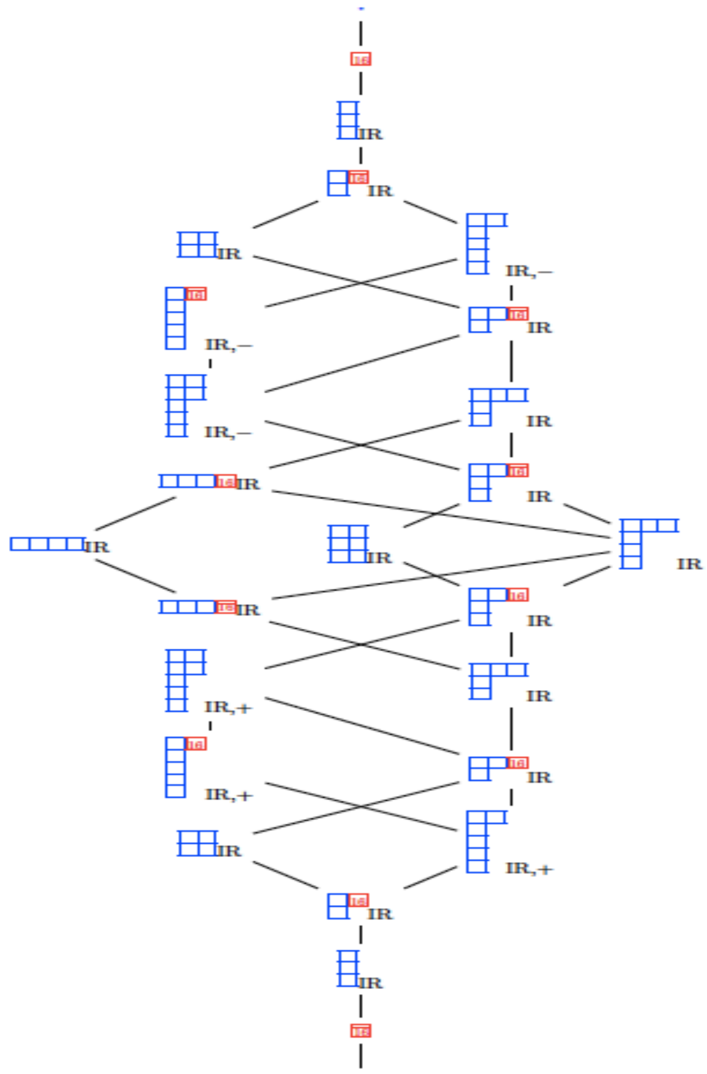
BROWN
THEORETICAL PHYSICS CENTER

$\mathcal{V} =$

- Level – 8
- Level – 9
- Level – 10
- Level – 11
- Level – 12
- Level – 13
- Level – 14
- Level – 15
- Level – 16

