Astrophysical Observational Signatures of Dynamical Chern-Simons Gravity

Nicolás Yunes
University of Illinois at Urbana-Champaign

Workshop on Chern-Simons and Other Topological Field Theories
Mathematical Sciences Research Institute
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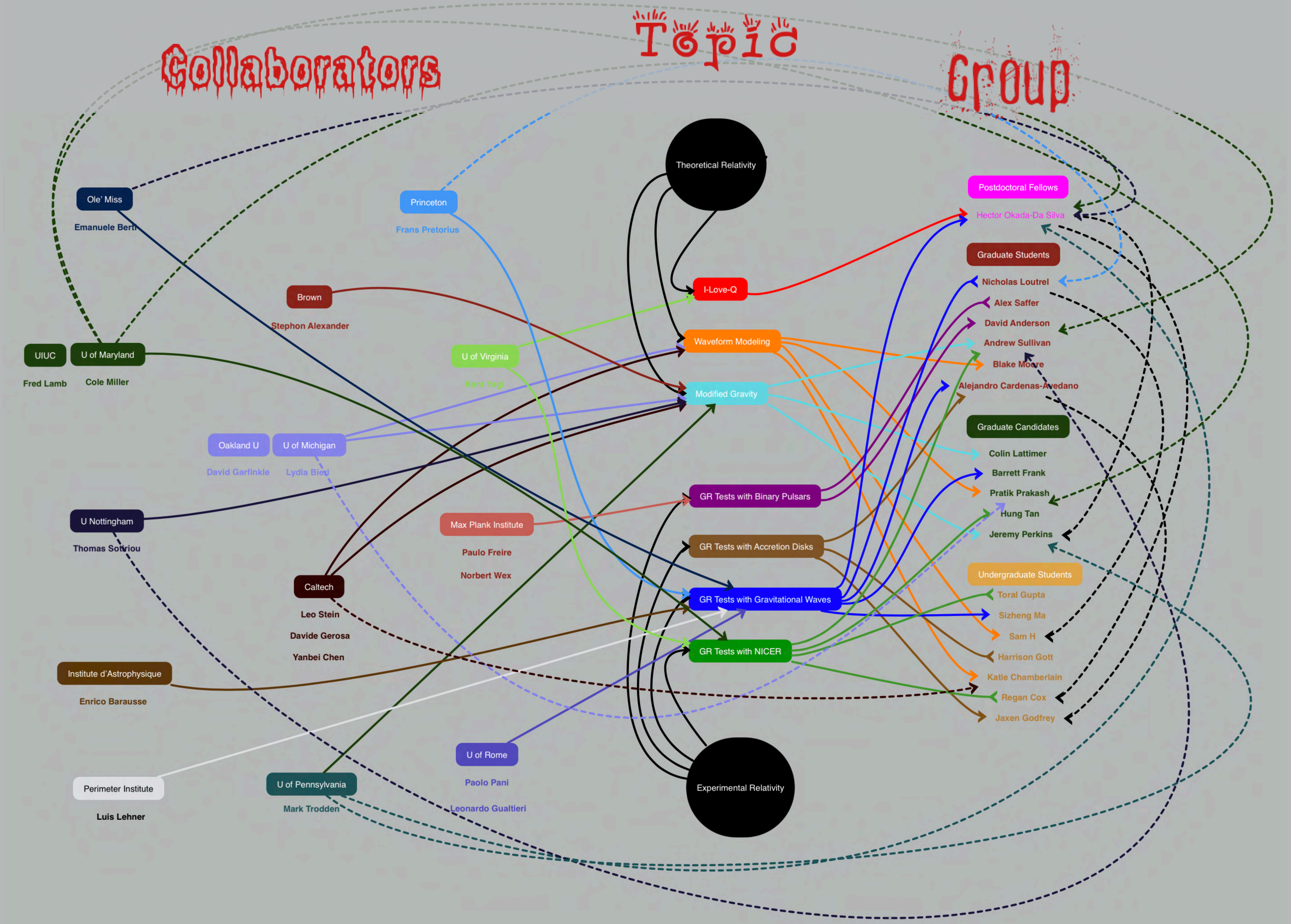
Who are “you” ?

The sum of my parts

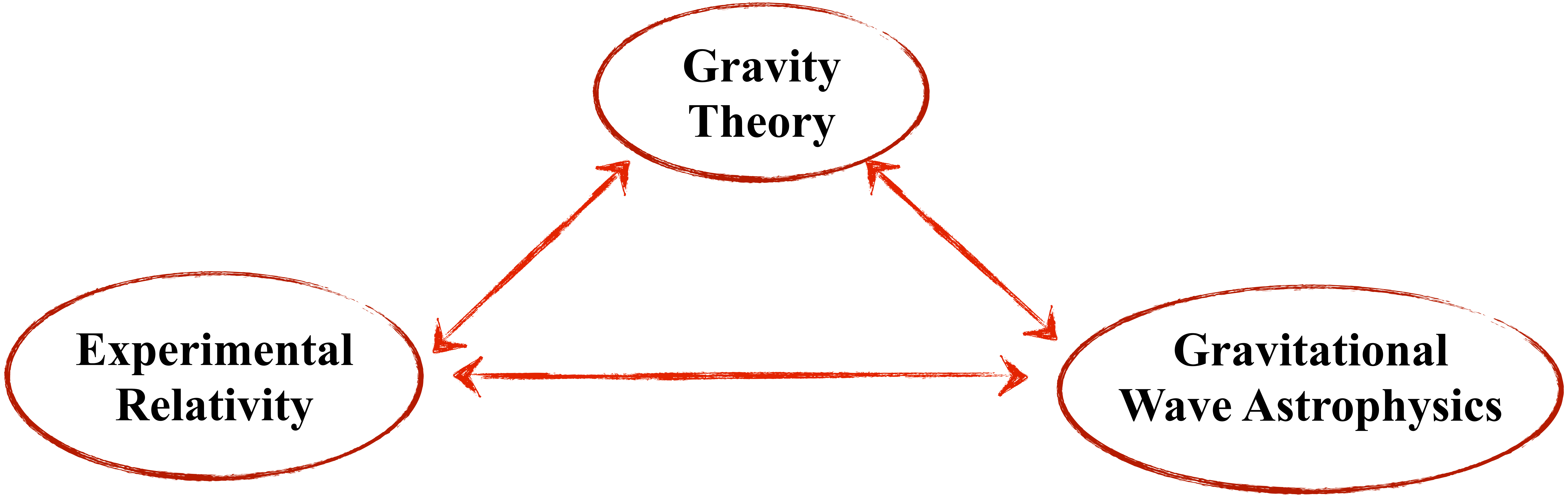
Illinois Center for Advanced Studies of the Universe

Nico's Adinkra



Nico's Adinkra

What is it that you do?



Observational signatures of dynamical Chern-Simons theory in extreme gravity astrophysical environments

Roadmap


Dynamical Chern-Simmons Gravity
Stationary Black Holes
Compact Binaries & Gravitational Waves
Frodo and Sam
ARNOR
ERIADOR
IRON HILLS
MIRKWOOD
RHOVANION
RHÛN
The SEA of RHÛN
ROHAN
GONDOR
MORDOR
ERED NIMRAIS
BELEGAER
(the Great Sea)

Dynamical Chern-Simons Gravity

Lagrangian Density

[e.g. Jackiw & Pi, PRD 68 (2003), Alexander & Yunes, Phys. Rept 480 (2009)]

$$L \sim \underbrace{R}_{\text{Einstein-Hilbert}} - \underbrace{\frac{1}{2}(\nabla_a \vartheta)(\nabla^a \vartheta)}_{\text{dynamical pseudo-scalar field}} + \underbrace{\alpha_{\rm dCS}\, \vartheta\, R^* R}_{\text{non-minimal Pontryagin coupling}}$$

Field Equations

$$\boxed{\begin{gathered} G_{ab} + \alpha_{\rm dCS} C_{ab} = 8\pi T^{\rm mat}_{ab} + 8\pi T^{\vartheta}_{ab} \\ \Box \vartheta = \alpha_{\rm dCS}\, R^* R \end{gathered}}$$

$$C_{ab} = (\nabla_c \vartheta)\ \epsilon^{cde}{}_{(a} \nabla_{|e|} R_{b)d} + (\nabla_{cd}\vartheta)\ {}^*R^d{}_{(ab)}{}^c \longrightarrow C_{ab} = 8\pi\,(\nabla_c \vartheta)\ \epsilon^{cde}{}_{(a} \nabla_{|e|} \bar{T}_{b)d} + (\nabla_{cd}\vartheta)\ {}^*R^d{}_{(ab)}{}^c$$

$$T^{\vartheta}_{ab} = (\nabla_a \vartheta)\,(\nabla_b \vartheta) - \frac{1}{2} g_{ab}\,(\nabla^c \vartheta)\,(\nabla_c \vartheta)$$

well-posed as initial value problem upon EFT order-reduction

[Delsate, et al PRD 91 (2015)]

Who ordered that?

10D heterotic string theory
(in Einstein frame)

$$S = \int d^{10}x\sqrt{g_{10}}\left[\mathcal{R} - \frac{1}{2}\partial_a\phi\partial^a\phi - \frac{1}{12}e^{-\phi}\, H_{abc}H^{abc} - \frac{1}{4}e^{\frac{-\phi}{2}}\mathrm{Tr}(F_{ab}F^{ab})\right]$$

dilaton
(assumed stabilized)

Kalb-Ramond 3-form

$$H_3 = dB_2 - \frac{1}{4}\left(\Omega_3(A) - \alpha'\Omega_3(\omega)\right)$$

Kalb-Ramond
(2-form) field

Chern-Simons
3-form

$$\Omega_3(A) = Tr\left(dA \wedge A + \frac{2}{3}A \wedge A \wedge A\right)$$

4D compactification
(to N=1 SUGRA)

$$ds^2_{10} = ds^2_4 + g_{mn}dy^m dy^n$$

$$S_{4d} = \frac{2}{\alpha'}\int d^4x\sqrt{-g}\left(\mathcal{R}_4 + A - \frac{1}{12}e^{-\phi}H_{abc} \wedge *H^{abc} - \frac{1}{4}e^{\frac{-\phi}{2}}\mathrm{Tr}(F_{ab}F^{ab})\right)$$

Simplify and Integrate by Parts

$$d\theta = *dB_2$$

$$(dB_2 + \Omega_3) \wedge (*dB_2 + *\Omega_3) \sim dB_2 \wedge *dB_2 + dB_2 \wedge *\Omega_3 + \ldots \sim *d\theta \wedge d\theta + d\theta \wedge \Omega_3 + \ldots$$

$$dH_3 = d\Omega_3 = R \wedge R$$

$$\boxed{S_{4d} \sim \int d^4x\,[\mathcal{R}_4 + *d\theta \wedge d\theta + \theta\, R \wedge R]}$$

[Alexander & Gates, JCAP 06 (2006),
Alexander & Yunes, Phys. Rept 480 (2009)]

Who ordered that? (cont'd)

Effective Field Theory of Inflation $\mathcal{L} = \mathcal{L}_0[g_{ab}, (\partial\varphi)^2] + \Delta\mathcal{L}[g_{ab}, (\partial\varphi)^4] + \mathcal{O}[g_{ab}, (\partial\varphi)^6]$

[Weinberg, PRD 77 (2008)]

single-field inflation

$$\mathcal{L}_0 = \sqrt{g}\left[-\frac{M_p^2}{2}R - \frac{M^2}{2}(\partial_a\varphi)(\partial^a\varphi) - M_p^2 U(\varphi)\right]$$

sum of all generally covariant terms with up to 4 spacetime derivatives

$$\begin{aligned}\Delta\mathcal{L} = \sqrt{g}[&f_1(\varphi)(g^{\mu\nu}\varphi_{,\mu}\varphi_{,\nu})^2 + f_2(\varphi)g^{\rho\sigma}\varphi_{,\rho}\varphi_{,\sigma}\Box\varphi \\ &+ f_3(\varphi)(\Box\varphi)^2 + f_4(\varphi)R^{\mu\nu}\varphi_{,\mu}\varphi_{,\nu} \\ &+ f_5(\varphi)Rg^{\mu\nu}\varphi_{,\mu}\varphi_{,\nu} + f_6(\varphi)R\Box\varphi + f_7(\varphi)R^2 \\ &+ f_8(\varphi)R^{\mu\nu}R_{\mu\nu} + f_9(\varphi)C^{\mu\nu\rho\sigma}C_{\mu\nu\rho\sigma}] \\ &+ f_{10}(\varphi)\epsilon^{\mu\nu\rho\sigma}C_{\mu\nu}{}^{\kappa\lambda}C_{\rho\sigma\kappa\lambda},\end{aligned}$$

A derivative expansion (in powers of M), so correction must be order-reduced through background EOM

$$\Delta\mathcal{L} = \sqrt{g}f_1(\varphi)[(\partial_a\varphi)(\partial^a\varphi)]^2 + \sqrt{g}f_9(\varphi)C_{abcd}C^{abcd} + \sqrt{g}f_{10}(\varphi)R^*R$$

Taylor expanding $f_{10}(\varphi) = f_{10}(\varphi_0) + f'_{10}(\varphi_0)(\varphi - \varphi_0) + \ldots$

$$\boxed{\sqrt{g}f'_{10}(\varphi_0)\varphi R^* R}$$

Who ordered that? (cont'd^2)

