Better Representation via Multimember Districts with a Supporting Election Method?

Duane Cooper
Morehouse College

SLMath Connections Workshop: Algorithms, Fairness, and Equity

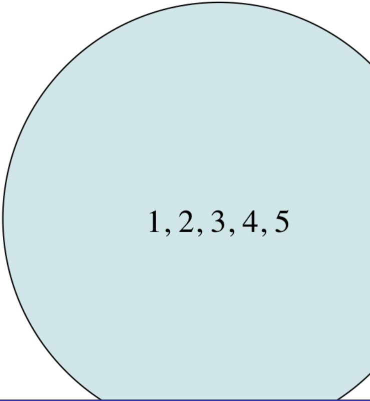
24 August 2023

Overview

1. **Minute 1:** Multimember Districts and Cumulative Voting
2. **Minute 2:** Quantifying Fairness to Groups
3. **Minute 3:** Quantifying Fairness to Individuals

An Example

- Population 500, 5 member council
- 400 White, 100 Black
- Racially polarized
- Multimember electoral district



Generalized Plurality Voting vs. Cumulative Voting

Walls	400	+
Wells	400	+
Will	400	+
Wont	400	+
Wye	400	+
Brown	100	

Generalized Plurality Voting vs. Cumulative Voting

Walls	400	+
Wells	400	+
Will	400	+
Wont	400	+
Wye	400	+
Brown	100	

Brown	100 x 5 = 500
Walls	
Wells	
Will	400 x 5 = 2000
Wont	
Wye	

Quantifying Fairness to Groups

- Fair division of representative bodies: Apportionment
- Webster's Method (Sainte-Laguë, "major fractions"): minimizes the absolute difference between groups in per capita representation
- Jefferson's Method (d'Hondt, "greatest divisors"): used in U.S. till 1840s; slightly favors larger states/larger populations

Cumulative Voting: Not Quite "Webster-fair"

Theorem 1

Consider a population of size P. Consider a minority fraction of the population, $\frac{x}{P}$, chosen from the uniform distribution on $(0, \frac{1}{2}) \cap \mathbb{Q}$, and suppose the remaining $\frac{P-x}{P}$ constitute the population's majority. Then, in an election for n representatives of the population under cumulative voting, the probability that the minority is **unable** *to elect its Webster-fair share of the n seats is*

$$\frac{1}{4} \cdot \left(1 - \frac{1}{1 + 2\lfloor \frac{n}{2} \rfloor}\right)$$

Moreover, if the minority's Webster-fair share is $k_w \geq 1$, then it has the voting strength to elect either k_w or $k_w - 1$ representatives.

Cumulative Voting: "Jefferson-fair"

Theorem 2

Consider a population of size P partitioned into two subgroups of size x and $P - x$, with a representative body of n seats to be determined. The number of seats each group can be assured under cumulative voting is equivalent to the number of seats each group would be assigned by Jefferson's method of apportionment.

Quantifying Fairness to Individuals

Never mind!

My three minutes are expiring, I'm sure!