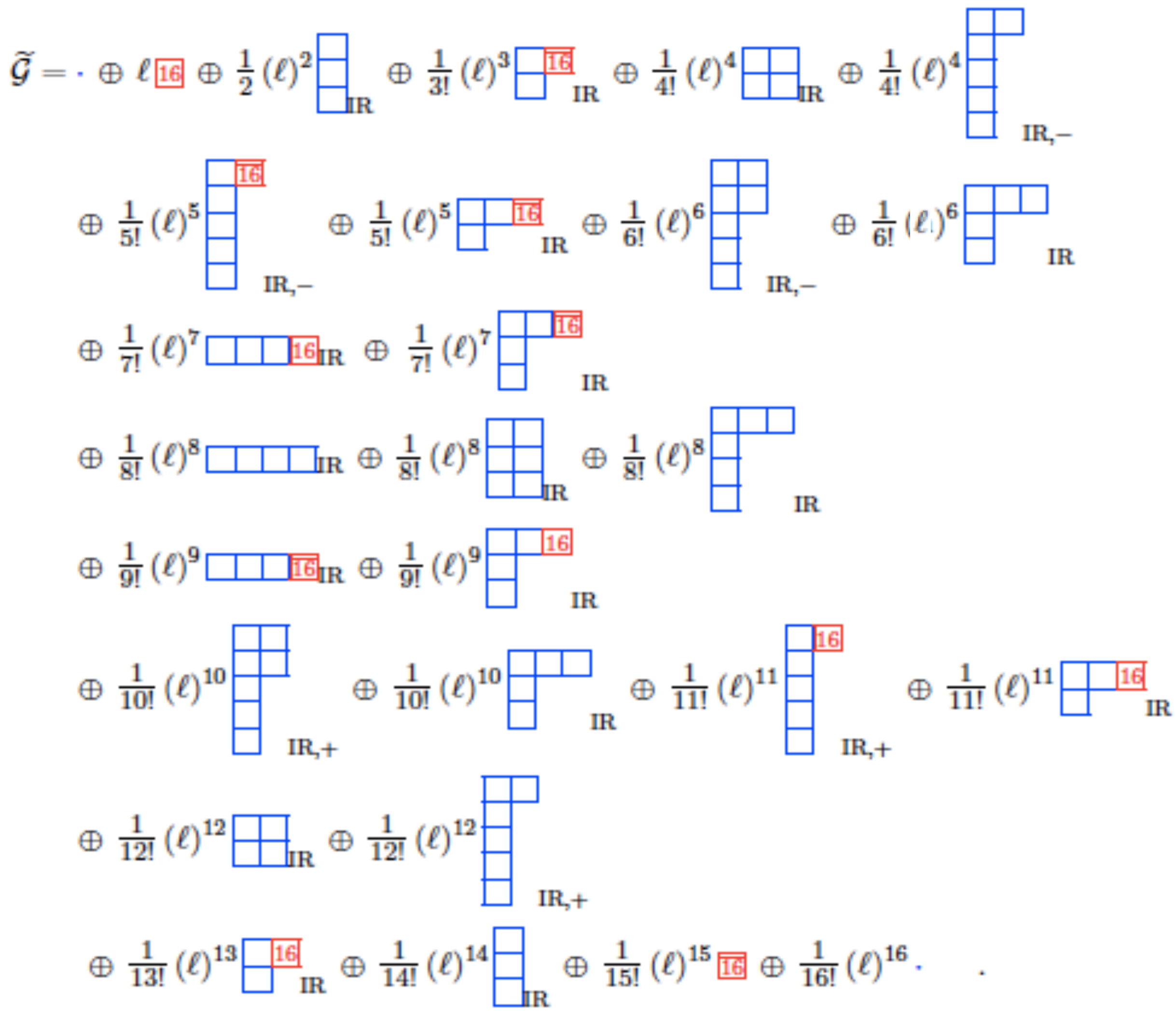


BROWN
THEORETICAL PHYSICS CENTER



10D, N =1
Adynkra Graph in
Dynkin Label/
Young Tableaux
Form

BROWN
THEORETICAL PHYSICS CENTER



BROWN
THEORETICAL PHYSICS CENTER

$$\widehat{\mathcal{G}}(x) = \Phi(x) + \ell\,\boxed{16}\,\Psi_{\alpha}(x) + \tfrac{1}{2}(\ell)^2\,\ydiagram{1,1,1}_{\rm IR}\,\Phi_{\{\underline{a}_1\underline{b}_1\underline{c}_1\}}(x) + \tfrac{1}{3!}(\ell)^3\,\begin{ytableau} ~ & 16 \\ ~ \end{ytableau}_{\rm IR}\,\Psi_{\{\underline{a}_1\underline{b}_1\}}{}^{\alpha}(x)$$

$$+ \tfrac{1}{4!}(\ell)^4\,\ydiagram{2,2}_{\rm IR}\,\Phi_{\{\underline{a}_1\underline{b}_1,\underline{a}_2\underline{b}_2\}}(x) + \tfrac{1}{4!}(\ell)^4\,\ydiagram{2,1,1,1,1}_{{\rm IR},-}\,\Phi_{\{\underline{a}_2|\underline{a}_1\underline{b}_1\underline{c}_1\underline{d}_1\underline{e}_1\}^{+}}(x)$$

$$+ \tfrac{1}{5!}(\ell)^5\,\begin{ytableau} ~ & 16 \\ ~ \\ ~ \\ ~ \\ ~ \end{ytableau}_{{\rm IR},-}\,\Psi_{\{\underline{a}_1\underline{b}_1\underline{c}_1\underline{d}_1\underline{e}_1\}^{+}}{}^{\alpha}(x) + \tfrac{1}{5!}(\ell)^5\,\begin{ytableau} ~ & ~ & 16 \\ ~ \end{ytableau}_{\rm IR}\,\Psi_{\{\underline{a}_2|\underline{a}_1\underline{b}_1\}}{}^{\alpha}(x)$$

$$+ \tfrac{1}{6!}(\ell)^6\,\ydiagram{2,2,1,1,1}_{{\rm IR},-}\,\Phi_{\{\underline{a}_2\underline{b}_2|\underline{a}_1\underline{b}_1\underline{c}_1\underline{d}_1\underline{e}_1\}^{+}}(x) + \tfrac{1}{6!}(\ell)^6\,\ydiagram{3,1,1}_{\rm IR}\,\Phi_{\{\underline{a}_2,\underline{a}_3|\underline{a}_1\underline{b}_1\underline{c}_1\}}(x)$$