Loop quantum gravity (by promoting the Barbero-Immirzi parameter to a field and adding fermions)

[Taveras & Yunes, PRD 78 (2008),
Mercuri, PRL 103 (2009)]

Particle Physics (e.g. gravitational ABJ anomaly, loop corrections in gravity + fermion qft, Yang-Mills theories)

[Mariz et al, PRD 70 (2004),
Gomes, PRD 78 (2008)]

EFTs of Gravity (e.g. quadratic gravity, parity-violating gravity)

[Yunes & Stein, PRD 83 (2011),
Crisostomi et al, PRD 97 (2018)]

For the rest of this talk, we focus on dynamical Chern-Simons gravity as an EFT

$$L \sim R - \frac{1}{2}(\nabla_a \vartheta)(\nabla^a \vartheta) + \alpha_{\rm dCS}\, \vartheta\, R^* R$$

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Spherically Symmetric Black Holes

EFT treatment ⟶ **GR Deformations** $g_{ab} = g_{ab}^{(\mathrm{GR})} + \alpha_{\mathrm{dCS}}^2 \delta g_{ab}$ $\quad \vartheta = \alpha_{\mathrm{dCS}} \delta\vartheta$

Static + Vacuum + Spherical Symmetry = Schwarzschild

[Jackiw & Pi, PRD 68 (2003), Grumiller & Yunes, PRD 77 (2008)]

$$\Box\vartheta = \alpha_{\mathrm{dCS}} \cancel{{}^*RR} \longrightarrow \vartheta = \vartheta_{\mathrm{H}} + \vartheta_{\mathrm{P}} = 0$$

$$G_{ab} + \alpha_{\mathrm{dCS}} \cancel{C_{ab}} = 8\pi \cancel{T_{ab}^{\mathrm{mat}}} + 8\pi \cancel{T_{ab}^{\vartheta}} \longrightarrow g_{ab} = g_{ab}^{\mathrm{Schw}}$$

$$C_{ab} = 8\pi \left(\nabla_c \vartheta\right) \, \epsilon^{cde}{}_{(a} \nabla_{|e|} \bar{T}_{b)d} + \left(\nabla_{cd}\vartheta\right) \, {}^*R^d{}_{(ab)}{}^c$$

~~Birkhoff Theorem~~ = Vacuum + Spherical Symmetry = Schwarzschild

[Alexander & Yunes, Phys. Rept 480 (2009)]

Axially Symmetric Black Holes

EFT treatment $\longrightarrow$ **GR Deformations** $g_{ab} = g_{ab}^{(\mathrm{GR})} + \alpha_{\mathrm{dCS}}^2 \delta g_{ab}$ $\quad \vartheta = \alpha_{\mathrm{dCS}} \delta\vartheta$

Stationary + Vacuum + Axially Symmetric ≠ Kerr

[Yunes & Pretorius, PRD 79 (2009)]

$$\Box_{\mathrm{GR}} \vartheta = \alpha_{\mathrm{dCS}} \left({}^*R R\right)_{\mathrm{GR}} = 96 \alpha_{\mathrm{dCS}} \frac{a M^2 r}{\left(r^2 + a^2 \cos^2\theta\right)^{12}} \cos\theta \left(r^2 - 3a^2\cos^2\theta\right)\left(3r^2 - a^2\cos^2\theta\right)$$

Slow Rotation expansion $g(r,\theta) = \sum_{n=0} \left(\frac{a}{M}\right)^n g_n(r,\theta)$

$$\vartheta = \frac{5}{8} \alpha_{\mathrm{dCS}} \frac{\cos\theta}{r^2} \left(1 + \frac{2M}{r} + \frac{18 M^2}{5 r^2}\right) + \mathcal{O}(\alpha_{\mathrm{dCS}} a^3)$$

$$G_{ab} + \alpha_{\mathrm{dCS}} C_{ab}^{(\mathrm{GR})} = \cancel{8\pi T_{ab}^{\mathrm{mat}}} + 8\pi T_{ab}^{\vartheta}$$

$$ds^2 = ds_{\mathrm{K}}^2 + 20\pi \alpha_{\mathrm{dCS}}^2 \frac{a}{r^4} \left(1 + \frac{12M}{7r} + \frac{27M^2}{10 r^2}\right) \sin^2\theta \, dt \, d\phi + \mathcal{O}(\alpha_{\mathrm{dCS}}^2 a^2)$$

Axially Symmetric Black Holes (cont'd)

Higher-order in spin solutions in slow-rotation expansion

[Hartle & Thorne, ApJ 153 (1968)]

✓ Second-order in rotation

[Yagi, Yunes, Tanaka, PRD 86 (2012)]

$$g_{tt} = -1 + \frac{2M}{r} + \chi^2 \left(\frac{M^3 \left(2M^2 + Mr - r^2 + \left(6M^2 - Mr - 3r^2\right)\cos(2\theta)\right)}{2r^5} + \frac{M^3 \zeta \left(-338688 M^7 + 490728 M^6 r - 355740 M^5 r^2 - 176620 M^4 r^3 - 116540 M^3 r^4 + 3494 M^2 r^5 + 4221 M r^6 + 4221 r^7 - 3 \times \left(338688 M^7 - 194376 M^6 r + 44940 M^5 r^2 - 13500 M^4 r^3 + 18540 M^3 r^4 - 4474 M^2 r^5 - 4221 M r^6 - 4221 r^7\right)\cos(2\theta)\right)}{75264\, r^{10}} \right)$$

$$g_{rr} = \frac{1}{1 - \frac{2M}{r}} + \chi^2 \left(-\frac{M^3 \left(10M^2 - 3Mr + r^2 + 3 \times \left(10M^2 - 7Mr + r^2\right)\cos(2\theta)\right)}{2r^3 (-2M + r)^2} + \frac{M^3 \zeta \left(1693440 M^7 + 611240 M^6 r - 109900 M^5 r^2 - 220900 M^4 r^3 - 66940 M^3 r^4 - 5042 M^2 r^5 - 1043 M r^6 + 1407 r^7 + (2M - r) \times \left(2540160 M^6 + 472332 M^5 r + 159180 M^4 r^2 - 154740 M^3 r^3 - 20000 M^2 r^4 - 10213 M r^5 - 4221 r^6\right)\cos(2\theta)\right)}{25088\, r^8 (-2M + r)^2} \right)$$

$$g_{\theta\theta} = r^2 + \chi^2 \left(-\frac{M^3 (2M + r) \times (1 + 3\cos(2\theta))}{2r^2} + \frac{M^3 \left(338688 M^6 + 80808 M^5 r + 67380 M^4 r^2 + 10360 M^3 r^3 + 18908 M^2 r^4 + 9940 M r^5 + 4221 r^6\right) \zeta (1 + 3\cos(2\theta))}{75264\, r^7} \right)$$

$$g_{\phi\phi} = r^2 \sin(\theta)^2 + \chi^2 \left(-\frac{M^3 (2M + r) \times (1 + 3\cos(2\theta)) \sin(\theta)^2}{2r^2} + \frac{M^3 \left(338688 M^6 + 80808 M^5 r + 67380 M^4 r^2 + 10360 M^3 r^3 + 18908 M^2 r^4 + 9940 M r^5 + 4221 r^6\right) \zeta (1 + 3\cos(2\theta)) \sin(\theta)^2}{75264\, r^7} \right)$$

$$g_{t\phi} = \chi \left(-\frac{2M^2 \sin(\theta)^2}{r} + \frac{M^5 \left(189 M^2 + 120 M r + 70 r^2\right) \zeta \sin(\theta)^2}{112\, r^6} \right)$$

Axially Symmetric Black Holes (cont'd)

Higher-order in spin solutions in slow-rotation expansion → **Hairy black hole solutions**

[Hartle & Thorne, ApJ 153 (1968)]

✓ **Second-order in rotation**

[Yagi, Yunes, Tanaka, PRD 86 (2012)]

✓ **Fifth-order in rotation**

[Maselli, et al ApJ 843 (2017)]

✗ **Extremal solutions**

[McNees, Stein, Yunes, CQG 33 (2016)]

✓ **Numerical solutions for arbitrary rotation**

[Delsate, Herdeiro, Radu, Phys Lett B 787 (2018), Sullivan, Yunes, Sotiriou, PRD (2020)]

Properties:

(magnetic, i.e. $1/r^2$) scalar hair

Perturbed horizon and ergosphere

No naked singularities or closed time-like curves

Petrov type I, no Killing tensor

[Owen, Yunes, Witek, PRD 103 (2021)]

Could there be chaos in geodesic motion for test particles in orbit around dynamical Chern-Simons black holes?

Black hole stability and hair loss

All these black holes are stable to linear perturbations

Non-rotating case

[Cardoso & Gualtieri, PRD 80 (2009), Molina, et al PRD 81 (2010)]

Slowly-Rotating case

[Wagle, Yunes, Silva, PRD (2021)]

- Polar and Axial sectors decouple
- Scalar couples to Axial sector only
- Breakage of isospectrality

Quasi-normal modes of black holes carry a dynamical Chern-Simons signature

All dynamical Chern-Simons gravity compact objects have no "hair"

[Yagi, Stein, Yunes, Tanaka, PRD 87 (2013), Wagle, Yunes, Garfinkle, Bieri, CQG 36 (2019)]

No-hair Theorem: Scalar hair (i.e. a monopole $1/r$ scalar) is not sourced in stationary, asymptotically flat, axially-symmetric (vacuum/punctured or non-vacuum/non-punctured) spacetimes of dynamical Chern-Simons gravity.

Dynamical Chern-Simons gravity evades all Solar System and binary pulsar constraints

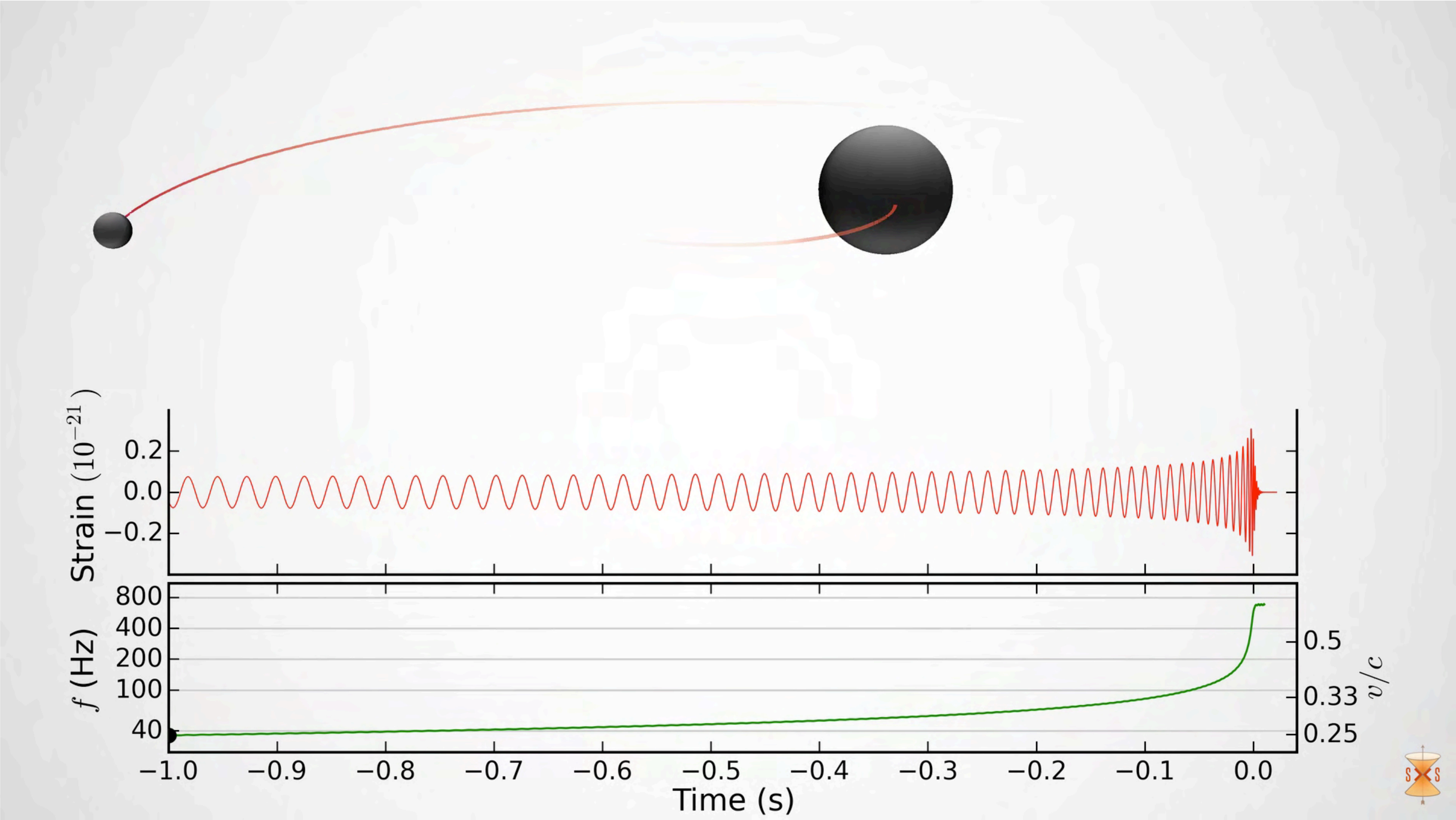
Roadmap

Dynamical Chern-Simmons Gravity

Stationary Black Holes

Compact Binaries & Gravitational Waves

How do you build a gravitational wave model of coalescence?



Newman-Penrose analysis

[Wagle, Safer, Yunes, PRD 100 (2019)]

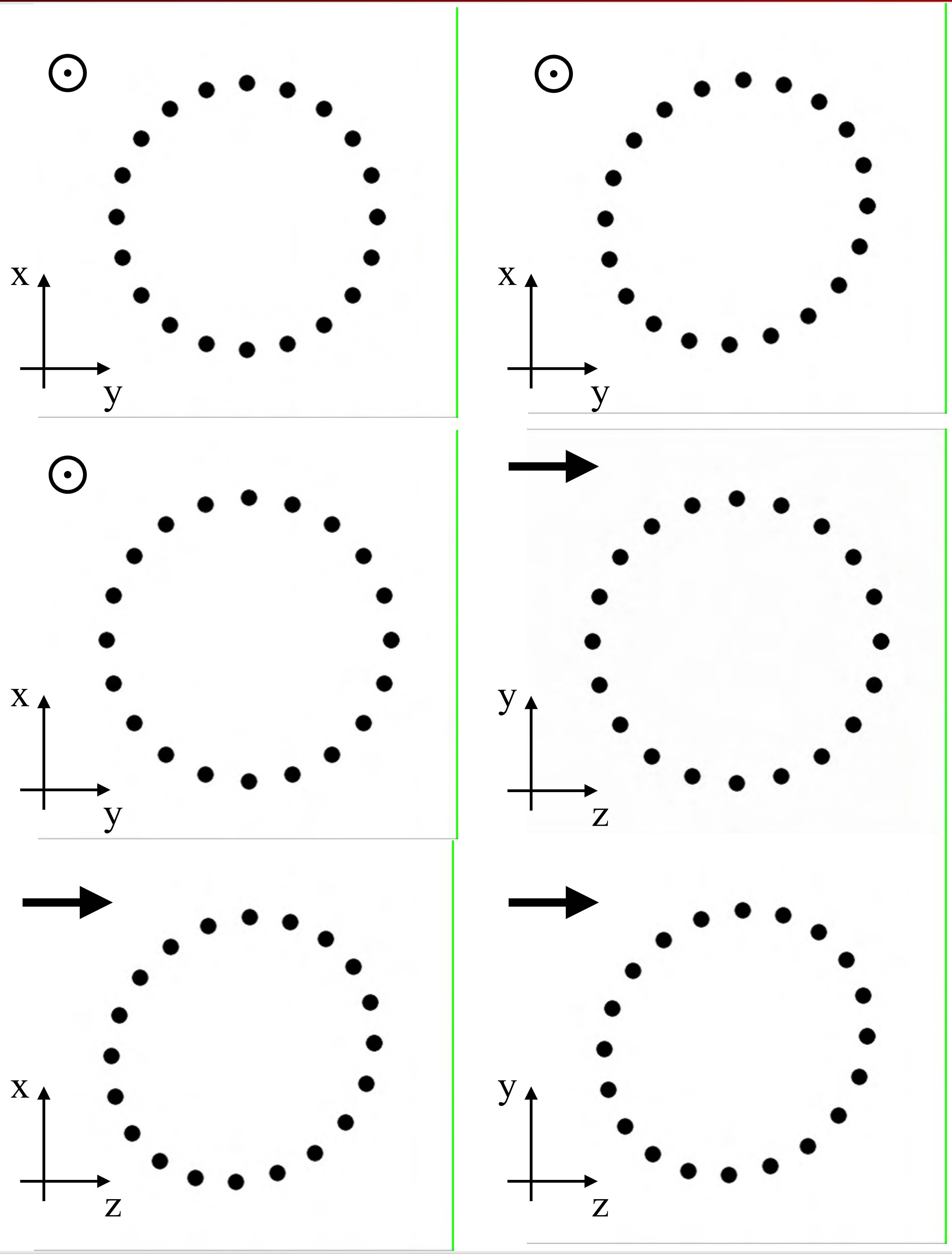
[Animations by Kristen Schumacher]

Only Ψ_4 and Φ_{00} are excited so

$$\text{Class } N_3 : \ \Psi_2 \equiv \Psi_3 = 0, \Psi_4 \neq 0, \Phi_{22} \neq 0.$$

But $\Psi_4 \sim 1/r$, while $\Phi_{00} \sim 1/r^2$

$$\boxed{\text{Class } N_2 : \ \Psi_2 \equiv \Psi_3 \equiv \Phi_{22} = 0, \Psi_4 \neq 0}$$



Amplitude Birefringence

[Alexander, Finn, Yunes, PRD 78 (2008), Yunes, et al, PRD 82 (2010)]

Use a plan wave ansatz $h_{\mu\nu} = A_{\mu\nu} e^{-i[\phi(t) - k_i x^i]}$

Field equations lead to $i\ddot{\phi}_{\rm R,L} + 3iH\dot{\phi}_{\rm R,L} + \dot{\phi}^2_{\rm R,L} - k_i k^i = i\lambda_{\rm R,L}\, \dot{\phi}_{\rm R,L}\, g(\dot{\vartheta}, \ddot{\vartheta})$

$$\boxed{h_{\rm R,L} = h^{\rm GR}_{\rm R,L} e^{-i\lambda_{\rm R,L}\delta\phi} \sim h^{\rm GR}_{\rm R,L}\left[1 + \frac{1}{2}\lambda_{\rm R,L} \int g(\dot{\vartheta}, \ddot{\vartheta}) dt\right]}$$

Gravitational waves generation in dCS gravity

I. Series expand field equations in weak-field, slow-motion (PN). Solve!

[Yunes & Pretorius, PRD 79, 2009;
Yagi, Yunes, Tanaka, PRD 86, 2012]

$$\Box_\eta \vartheta \sim \alpha \epsilon^{abuv} h_{ad,gb} h_v{}^{[g,d]}{}_u$$

$$\vartheta \sim \alpha \epsilon_{ijk} \nabla^{-2} \left(U_{,im} V_{k,jm} \right)$$

(Cannot use point-particle approx;
spinning CS BHs have scalar charge)

II. Calculate effective GW stress-energy.

[Stein & Yunes, PRD 83, 2011]

$$T_{ab}^{GW} \sim \left\langle h_{cd,(a} h^{cd}{}_{,b)} \right\rangle_{swa}$$

III. From the near-zone solution, construct the Hamiltonian.

[Yagi, Yunes, Tanaka, PRL 109 (2012)]

$$E = -\frac{\eta M^2}{r_{12}} \left[1 + 1PN + \ldots + 3PN + \mathcal{O}\left(\alpha^2 \frac{S^2}{m^2} \frac{m^2}{r_{12}^2} \right) \right]$$

IV. From the far-zone solution, construct the RR force (fluxes).

[Yagi, Stein, Yunes, Tanaka, PRD 85, 2012]

$$\dot{E} = -\frac{32}{5} \left(\frac{M}{r_{12}} \right)^5 \left[1 + 1PN + \ldots + 3.5PN + \mathcal{O}\left(\alpha^2 \frac{S^2}{m^2} \frac{m^2}{r_{12}^2} \right) \right]$$

V. From E and Edot, find the equations of motion with dissipation

[Yagi, Yunes, Tanaka, PRL 109 (2012)]

$$\dot{f} \sim f^{11/3} \left[1 + 1PN + \ldots + 3.5PN + \mathcal{O}\left(\alpha^2 \frac{S^2}{m^2} f^{4/3} \right) \right]$$

VI. Understand the propagation of metric perturbations.

[Sopuerta & Yunes, PRD 84, 2011]

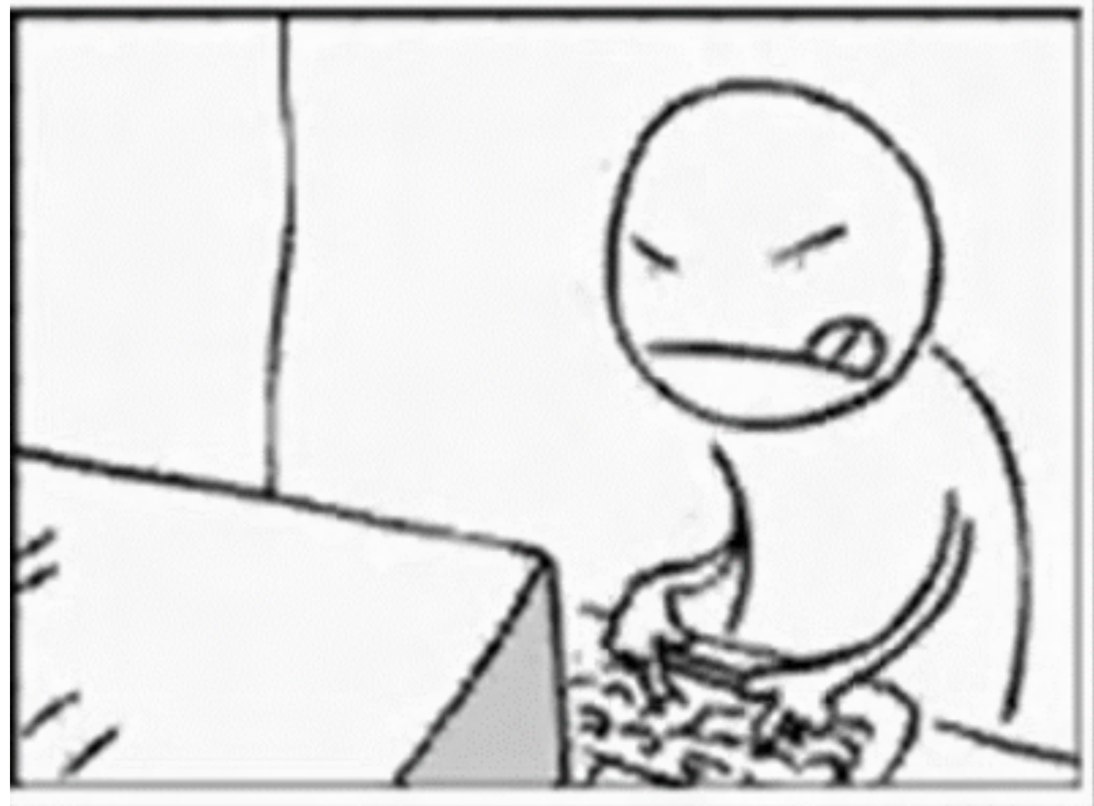
$$E_g^2 = p_g^2$$

VII. Construct the response function and Fourier transform it.

[Yagi, Yunes, Tanaka, PRL 109 (2012)]

$$\boxed{\tilde{h} \sim \mathcal{A} \frac{\mathcal{M}^{5/6}}{D_L} f^{-7/6} e^{i \left\{ \frac{3}{128} (\pi \mathcal{M} f)^{-5/3} \left[1 + 1PN + \ldots + 3.5PN + \mathcal{O}\left(\alpha^2 \frac{S^2}{M^2} f^{4/3} \right) \right] \right.}}$$

AAAAH
HHHH!!!!!



Main effects of dCS gravitational waves

I. Spinning BHs are not Kerr because they excite a scalar field

[Yunes & Pretorius, PRD 79 (2009), Yagi, Yunes & Tanaka PRD 86 (2012), Maselli et al, ApJ 843 (2017), McNees, Stein & Yunes, CQG 33 (2016)]

II. Binary BH spacetime has **two** scalar fields anchored with each black hole

[Yagi, Stein, Yunes & Tanaka, PRD 85 (2012), Yagi, Stein, Yunes & Tanaka PRD 87 (2013)]

IIa. Modified spacetime leads to modified Hamiltonian (and thus orbital energy)

IIb. Anchored scalar fields move with the black holes and emit scalar waves that remove energy from the system.