$$+ \tfrac{1}{7!}(\ell)^7\,\begin{ytableau} ~ & ~ & ~ & 16 \end{ytableau}_{\rm IR}\,\Psi_{\{\underline{a}_1,\underline{a}_2,\underline{a}_3\}\alpha}(x) + \tfrac{1}{7!}(\ell)^7\,\begin{ytableau} ~ & ~ & 16 \\ ~ \\ ~ \end{ytableau}_{\rm IR}\,\Psi_{\{\underline{a}_2|\underline{a}_1\underline{b}_1\underline{c}_1\}}{}^{\alpha}(x)$$

$$+ \tfrac{1}{8!}(\ell)^8\,\ydiagram{4}_{\rm IR}\,\Phi_{\{\underline{a}_1,\underline{a}_2,\underline{a}_3,\underline{a}_4\}}(x) + \tfrac{1}{8!}(\ell)^8\,\ydiagram{2,2,2}_{\rm IR}\,\Phi_{\{\underline{a}_1\underline{b}_1\underline{c}_1,\underline{a}_2\underline{b}_2\underline{c}_2\}}(x)$$

$$+ \tfrac{1}{8!}(\ell)^8\,\ydiagram{3,1,1,1}_{\rm IR}\,\Phi_{\{\underline{a}_2,\underline{a}_3|\underline{a}_1\underline{b}_1\underline{c}_1\underline{d}_1\}}(x)$$

$$+ \tfrac{1}{9!}(\ell)^9\,\begin{ytableau} ~ & ~ & ~ & 16 \end{ytableau}_{\rm IR}\,\Psi_{\{\underline{a}_1,\underline{a}_2,\underline{a}_3\}}{}^{\alpha}(x) + \tfrac{1}{9!}(\ell)^9\,\begin{ytableau} ~ & ~ & 16 \\ ~ \\ ~ \end{ytableau}_{\rm IR}\,\Psi_{\{\underline{a}_2|\underline{a}_1\underline{b}_1\underline{c}_1\}\alpha}(x)$$

$$+ \tfrac{1}{10!}(\ell)^{10}\,\ydiagram{2,2,1,1,1}_{{\rm IR},+}\,\Phi_{\{\underline{a}_2\underline{b}_2|\underline{a}_1\underline{b}_1\underline{c}_1\underline{d}_1\underline{e}_1\}^{-}}(x) + \tfrac{1}{10!}(\ell)^{10}\,\ydiagram{3,1,1}_{\rm IR}\,\widehat{\Phi}_{\{\underline{a}_2,\underline{a}_3|\underline{a}_1\underline{b}_1\underline{c}_1\}}(x)$$

$$+ \tfrac{1}{11!}(\ell)^{11}\,\begin{ytableau} ~ & 16 \\ ~ \\ ~ \\ ~ \\ ~ \end{ytableau}_{{\rm IR},+}\,\Psi_{\{\underline{a}_1\underline{b}_1\underline{c}_1\underline{d}_1\underline{e}_1\}^{-}\alpha}(x) + \tfrac{1}{11!}(\ell)^{11}\,\begin{ytableau} ~ & ~ & 16 \\ ~ \end{ytableau}_{\rm IR}\,\Psi_{\{\underline{a}_2|\underline{a}_1\underline{b}_1\}\alpha}(x)$$