III. Dynamical Chern-Simons induces a **2PN** correction to the orbital evolution and GWs

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$1/r^2$

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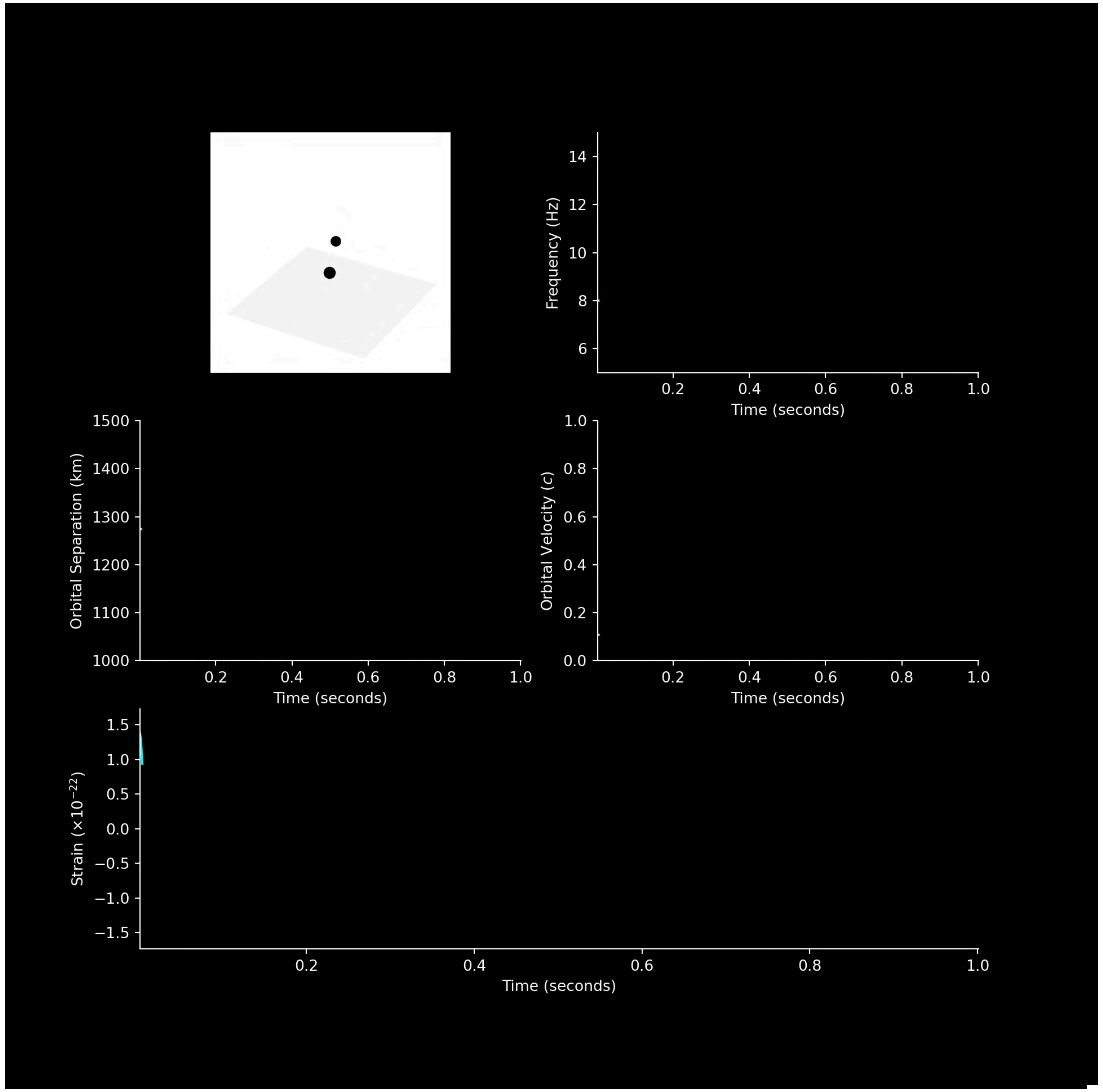
Gravitational wave signatures of dCS gravity



Gravitational waves can encode dynamical Chern-Simons signatures!

[Yagi, Yunes & Tanaka, *Phys.Rev.Lett.* 109 (2012) 251105,
Nair, Perkins, Silva & Yunes, *Phys.Rev.Lett.* 123 (2019) 19, 191101.]

Movies by S. Perkins:
(m_1,m_2) = (2.8,7.2) M_{Sun}
(c_1,c_2) = (0.9,-0.9)
z = 0.1

Conclusions

Dynamical Chern-Simons gravity cannot be constrained by Solar System or binary pulsars

Dynamical Chern-Simons gravity black holes are (linearly) stable but not Kerr

Dynamical Chern-Simons gravity compact binaries inspiral and ringdown faster than in GR

Gravitational waves from coalescing black holes may detect or constrain dynamical Chern-Simons gravity

But what's next?

Outlook: The data angle

Detectors with higher sensitivity (2022-2025)

Many more detections (stacking)

A few very loud detections (SNRs in low 100s)

Spin-precessing binaries constraining dCS gravity?

Quasi-normal ringdown tests?

GW polarization tests (LISA needed for full test)

New detectors (3G and LISA) (2035-2045)

New tests of GR

EMRIs in dCS?

Super-radiance in dCS?

Outlook: The theory angle

Theory Concerns

- Gravitational Collapse in dCS? Singularity theorems?
- Area theorem in dCS? Black hole thermodynamics?
- Exact rotating black hole solution? AdS black holes?

Modeling Concerns

- Quasinormal frequencies for moderate or large spin black holes?
- Spin-precessing inspiral model?
- Merger model? Will this matter?

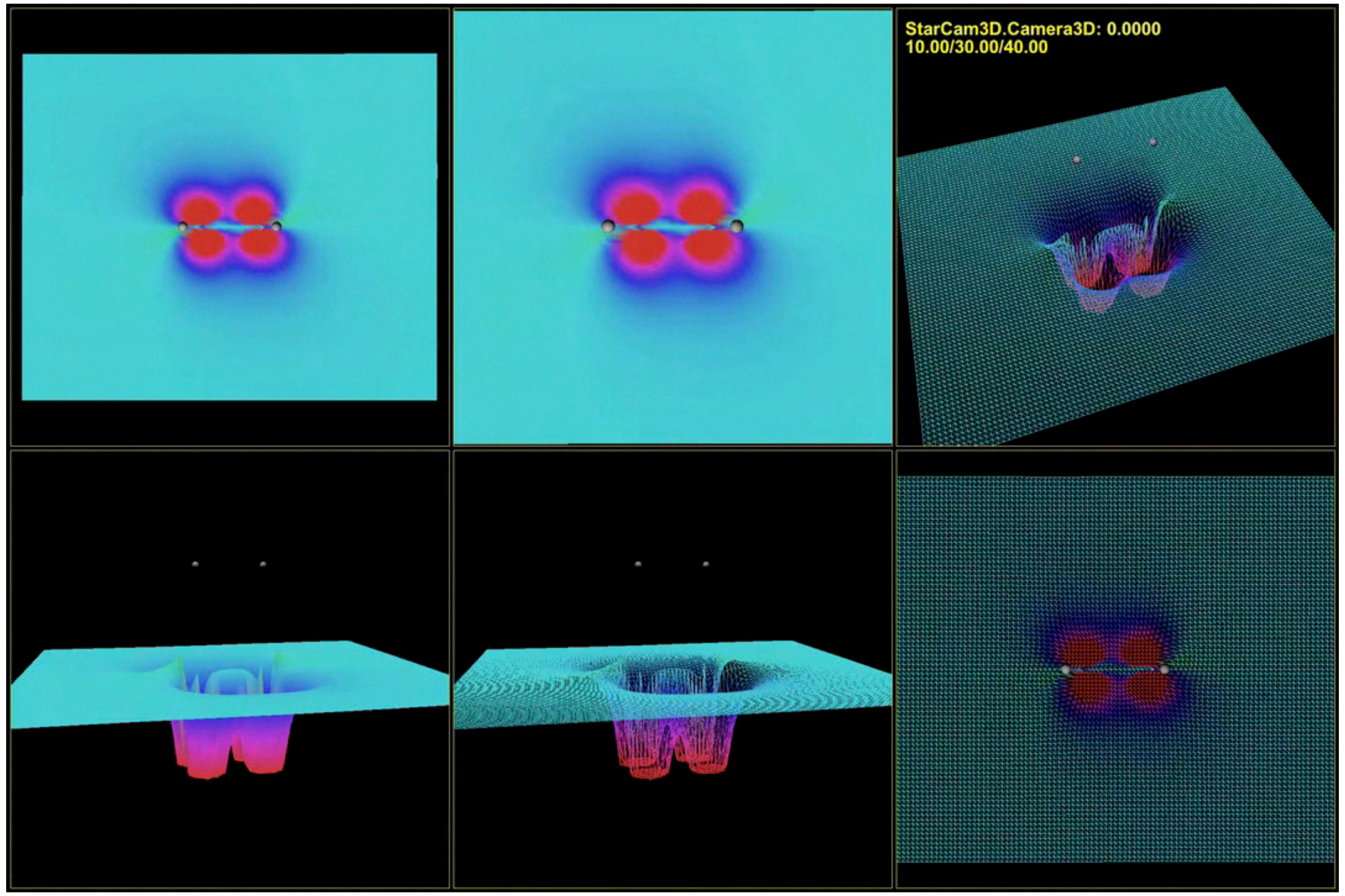
"Only put off until tomorrow what you are willing to die having left undone"

Pablo PicCATso

Thank You

What are gravitational waves and how are they generated?

[RIT Group]



eXtreme Gravity: where gravity is
(a) very strong,
(b) non-linear
(c) dynamical

Gravitational Waves (GWs): Wave-like perturbation of the grav. field.

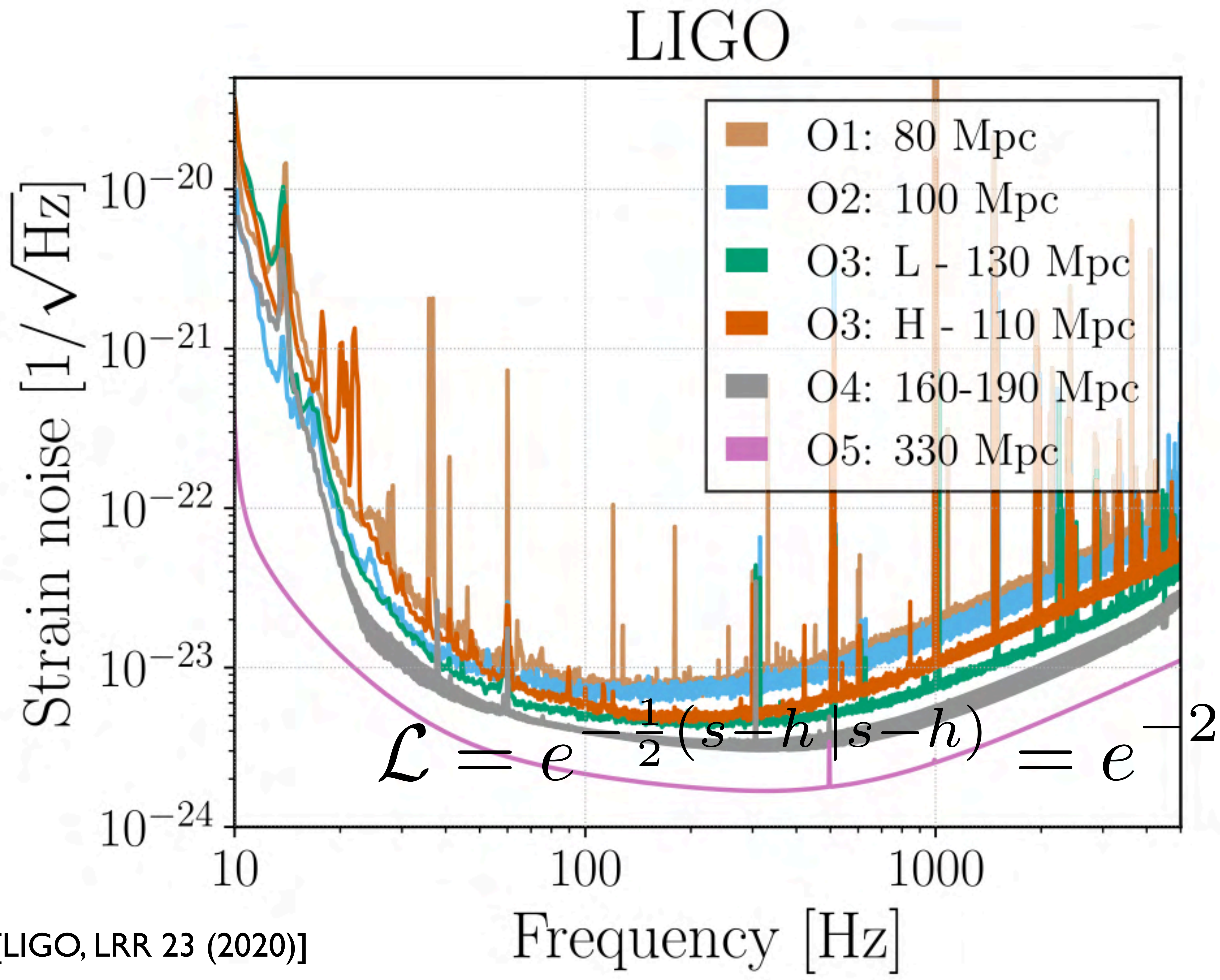
Generation of GWs: Accelerating masses (t-variation in multipoles)

Propagation of GWs: Light speed, weakly interacting, 1/R decay.

GW Spectrum: Kepler 3rd Law: $\frac{f}{2\pi} = \sqrt{\frac{m_{\rm tot}}{r_{12}^3}} \sim \frac{1}{m_{\rm tot}}$, $E_{\rm rad} \sim \%\, m_{\rm tot}$ in about 10^{79} gravitons

Example: Binary BH merger, $E_{\rm rad} \sim 10^{46}\, J \left(\frac{\epsilon}{1\%}\right)\left(\frac{M}{10 M_\odot}\right) \sim 10^2 E_{\rm SN}$

How do you extract information from gravitational waves?



[LIGO, LRR 23 (2020)]

Gravitational Wave Physics requires accurate models!

ıood function.

ı

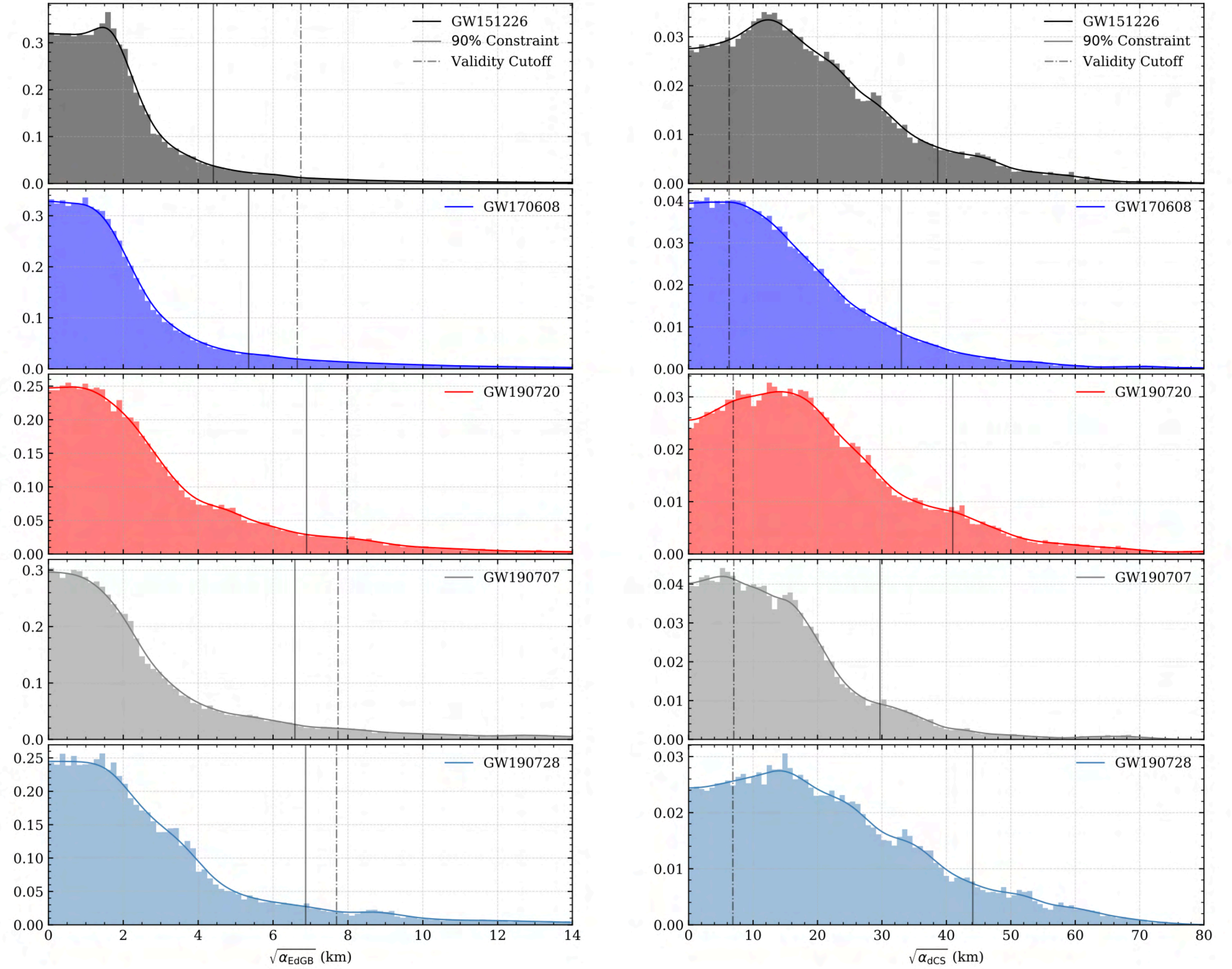
template param that characterize system

$$\mathcal{L} = e^{-\frac{1}{2}(s-h|s-h)} = e^{-2\int \frac{|\tilde{s}-\tilde{h}(f,\lambda^{\mu})|^2}{S_n(f)} df}$$

ise density)

template (projection of GW metric perturbation)

Gravitational Wave Constraints on scalar Gauss-Bonnet Gravity

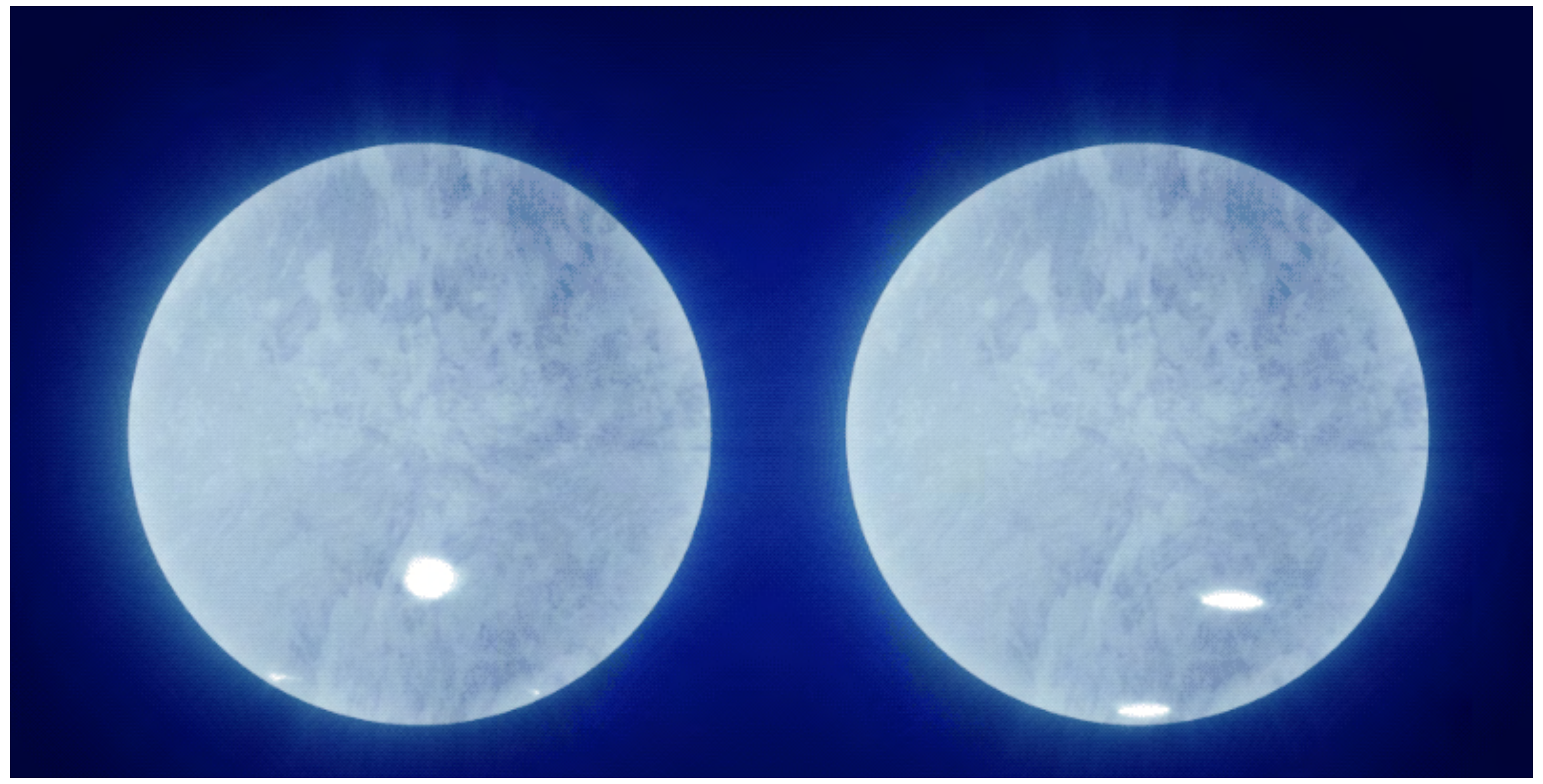


First LIGO constraints on quadratic gravity!

$$\sqrt{\alpha_{\rm EdGB}} \leq 1.7 \text{ km}$$

[Nair, Perkins, Silva & Yunes, *Phys.Rev.Lett.* 123 (2019) 19, 191101, Nair, Perkins, Silva & Yunes, *Phys.Rev.D* 104 (2021) 2, 024060]

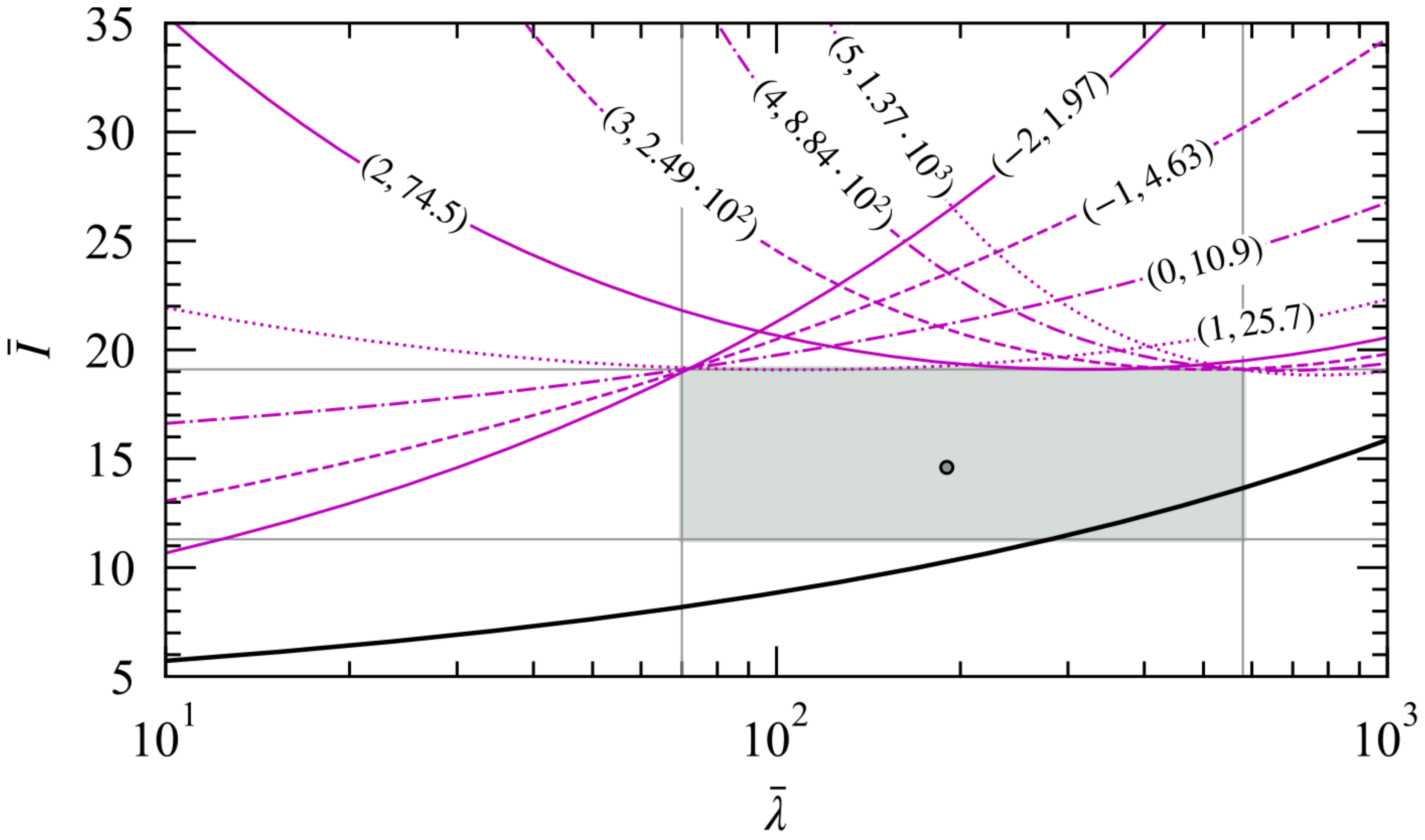
Multi-Messanger Constraints on dynamical Chern-Simons Gravity



[NICER + Miller, et al, Astrophys.J.Lett. 887 (2019)]