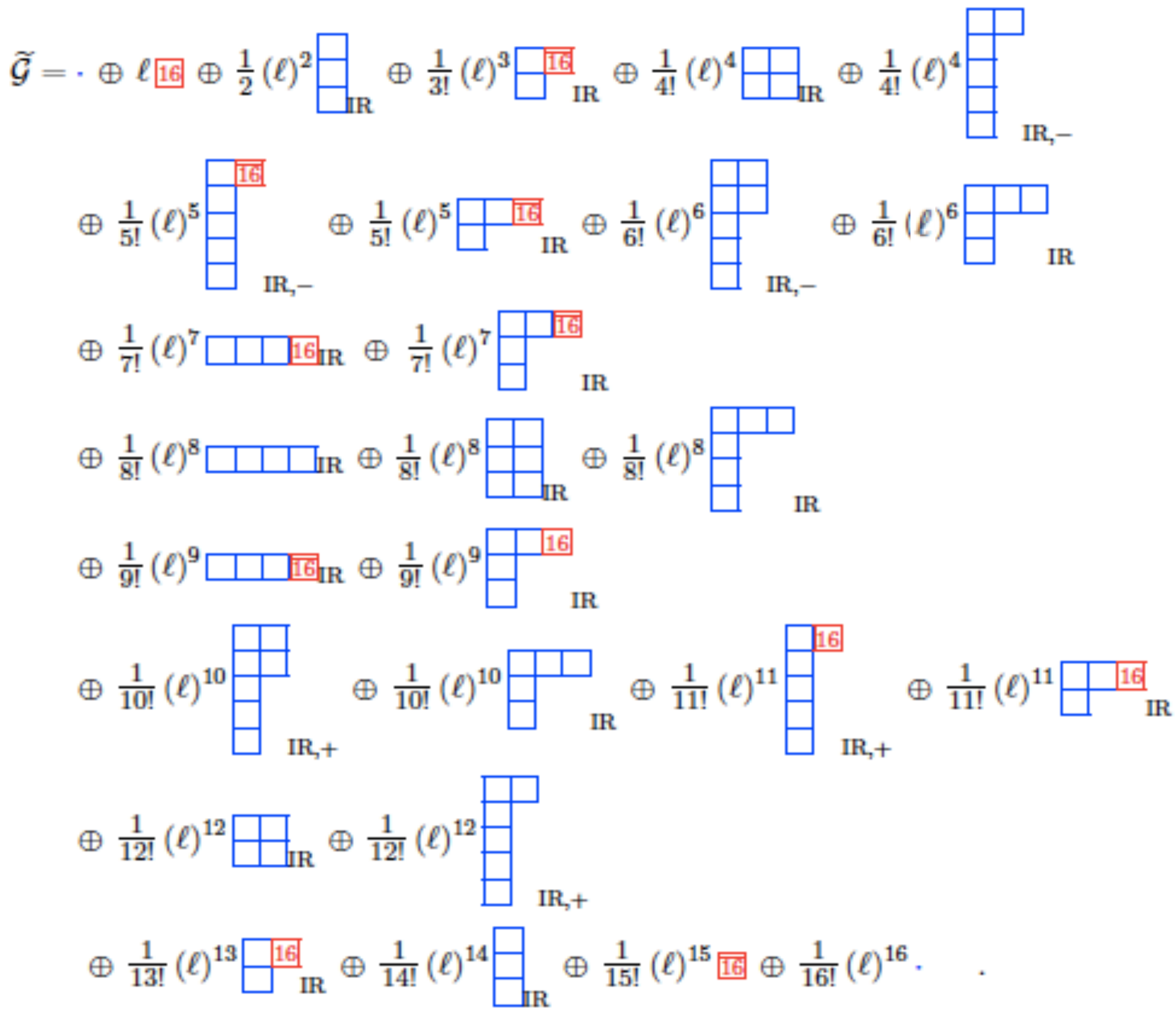
$$+ \tfrac{1}{12!}(\ell)^{12}\,\ydiagram{2,2}_{\rm IR}\,\widehat{\Phi}_{\{\underline{a}_1\underline{b}_1,\underline{a}_2\underline{b}_2\}}(x) + \tfrac{1}{12!}(\ell)^{12}\,\ydiagram{2,1,1,1,1}_{{\rm IR},+}\,\Phi_{\{\underline{a}_2|\underline{a}_1\underline{b}_1\underline{c}_1\underline{d}_1\underline{e}_1\}^{-}}(x)$$

$$+ \tfrac{1}{13!}(\ell)^{13}\,\begin{ytableau} ~ & 16 \\ ~ \end{ytableau}_{\rm IR}\,\Psi_{\{\underline{a}_1\underline{b}_1\}\alpha}(x) + \tfrac{1}{14!}(\ell)^{14}\,\ydiagram{1,1,1}_{\rm IR}\,\widehat{\Phi}_{\{\underline{a}_1\underline{b}_1\underline{c}_1\}}(x) + \tfrac{1}{15!}(\ell)^{15}\,\boxed{16}\,\Psi^{\alpha}(x)$$

$$+ \tfrac{1}{16!}(\ell)^{16}\,\widehat{\Phi}(x) \quad ,$$

IDENTIFYING THE SUPERFIELD GENOME

BROWN
THEORETICAL PHYSICS CENTER



$$\mathcal{G} \;=\; 1 \,\oplus\, \ell\,\{(\square)\times[a_1,b_1,c_1,d_1,e_1]\} \;\oplus\; \bigoplus_{p=2}^{16}\frac{1}{p!}\,(\ell)^p\left\{\left(\square\,(\wedge\square)^{p-2}\wedge\square\right)\times[a_p,b_p,c_p,d_p,e_p]\right\} \quad ,$$

which can be expanded to,

$$\begin{aligned}\mathcal{G} \;=\; & \ell\,\{(\square)\times[a_1,b_1,c_1,d_1,e_1]\} \\ & \oplus\bigoplus_{p=1}^{7}\frac{1}{p!}\,(\ell)^{2p+1}\left\{\left(\square\,(\wedge\square)^{2p-1}\wedge\square\right)\times[a_{2p+1},b_{2p+1},c_{2p+1},d_{2p+1},e_{2p+1}]\right\} \\ & \oplus 1\oplus\bigoplus_{p=1}^{8}\frac{1}{(2p)!}\,(\ell)^{2p}\left\{\left(\square\,(\wedge\square)^{2(p-1)}\wedge\square\right)\times[a_{2p},b_{2p},c_{2p},d_{2p},e_{2p}]\right\} \quad ,\end{aligned}$$