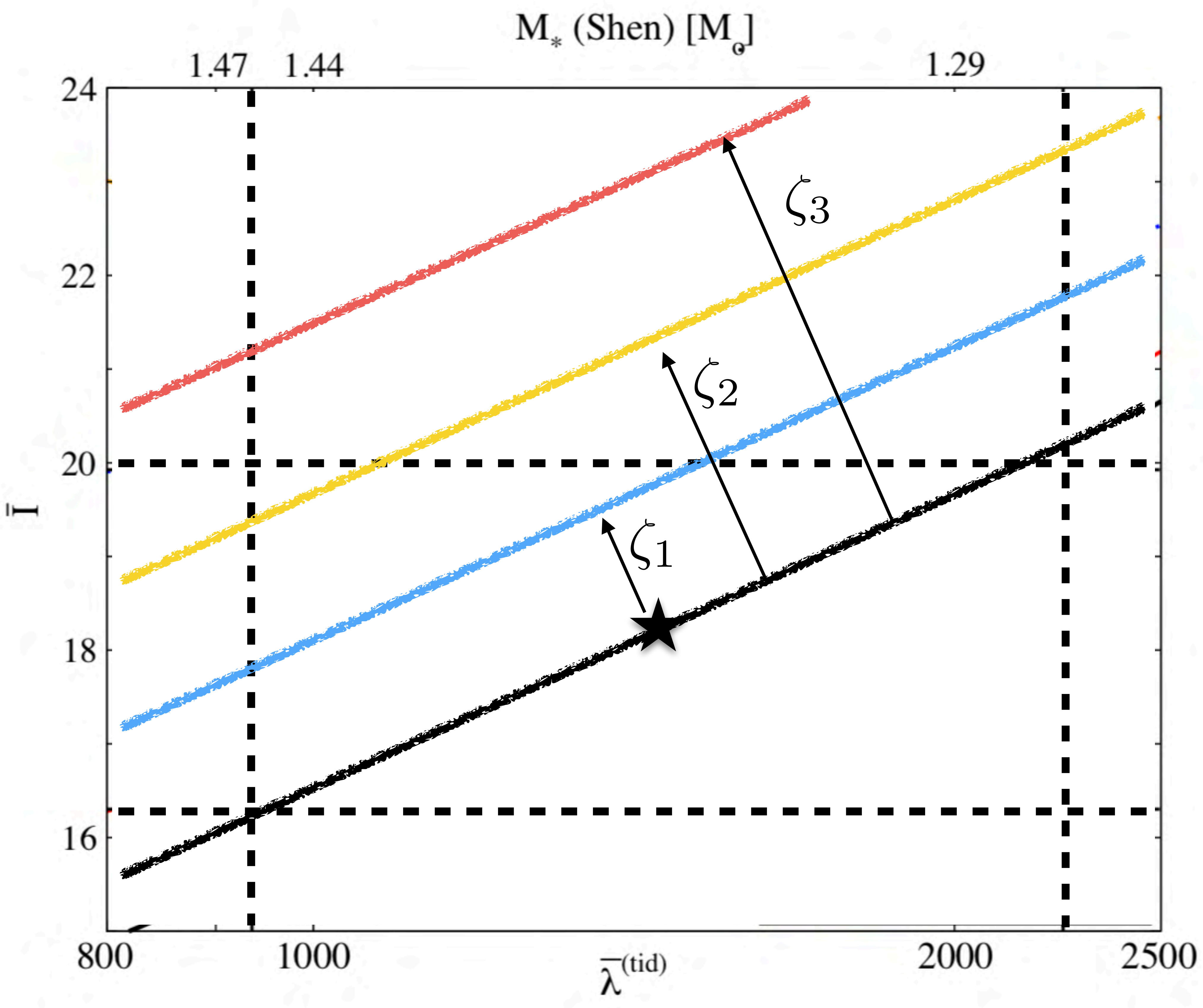
First Multi-Messenger constraints on dynamical Chern-Simons gravity!

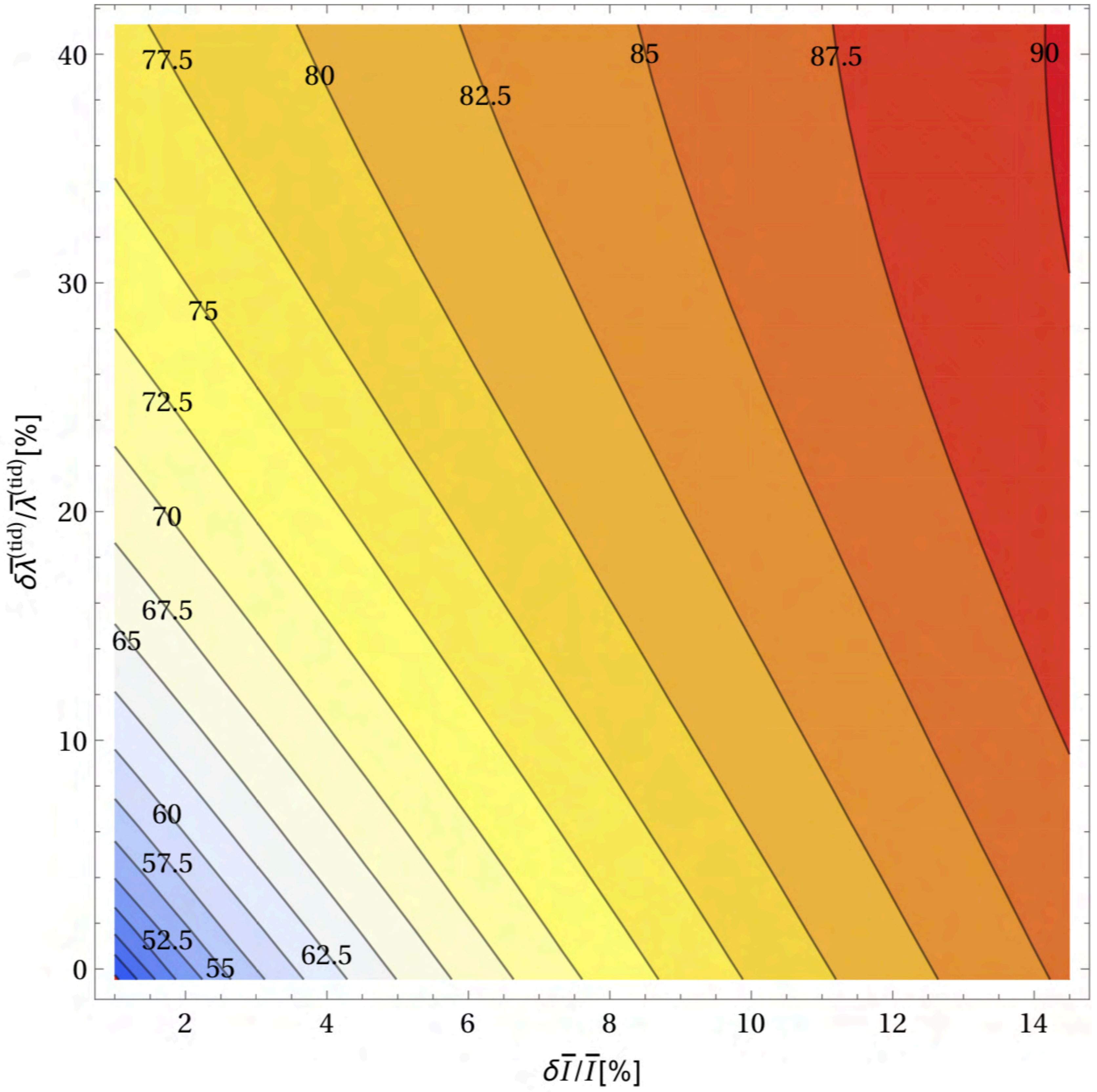
$$\sqrt{\alpha_{\rm dCS}} \leq 8.5 \text{ km}$$



[Yagi & Yunes, *Science* 341 (2013) 365-368.
Silva, Holgado, Cárdenas-Avendaño & Yunes, accepted in Phys. Rev. Letts (Editor's suggestion)]

Multi-Messanger (I-Love-Q) Tests of General Relativity





potential for constraints 10^6 - 10^8 times better than Solar System bounds!!

I-Love test could lead to strong tests of General Relativity

[Yagi & Yunes, *Science* 341 (2013), Gupta, Yagi & Yunes, CQG+ 35 ('17)]

Parameterized post-Einsteinian test

The parameterized post-Einsteinian Framework

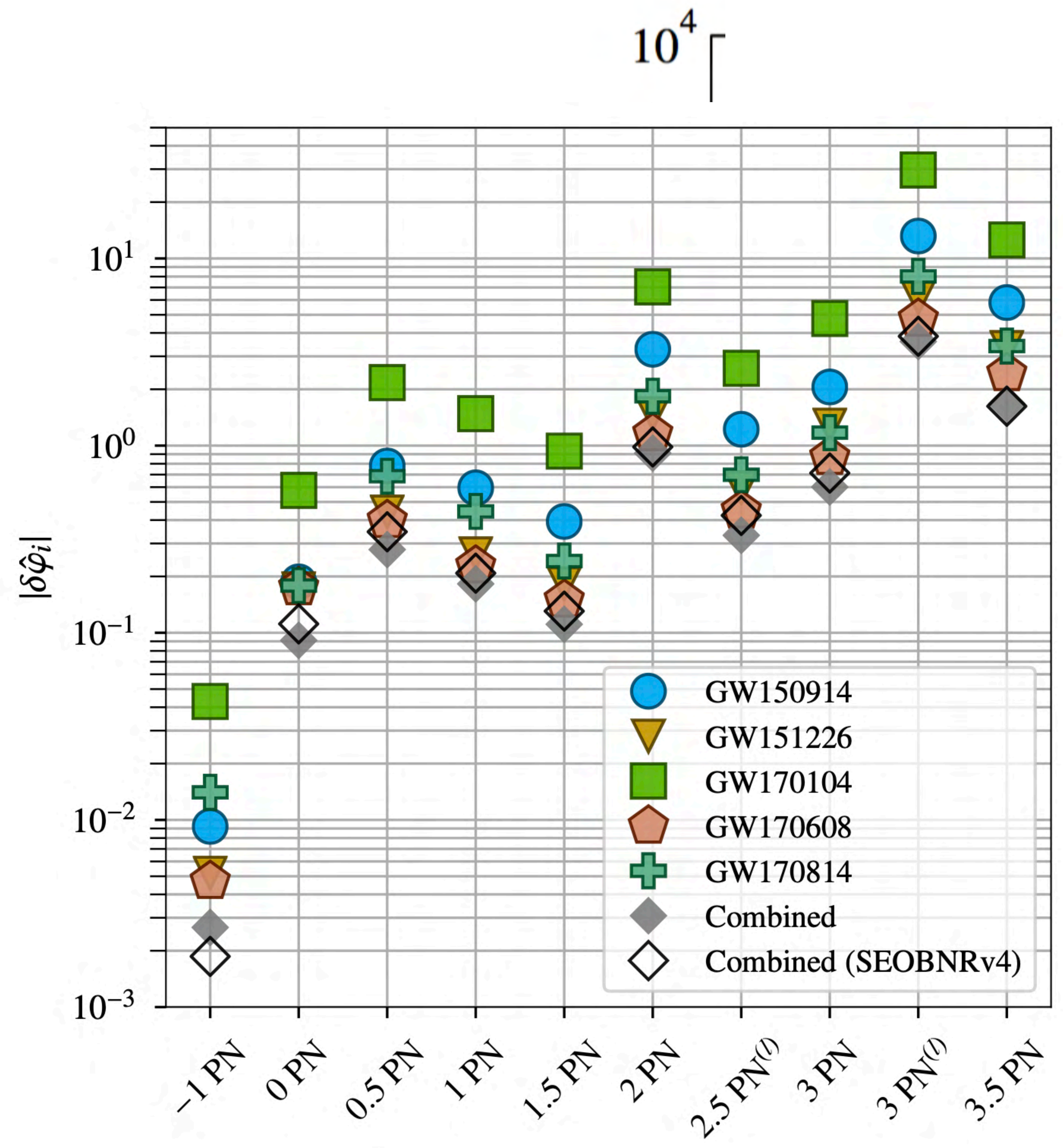
$$\tilde{h}(f) = \tilde{h}_{GR}(f)\,(1+\alpha f^{a})\, e^{i\beta f^{b}}$$

[Yunes & Pretorius, PRD 80 ('09)]

Theoretical Effect	Theoretical Mechanism	Theories	ppE b	Order	Mapping
Scalar Dipolar Radiation	Scalar Monopole Field Activation	EdGB [140, 142, 149, 150]	-7	-1PN	$\beta_{\rm EdGB}$ [140]
	BH Hair Growth	Scalar-Tensor Theories [59, 151]	-7	-1PN	$\beta_{\rm ST}$ [59, 151]
Anomalous Acceleration	Extra Dimension Mass Leakage	RS-II Braneworld [152, 153]	-13	-4PN	$\beta_{\rm ED}$ [141]
	Time-Variation of G	Phenomenological [137, 154]	-13	-4PN	$\beta_{\dot{G}}$ [137]
Scalar Quadrupolar Radiation Scalar Dipole Force Quadrupole Moment Deformation	Scalar Dipole Field Activation due to Gravitational Parity Violation	dCS [140, 155]	-1	$+2$PN	$\beta_{\rm dCS}$ [146]
Scalar/Vector Dipolar Radiation Modified Quadrupolar Radiation	Vector Field Activation due to Lorentz Violation	EA [109, 110], Khronometric [111, 112]	-7 -5	-1PN 0PN	$\beta_{\AE}^{(-1)}$ [113] $\beta_{\AE}^{(0)}$ [113]
Modified Dispersion Relation	GW Propagation/Kinematics	Massive Gravity [156–159]	-3	$+1$PN	$\beta_{\rm MDR}$ [145, 156]
		Double Special Relativity [160–163]	$+6$	$+5.5$PN	
		Extra Dim. [164], Horava-Lifshitz [165–167],	$+9$	$+7$PN	
		gravitational SME ($d=4$) [179]	$+3$	$+4$PN	
		gravitational SME ($d=5$) [179]	$+6$	$+5.5$PN	
		gravitational SME ($d=6$) [179]	$+9$	$+7$PN	
		Multifractional Spacetime [168–170]	3–6	4–5.5PN	

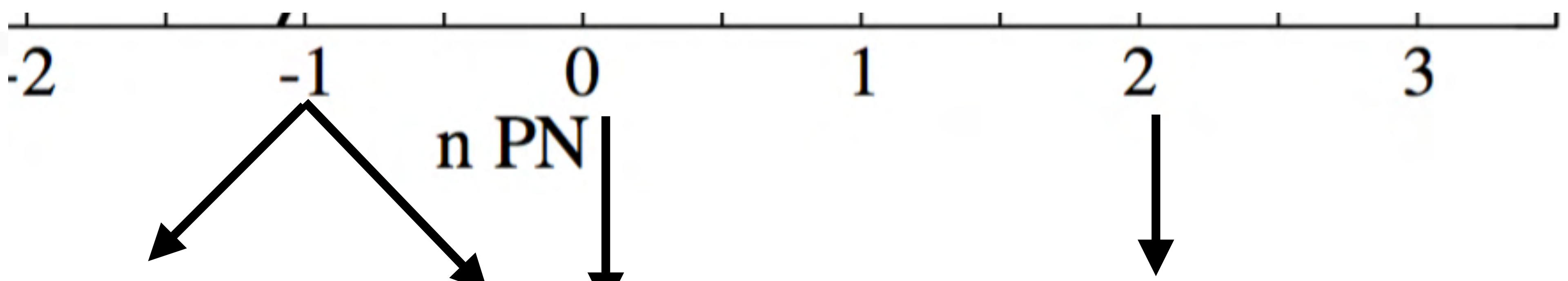
[MSU: Cornish et al PRD 84 ('11), Sampson et al PRD 87 ('13), Sampson, et al PRD 88 ('13), Sampson et al PRD 89 ('14), Nikhef: Del Pozzo et al PRD 83 ('11), Li et al PRD 85 ('12), Agathos et al PRD 89 ('14), Del Pozzo et al CQG ('14).]

ppE constraints



[LIGO, PRD 100 '(19)]

$$\tilde{h}(f) = \tilde{h}_{GR}(f)\,(1 + \alpha f^a)\, e^{i\beta f^b}$$



Anomalous Acceleration

Scalar Dipole Radiation

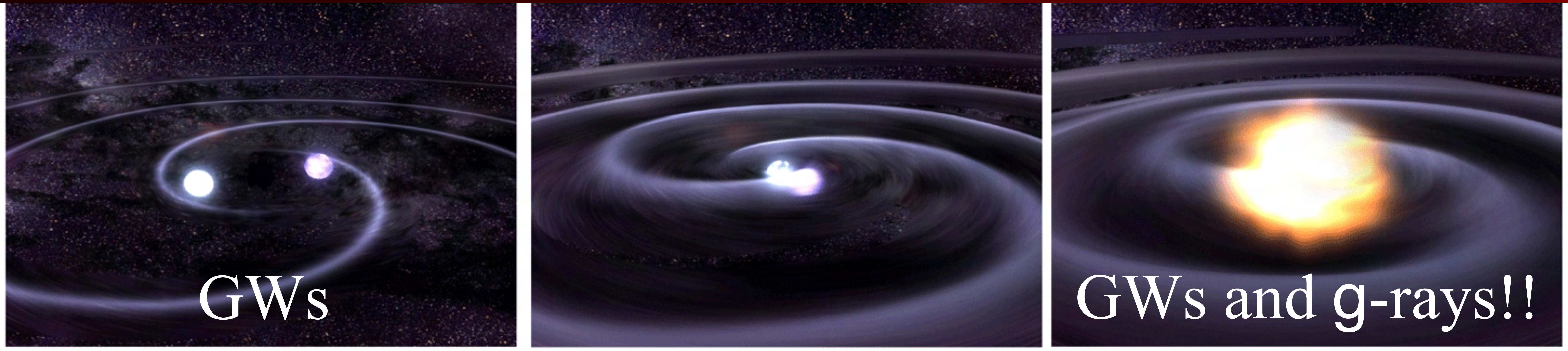
Lorentz Violation

Parity Violation

[Yunes, Yagi, Pretorius, PRD 94 '(16), Editor's suggestion]

And one more thing, the speed of gravity

[Nishizawa & Nakamura, PRD 90 ('14)]
[LIGO ApJ L 848 ('17)]



GW detection gives you distance D (~26Mpc) and thus, an arrival time $T_g = D/v_g$

Short GRB + galaxy identification (w/LIGO+Virgo) gives you distance D, so $T_\gamma = D/c + \tau_{\rm int}$

If $t_{\rm int} = 0$ ⟶ GW travelled faster than g to arrive 1.7 s before g ⟶ $\Delta t_{\rm obs} = \frac{D}{c} - \frac{D}{v_g}$ ⟶ $\frac{v_g}{c} \sim 1 + \frac{c\,\Delta t_{\rm obs}}{D}$

If $t_{\rm int} = 10$ secs ⟶ GW travelled slower than g to allow g to catch up to a 1.7 sec delay ⟶ $\Delta t_{\rm obs} = \frac{D}{v_g} - \frac{D}{c} + \tau_{\rm int}$ ⟶ $\frac{v_g}{c} \sim 1 - \frac{c\,(\tau_{\rm int} - \Delta t_{\rm obs})}{D}$

$$-3 \times 10^{-15} < \frac{v_g}{c} - 1 < 7 \times 10^{-16}$$

$$\ddot{h}_{ij} + (3 + \alpha_M) H \dot{h}_{ij} + (1 + \alpha_T) k^2 h_{ij} = 0\,,$$

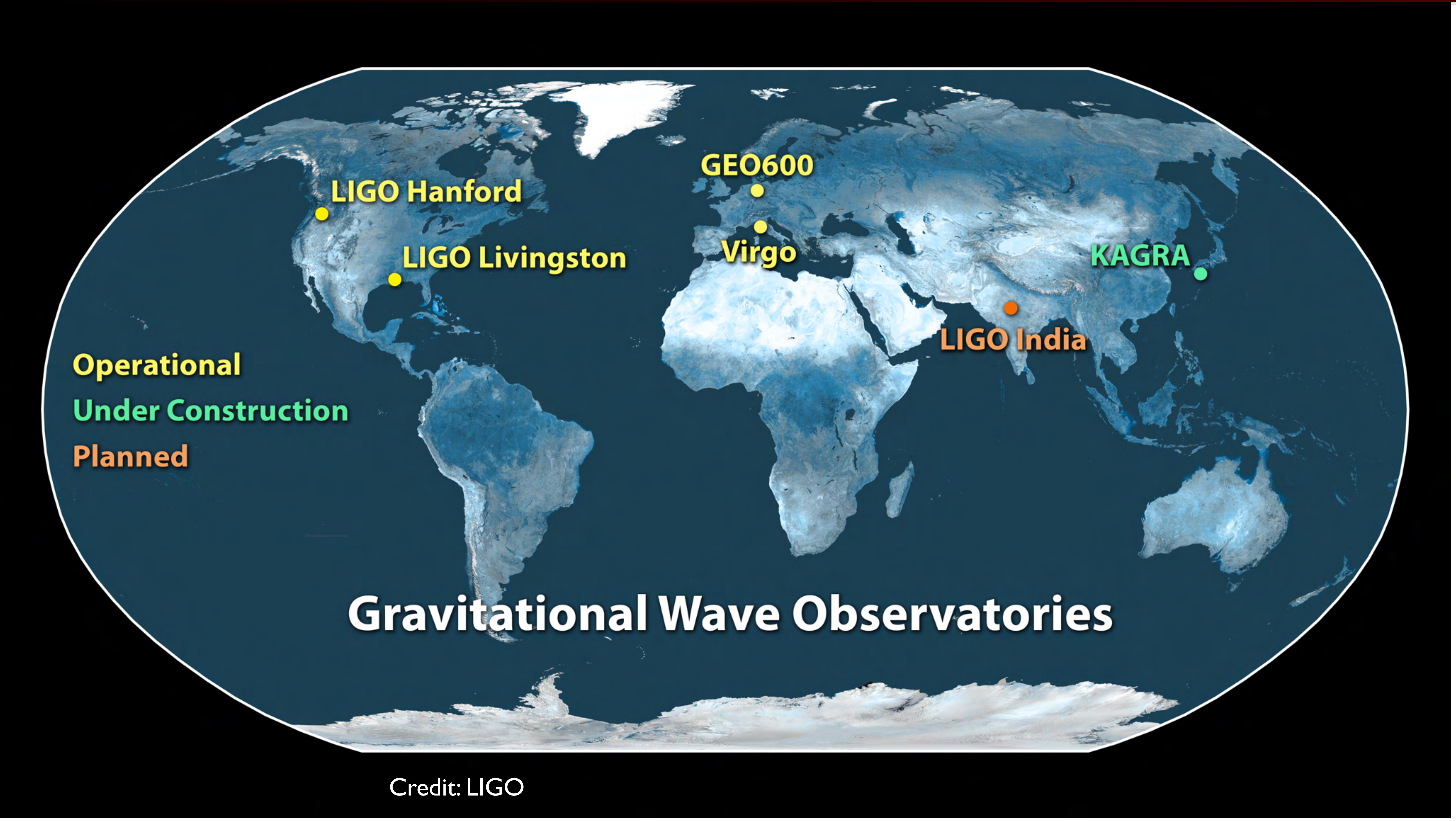
$$i\ddot{\phi} + (3 + \alpha_M) i H \dot{\phi} + \dot{\phi}^2 - (1 + \alpha_T) k_i k^i = 0,$$

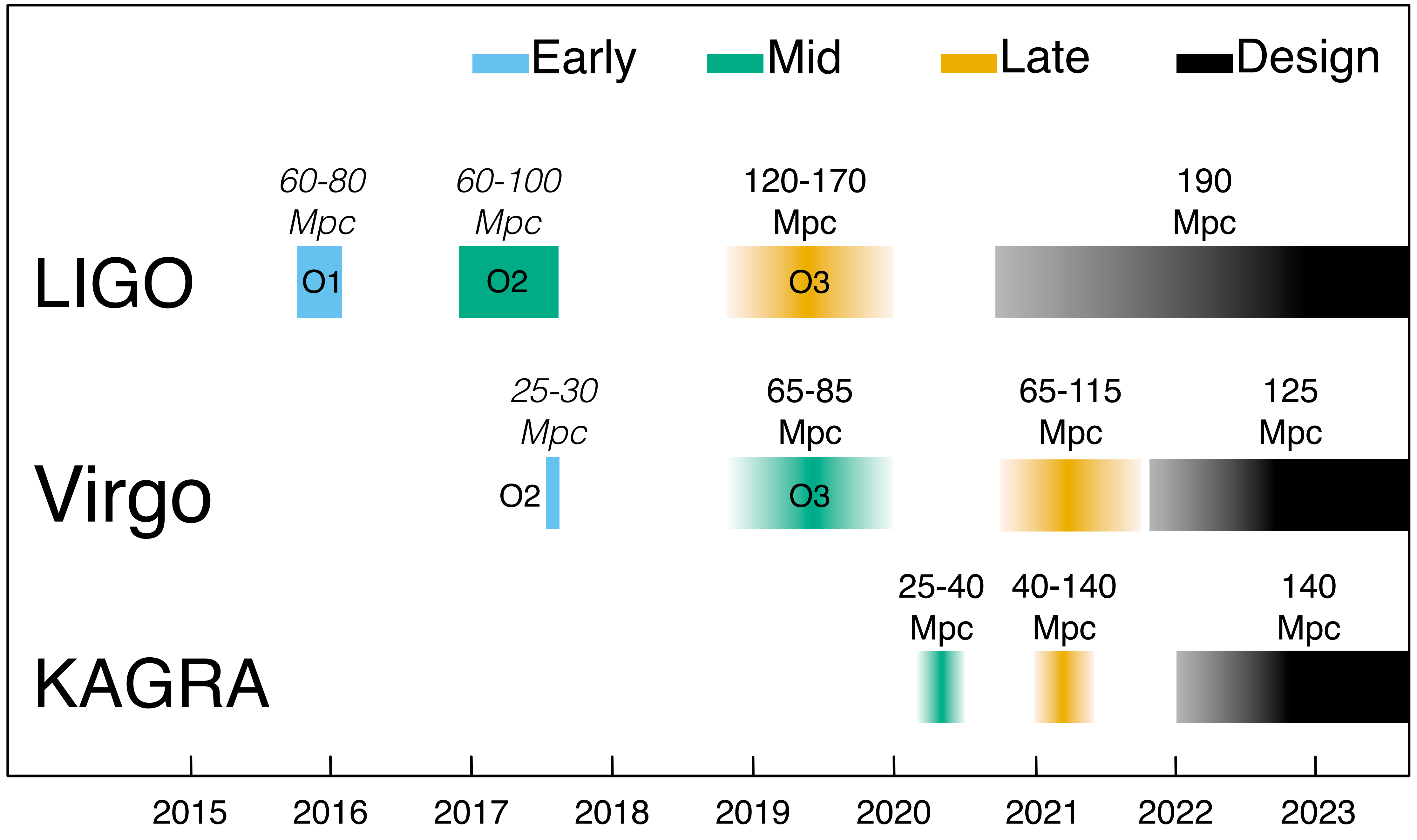
"Dead" Theories: Quartic/ Quintic Galileon, Fab Four, quadratic/cubic DHOST, etc.