The adynkra shown in Figure 1 can be expressed totally in a field-independent manner and purely in terms of group-theoretical constructs mathematically in terms of $\mathcal{G}$ with the definition

$$\mathcal{G} \;=\; 1 \,\oplus\, \ell\,\{(\Box) \times [a_1, b_1, c_1, d_1, e_1]\} \,\oplus\, \bigoplus_{p=2}^{16} \tfrac{1}{p!}\,(\ell)^p\,\{(\Box\,(\wedge\Box)^{p-2}\wedge\Box) \times [a_p, b_p, c_p, d_p, e_p]\} \quad ,$$

and where a number of definitions must be understood and these include:

(a.) $\Box$ denotes the SYT i

(b.) the $\wedge$ product denotes the usual rule for multiplying two tableaux, but restricted so that only single column resultants are kept,

(c.) $[a_p, b_p, c_p, d_p, e_p]$ denotes a Dynkin Label for an irrep in $\mathfrak{so}(10)$ where the quantities a_p, b_p, c_p, d_p, and e_p are a set of integers,

(d.) $\mathcal{A} \times [a_p, b_p, c_p, d_p, e_p] = [a_p, b_p, c_p, d_p, e_p]$ where $\mathcal{A}$ is a single column SYT containing the irrep $[a_p, b_p, c_p, d_p, e_p]$ otherwise $\mathcal{A} \times [a_p, b_p, c_p, d_p, e_p] = 0$,

(e.) $\mathcal{A} \times [a_p, b_p, c_p, d_p, e_p] = \mathrm{m}\,[a_p, b_p, c_p, d_p, e_p]$ if instead $\mathcal{A}$ contains the representation $[a_p, b_p, c_p, d_p, e_p]$ m-times, and finally

(f.) $\{\mathcal{A} \times [a_p, b_p, c_p, d_p, e_p]\}$ is a notation implying independent sums to be taken over all possible values of a_p, b_p ,c_p ,d_p, and e_p.

BROWN
THEORETICAL PHYSICS CENTER

LOOKING BACK AT LESSONS FROM BASIC LESSONS FROM 4D, N = 1 SUPERFIELD SUPERGRAVITY FOR ADYNKRA SUPERGRAVITY GAUGE GROUPS

4D, N = 1 BASIC GENOME OPERATORS

$$\mathcal{G}^{(L)}\left[\ell\,\boxed{2}\right] = \exp\left[\ell\,\boxed{2}\right] \quad ,$$
$$\mathcal{G}^{(R)}\left[\bar{\ell}\,\boxed{\bar{2}}\right] = \exp\left[\bar{\ell}\,\boxed{\bar{2}}\right] \quad ,$$

4D, N = 1 SUPERFIELD GENOMES

$$\mathcal{G}\left[\ell, \bar{\ell}, \boxed{2}, \boxed{\bar{2}}\right] [\mathrm{YT}] \equiv \mathcal{G}^{(L)}\left[\ell\,\boxed{2}\right] \mathcal{G}^{(R)}\left[\bar{\ell}\,\boxed{\bar{2}}\right] [\mathrm{YT}] \quad ,$$

where $[\mathrm{YT}]$ is any bosonic Dynkin Label/Young Tableux

$$\mathcal{G}\left[\ell, \bar{\ell}, \boxed{2}, \boxed{\bar{2}}\right] [\mathrm{YT}] \equiv \mathcal{G}^{(L)}\left[\ell\,\boxed{2}\right] \mathcal{G}^{(R)}\left[\bar{\ell}\,\boxed{\bar{2}}\right] [\mathrm{YT}] \quad ,$$

where $[\mathrm{YT}]$ is any fermionic Dynkin Label/Young Tableux

4D, N = 1 CHIRAL SUPERFIELD GENOMES

$$\mathcal{G}[\ell,0,\boxed{2},\boxed{\bar{2}}]\ \cdot\ = \mathcal{G}^{(L)}[\ell\boxed{2}]\ \cdot\ = \ \cdot\ \oplus\ \ell\boxed{2}\ \oplus\ \frac{1}{2!}(\ell)^2\ \cdot\ ,$$

$$\mathcal{G}[\ell,0,\boxed{2},\boxed{\bar{2}}]\ \boxed{2} = \mathcal{G}^{(L)}[\ell\boxed{2}]\ \boxed{2} = \boxed{2}\ \oplus\ \ell\,\Big(\ \cdot\ \oplus \begin{smallmatrix}\square\\\square\end{smallmatrix}_{\text{IR},-}\Big)\ \oplus\ \frac{1}{2!}(\ell)^2\,\boxed{2}\ ,$$

$$\mathcal{G}[\ell,0,\boxed{2},\boxed{\bar{2}}]\ \boxed{\bar{2}} = \mathcal{G}^{(L)}[\ell\boxed{2}]\ \boxed{\bar{2}} = \boxed{\bar{2}}\ \oplus\ \ell\,\square_{\text{IR}}\ \oplus\ \frac{1}{2!}(\ell)^2\,\boxed{\bar{2}}\ ,$$

4D, N = 1 GENERAL SUPERFIELD GENOMES

$$\begin{aligned}\mathcal{G}[\ell,\bar{\ell},\boxed{2},\boxed{\bar{2}}]\ \cdot\ = &\ \cdot\ \oplus\ \ell\boxed{2}\ \oplus\ \bar{\ell}\boxed{\bar{2}}\ \oplus\ \frac{1}{2!}(\ell)^2\ \cdot\ \oplus\ \frac{1}{2!}(\bar{\ell})^2\ \cdot\ \oplus\ \ell\bar{\ell}\,\square_{\text{IR}}\\ &\oplus\ \frac{1}{2!}(\ell)^2\,\bar{\ell}\,\boxed{\bar{2}}\ \oplus\ \frac{1}{2!}(\bar{\ell})^2\,\ell\,\boxed{2}\ \oplus\ \frac{1}{2!2!}(\ell)^2(\bar{\ell})^2\ \cdot\ ,\end{aligned}$$

$$\begin{aligned}\mathcal{G}[\ell,\bar{\ell},\boxed{2},\boxed{\bar{2}}]\ \boxed{2} = &\ \boxed{2}\ \oplus\ \ell\,\Big(\ \cdot\ \oplus \begin{smallmatrix}\square\\\square\end{smallmatrix}_{\text{IR},-}\Big)\ \oplus\ \bar{\ell}\,\square_{\text{IR}}\ \oplus\ \frac{1}{2!}(\ell)^2\boxed{2}\ \oplus\ \frac{1}{2!}(\bar{\ell})^2\boxed{2}\\ &\oplus\ \ell\bar{\ell}\,\Big(\boxed{\bar{2}}\ \oplus\ \boxed{}\boxed{2}_{\text{IR}}\Big)\ \oplus\ \frac{1}{2!}(\ell)^2\,\bar{\ell}\,\square_{\text{IR}}\ \oplus\ \frac{1}{2!}(\bar{\ell})^2\,\ell\,\Big(\ \cdot\ \oplus \begin{smallmatrix}\square\\\square\end{smallmatrix}_{\text{IR},-}\Big)\end{aligned}$$

The Component Fields of the 4D, N = 1 Supergravity Prepotential Superfield

$$H_{\underline{a}} = \begin{cases} \text{level} - 0: & h_{\underline{a}} \quad , \\ \text{level} - 1: & h_{\alpha\beta\dot{\beta}} \quad , \quad \bar{h}_{\dot{\alpha}\beta\dot{\beta}} \quad , \\ \text{level} - 2: & h^{(2)}{}_{\underline{a}} \quad , \quad \bar{h}^{(2)}{}_{\underline{a}} \quad , \quad h_{\underline{a}\underline{b}} \quad , \\ \text{level} - 3: & \bar{\psi}_{\underline{a}\dot{\beta}} \quad , \quad \psi_{\underline{a}\beta} \quad , \\ \text{level} - 4: & A_{\underline{a}} \quad . \end{cases}$$

The Dynkin Labels/Young Tableaux of the 4D, N = 1 Supergravity Prepotential Genome

$$H_{\underline{a}} = \begin{cases} \text{level}-0: & \Box_{\rm IR} \quad , \\ \text{level}-1: & \boxed{2} \oplus \boxed{\bar{2}} \oplus \Box\boxed{\bar{2}}_{\rm IR} \oplus \Box\boxed{2}_{\rm IR} \quad , \\ \text{level}-2: & \cdot \oplus \begin{array}{|c|}\hline \\ \hline \\ \hline \end{array}_{\rm IR,-} \oplus \begin{array}{|c|}\hline \\ \hline \\ \hline \end{array}_{\rm IR,+} \oplus (2)\Box_{\rm IR} \oplus \Box\Box_{\rm IR} \quad , \\ \text{level}-3: & \boxed{2} \oplus \boxed{\bar{2}} \oplus \Box\boxed{\bar{2}}_{\rm IR} \oplus \Box\boxed{2}_{\rm IR} \quad , \\ \text{level}-4: & \Box_{\rm IR} \quad . \end{cases}$$

$$\underline{a} \equiv \Box_{\rm IR} \quad , \quad \alpha \equiv \boxed{2} \quad , \quad \dot{\alpha} \equiv \boxed{\bar{2}} \quad ,$$