Caveats: [de Rham & Melville PRL 121 ('18)] + [Alexander & Yunes, PRD 97 ('18)]

[Ezquiaga & Zumalacarregui PRL 119 ('17)]
[Baker, Bellini, Ferreira, Lagos, Noller, Sawicki, PRL 119 ('17)]

The future of 2G detectors





Abbott, B.P. et al. Living Rev Relativ ('18)

2021 2025 2029 2033 2036

aLIGO
aVirgo
KAGRA

A+

LIGO-India

Voyager

Cosmic
Explorer

Einstein
Telescope

LISA, DECIGO

improved quantum noise
improved thermal coating
increased range to 140% wrt aLIGO

silicon mirrors and suspensions
low temperature (120K)
increased range to 200% wrt aLIGO

Moderate Improvements

New physics

Black Holes outside General Relativity

$$G_{\mu\nu} + \alpha C_{\mu\nu}[g, \phi] = T^{(\phi)}_{\mu\nu} \qquad \Box\phi = \alpha S[g, \phi]$$

Analytic Solutions

Use perturbation theory (EFT) techniques to find closed-form, analytic solutions for vacuum, stationary, axisymmetric spacetimes

Small-coupling + small spin

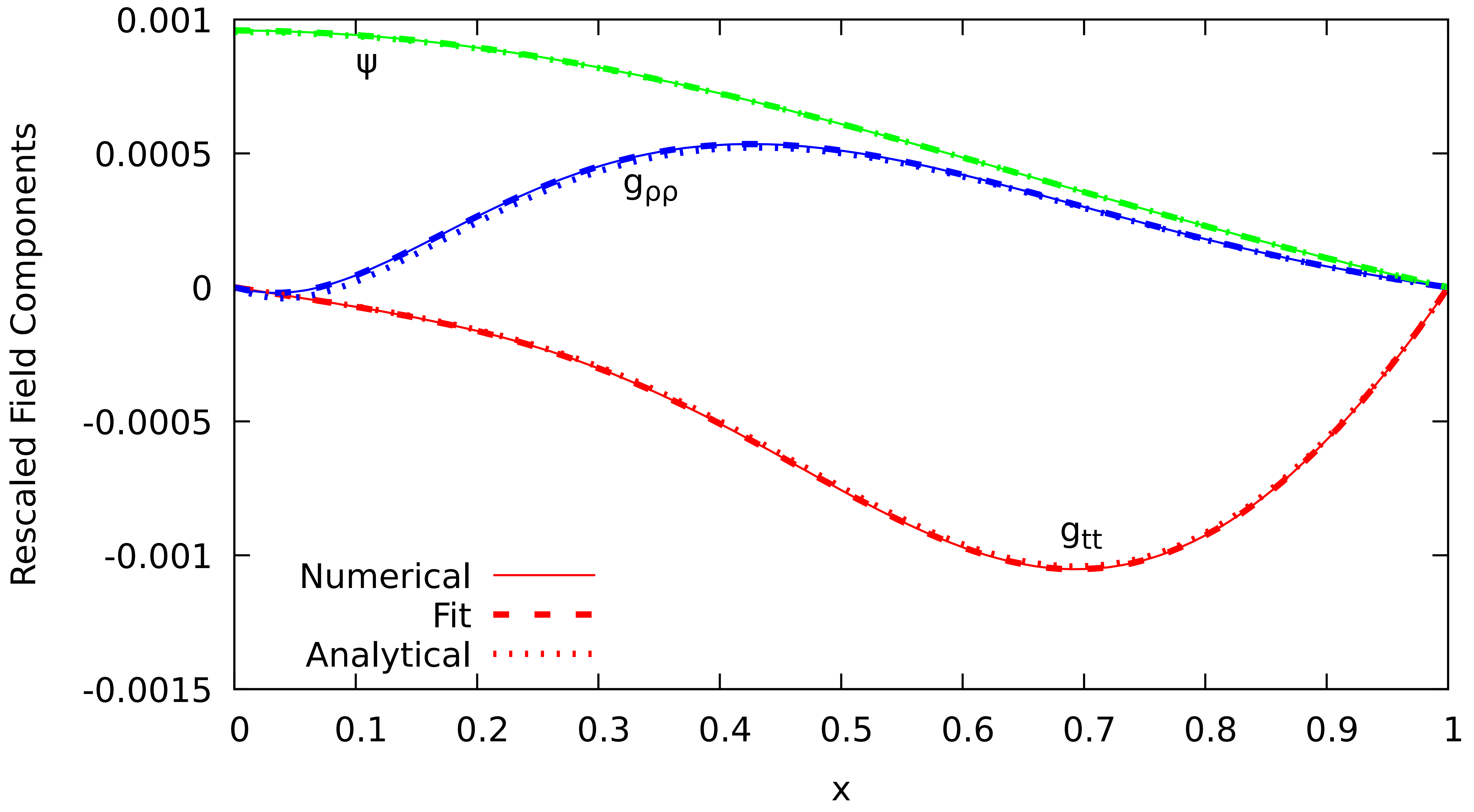
dCS: Yunes & Pretorius PRD 79 ('09), Yagi, Yunes & Tanaka PRD 86 ('12), …

EdGB: Yunes & Stein PRD 83 ('11), Ayzenberg & Yunes PRD 90 ('14), …

Small-coupling + extremal spin

dCS: Stein, Yunes & McNees CQG 33 ('16)

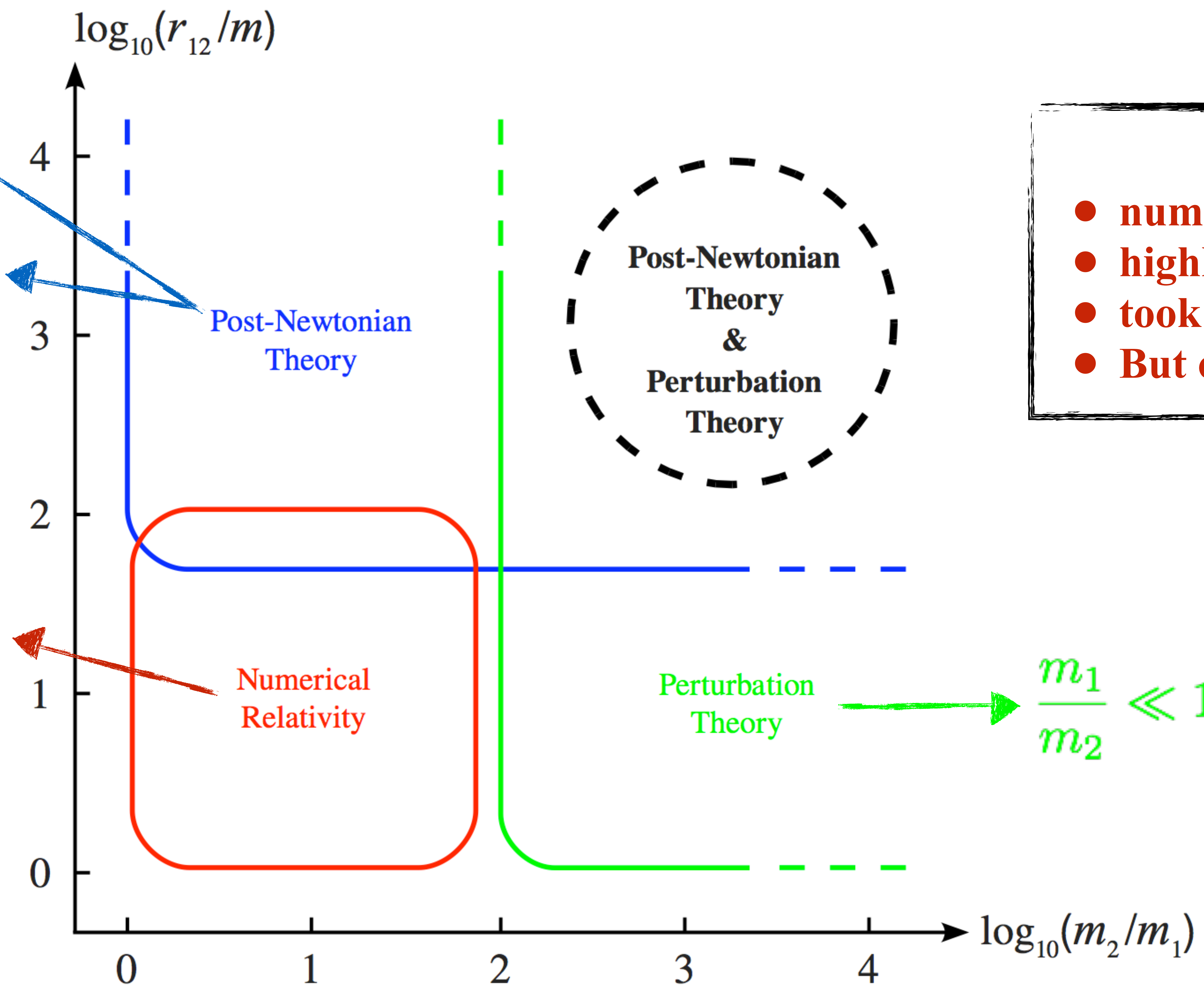
Numerical Solutions



EdGB: Sullivan, Yunes & Sotiriou, to appear soon

$$\frac{G m_{\rm tot}}{c^2 r_{12}} \ll 1$$

$$\frac{v}{c} \ll 1$$



$$\frac{m_1}{m_2} \ll 1$$

The GW models are

- **numerical and analytic**
- **highly accurate**
- **took over 50 years to develop**
- **But only for simple orbits**

[see e.g. Blanchet, Liv. Rev. in Rel.]

Mapping ppE constraints to theoretical physics

Theoretical Mechanism	GR Pillar	PN	$\lvert\beta\rvert$ GW150914	$\lvert\beta\rvert$ GW151226	Example Theory Constraints: Repr. Parameters	GW150914	GW151226	Current Bounds
Scalar Field Activation	SEP	-1	1.6×10^{-4}	4.4×10^{-5}	$\sqrt{\lvert\alpha_{\rm EdGB}\rvert}$ [km] $\lvert\dot\phi\rvert$ [1/sec]	— —	— —	10^7 [56], 2 [57–59] 10^{-6} [60]
Scalar Field Activation	SEP, PI	$+2$	1.3×10^{1}	4.1	$\sqrt{\lvert\alpha_{\rm dCS}\rvert}$ [km]	—	—	10^8 [61, 62]
Vector Field Activation	SEP, LI	0	7.2×10^{-3}	3.4×10^{-3}	(c_+, c_-) $(\beta_{\rm KG}, \lambda_{\rm KG})$	$(0.9, 2.1)$ $(0.42, -)$	$(0.8, 1.1)$ $(0.40, -)$	$(0.03, 0.003)$ [63, 64] $(0.005, 0.1)$ [63, 64]
Extra Dimensions	4D	-4	9.1×10^{-9}	9.1×10^{-11}	ℓ [μm]	5.4×10^{10}	2.0×10^{9}	10–10^3 [65–69]
Time-Varying G	SEP	-4	9.1×10^{-9}	9.1×10^{-11}	$\lvert\dot G\rvert$ [10^{-12}/yr]	5.4×10^{18}	1.7×10^{17}	0.1–1 [70–74]
Massive graviton	$m_g = 0$	$+1$	1.3×10^{-1}	8.9×10^{-2}	m_g [eV]	10^{-22} [19]	10^{-22} [5]	10^{-29}–10^{-18} [75–79]
Mod. Disp. Rel. (*Multifractional*)	LI	$+4.75$	1.1×10^{2}	2.6×10^{2}	E_*^{-1} [eV^{-1}] (time) E_*^{-1} [eV^{-1}] (space)	5.8×10^{-27} 1.0×10^{-26}	3.3×10^{-26} 5.7×10^{-26}	— 3.9×10^{-53} [80]
Mod. Disp. Rel. (*Modified Special Rel.*)	LI	$+5.5$	1.4×10^{2}	4.3×10^{2}	$\eta_{\rm dsrt}/L_{\rm Pl} > 0$ $\eta_{\rm dsrt}/L_{\rm Pl} < 0$	1.3×10^{22}	3.8×10^{22}	— 2.1×10^{-7} [80]
Mod. Disp. Rel. (*Extra Dim.*)	4D	$+7$	5.3×10^{2}	2.4×10^{3}	$\alpha_{\rm edt}/L_{\rm Pl}^2 > 0$ $\alpha_{\rm edt}/L_{\rm Pl}^2 < 0$	5.5×10^{62}	2.5×10^{63}	2.7×10^{2} [80] —
Mod. Disp. Rel. (*Standard Model Ext.*)	LI	$+4$	—	—	$\mathring{k}^{(4)}_{(I)} > 0$ $\mathring{k}^{(4)}_{(I)} < 0$	— 0.64	— 19	6.1×10^{-17} [80, 81] —
		$+5.5$	1.4×10^{2}	4.3×10^{2}	$\mathring{k}^{(5)}_{(V)} > 0$ [cm] $\mathring{k}^{(5)}_{(V)} < 0$ [cm]	1.7×10^{-12} [82]	3.1×10^{-11}	1.7×10^{-40} [80, 81] —
		$+7$	5.3×10^{2}	2.4×10^{3}	$\mathring{k}^{(6)}_{(I)} > 0$ [cm^2] $\mathring{k}^{(6)}_{(I)} < 0$ [cm^2]	7.2×10^{-4}	3.3×10^{-3}	3.5×10^{-64} [80, 81] —
Mod. Disp. Rel. (*Hořava-Lifshitz*)	LI	$+7$	5.3×10^{2}	2.4×10^{3}	$\kappa_{\rm hl}^4\mu_{\rm hl}^2$ [1/eV^2]	1.5×10^{6}	6.9×10^{6}	—
Mod. Disp. Rel. (*Lorentz Violation*)	LI	$+4$	—	—	c_+	0.7 [83]	0.998	0.03 [63, 64]

[Yunes, Yagi, Pretorius, PRD 94 '(16), Editor's suggestion]

What are the sources of gravitational waves?