BROWN
THEORETICAL PHYSICS CENTER

The Dynkin Labels/Young Tableaux of the 4D, N = 1 Supergravity Prepotential Genome

$$H_{\underline{a}} = \begin{cases} \mathrm{level}-0: & \Box \quad , \\ \mathrm{level}-1: & \Box \otimes \boxed{2} \ \oplus \ \Box \otimes \boxed{\bar{2}} \quad , \\ \mathrm{level}-2: & (2)\Box \ \oplus \ \Box \otimes \Box \quad , \\ \mathrm{level}-3: & \Box \otimes \boxed{\bar{2}} \ \oplus \ \Box \otimes \boxed{2} \quad , \\ \mathrm{level}-4: & \Box \quad . \end{cases}$$

$$\underline{a} \equiv \Box_{\mathrm{IR}} \quad , \quad \alpha \equiv \boxed{2} \quad , \quad \dot{\alpha} \equiv \boxed{\bar{2}} \quad ,$$

The Dynkin Labels/Young Tableaux Equivalence To Wess-Zumino Gauge Supergravity Component Fields

graviton

$$h_{\underline{ab}} \;=\; \square\!\square_{\mathrm{IR}} \oplus \begin{array}{c}\square\\[-4pt]\square\end{array}_{\mathrm{IR},+} \oplus \begin{array}{c}\square\\[-4pt]\square\end{array}_{\mathrm{IR},-} \oplus \;\cdot \quad ,$$

gravitino field

$$\psi_{\underline{a}\,\beta} \sim \square\!\boxed{2}_{\mathrm{IR}} \oplus \boxed{\bar{2}} \quad ,$$

axial vector auxiliary field

$$A_{\underline{a}} \sim \square_{\mathrm{IR}} \quad ,$$

BROWN
THEORETICAL PHYSICS CENTER

Acknowledgments

Research supported by the endowment of the Ford Foundation Professorship endowment at Brown University.

I acknowledge my postdoctoral researcher, Dr.Konstantinos Koutrolikos, and Ph.D. students, Yangrui Hu, Aleksander Cianciara. In addition, gratitude must be expressed to the efforts of SSTPRS

https://sites.brown.edu/sjgates/sstprs/

undergraduate research interns.

Finally, I wish to acknowledge my all collaborators as we have slowly worked our ways to deeper insights.

BROWN
THEORETICAL PHYSICS CENTER

Acknowledgments

Stephon Alexander
Joseph Buchbinder
Isaac Chappell, II
Michael Faux
James Gonzales
T. Hübsch
William Linch
Yangrui Hu
Charles Doran
Sze-Ning (Hazel) Mak
Konstantinos Koutrolikos
Boanne MacGregor
Joseph Phillips
James Parker
Lubna Rana
Rueben Polo-Sherk
Vincent Rodgers
Kevin Iga

(collaborators and students listed)

Research supported by the National Science Foundation, Brown University, and the University of Maryland over the last decade

High Energy Physics – Theory

[Submitted on 3 Nov 2019 (v1), last revised 30 Jan 2020 (this version, v3)]

Superfield Component Decompositions and the Scan for Prepotential Supermultiplets in 10D Superspaces

S. James Gates Jr., Yangrui Hu, S.-N. Hazel Mak

The first complete and explicit SO(1,9) Lorentz descriptions of all component fields contained in $\mathcal{N} = 1$, $\mathcal{N} = 2$A, and $\mathcal{N} = 2$B unconstrained scalar 10D superfields are presented. These are made possible by the discovery of the relation of the superfield component expansion as a consequence of the branching rules of irreducible representations in one ordinary Lie algebra into one of its Lie subalgebras. Adinkra graphs for ten dimensional superspaces are defined for the first time, whose nodes depict spin bundle representations of SO(1,9). An analog of Breitenlohner's approach is implemented to scan for superfields that contain graviton(s) and gravitino(s), which are the candidates for the prepotential superfields of 10D off-shell supergravity theories and separately abelian Yang-Mills theories are similarly treated. Version three contains additional content, both historical and conceptual, which broaden the reach of the scan in the 10D Yang-Mills case.

BROWN
THEORETICAL PHYSICS CENTER

arXiv.org > hep-th > arXiv:2002.08502

High Energy Physics – Theory

[Submitted on 20 Feb 2020 (v1), last revised 12 May 2020 (this version, v7)]

Adinkra Foundation of Component Decomposition and the Scan for Superconformal Multiplets in 11D, N = 1 Superspace

S. James Gates Jr., Yangrui Hu, S.-N. Hazel Mak

For the first time in the physics literature, the Lorentz representations of all 2,147,483,648 bosonic degrees of freedom and 2,147,483,648 fermionic degrees of freedom in an unconstrained eleven dimensional scalar superfield are presented. Comparisons of the conceptual bases for this advance in terms of component field, superfield, and adinkra arguments, respectively, are made. These highlight the computational efficiency of the adinkra-based approach over the others. It is noted at level sixteen in the 11D, N = 1 scalar superfield, the {65} representation of SO(1,10), the conformal graviton, is present. Thus, Adinkra-based arguments suggest the surprising possibility that the 11D, N = 1 scalar superfield alone might describe a Poincare supergravity prepotential in analogy to one of the off-shell versions of 4D, N = 1 superfield supergravity.

BROWN
THEORETICAL PHYSICS CENTER

arXi

arXiv.org > hep-th > arXiv:2002.08502

Hi

[Su

Su fo

S.

Hi

[Su

A a =

S.

arXiv.org > hep-th > arXiv:2006.03609

High Energy Physics – Theory

[Submitted on 5 Jun 2020 (v1), last revised 20 Jul 2020 (this version, v2)]

Advening to Adynkrafields: Young Tableaux to Component Fields of the 10D, N = 1 Scalar Superfield

S. James Gates Jr., Yangrui Hu, S.-N. Hazel Mak

Starting from higher dimensional adinkras constructed with nodes referenced by Dynkin Labels, we define "adynkras." These suggest a computationally direct way to describe the component fields contained within supermultiplets in all superspaces. We explicitly discuss the cases of ten dimensional superspaces. We show this is possible by replacing conventional θ-expansions by expansions over Young Tableaux and component fields by Dynkin Labels. Without the need to introduce σ-matrices, this permits rapid passages from Adynkras $\rightarrow$ Young Tableaux $\rightarrow$ Component Field Index Structures for both bosonic and fermionic fields while increasing computational efficiency compared to the starting point that uses superfields. In order to reach our goal, this work introduces a new graphical method, "tying rules," that provides an alternative to Littlewood's 1950 mathematical results which proved branching rules result from using a specific Schur function series. The ultimate point of this line of reasoning is the introduction of mathematical expansions based on Young Tableaux and that are algorithmically superior to superfields. The expansions are given the name of "adynkrafields" as they combine the concepts of adinkras and Dynkin Labels.

BROWN
THEORETICAL PHYSICS CENTER

arXiv.org > hep-th > arXiv:2002.08502

arXiv.org > hep-th > arXiv:2006.03609

arXiv.org > hep-th > arXiv:2007.07390

High Energy Physics – Theory

[Submitted on 14 Jul 2020]

Component Decompositions and Adynkra Libraries for Supermultiplets in Lower Dimensional Superspaces

S. James Gates Jr., Yangrui Hu, S.-N. Hazel Mak

We present Adynkra Libraries that can be used to explore the embedding of multiplets of component field (whether on-shell or partial on-shell) within Salam-Strathdee superfields for theories in dimension nine through four.

BROWN
THEORETICAL PHYSICS CENTER

arXiv.org > hep-th > arXiv:1904.01738

High Energy Physics – Theory

[Submitted on 3 Apr 2019 (v1), last revised 15 May 2019 (this version, v3)]

Adinkra Height Yielding Matrix Numbers: Eigenvalue Equivalence Classes for Minimal Four-Color Adinkras

S. James Gates Jr., Yangrui Hu, Kory Stiffler

An adinkra is a graph-theoretic representation of spacetime supersymmetry. Minimal four-color valise adinkras have been extensively studied due to their relations to minimal 4D, $\mathcal{N} = 1$ supermultiplets. Valise adinkras, although an important subclass, do not encode all the information present when a 4D supermultiplet is reduced to 1D. Eigenvalue equivalence classes for valise adinkra matrices exist, known as χ_o equivalence classes, where valise adinkras within the same χ_o equivalence class are isomorphic in the sense that adinkras within a χ_o-equivalence class can be transformed into each other via field redefinitions of the nodes. We extend this to non-valise adinkras, via Python code, providing a complete eigenvalue classification of "node-lifting" for all 36,864 valise adinkras associated with the Coxeter group BC_4. We term the eigenvalues associated with these node-lifted adinkras Height Yielding Matrix Numbers (HYMNs) and introduce HYMN equivalence classes. These findings have been summarized in a *Mathematica* notebook that can found at the HEPTHools Data Repository (this https URL) on GitHub.