D-BRANES, SUPERSYMMETRY, AND MIRROR SYMMETRY

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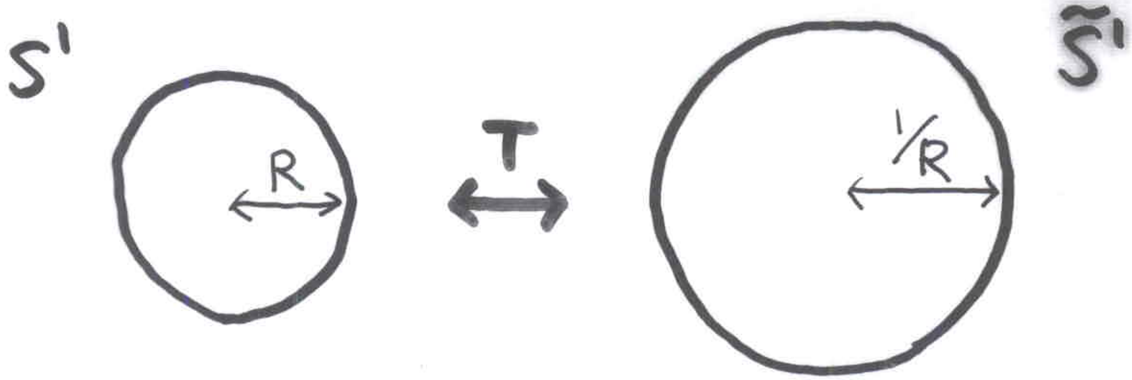
D-branes & T-duality

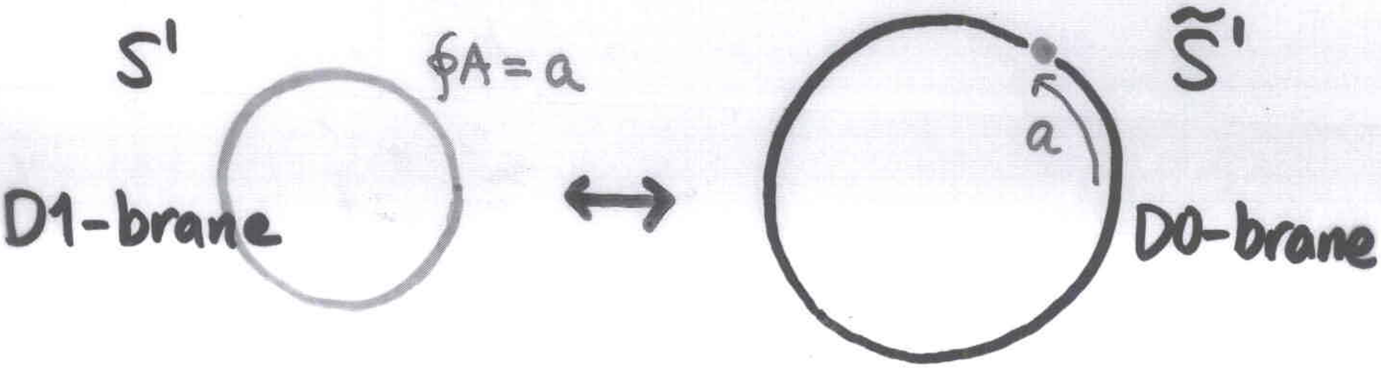


$$*dX = d\tilde{X}$$

Σ : World sheet with boundary

Neumann B.C. Dirichlet B.C.

$$*dX|_{\partial\Sigma} = 0 \leftrightarrow d\tilde{X}|_{\partial\Sigma} = 0$$



Wilson line $\leftrightarrow$ Position

Dai-Leigh-Polchinski
Horava

Q: How are D-branes transfored under Mirror Symmetry?

We restrict our attention to D-branes that preserve a $\frac{1}{2}$ of the supersymmetry of the bulk of the world sheet

↳ (2,2) supersymmetry
$Q_\pm, \overline{Q}_\pm$ $\qquad \{Q_\pm, \overline{Q}_\pm\} = H \pm P$

(mirror Symmetry : $Q_- \leftrightarrow \overline{Q}_-$)

two kinds of "$\frac{1}{2}$" : Ooguri Oz Yin

$$\left.\begin{aligned} Q_A &= \overline{Q}_+ + Q_- \\ Q_A^\dagger &= Q_+ + \overline{Q}_- \end{aligned}\right\} \text{A-branes}$$

$\updownarrow$ mirror

$$\left.\begin{aligned} Q_B &= \overline{Q}_+ + \overline{Q}_- \\ Q_B^\dagger &= Q_+ + Q_- \end{aligned}\right\} \text{B-branes}$$

X Kähler manifold $\begin{cases} \omega & \text{symplectic structure} \\ J & \text{complex structure} \end{cases}$

[LG model
$W : X \to \mathbb{C}$ superpotential]

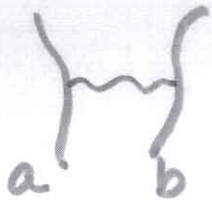
A D-brane wrapped on $\gamma \subset X$
supporting a $U(1)$ gauge potential A

is

<u>an A-brane</u> if $\gamma \subset (X, \omega)$ **Lagrangian**
A : **flat** $(F_A = 0)$
[$\mathrm{Im}\, W =$ constant on γ]

<u>a B-brane</u> if $\gamma \subset (X, J)$ **complex submanifold**
A : **holomorphic** $(F_A^{0,2} = 0)$
[$W =$ constant on γ]

$\mathbf{a} = (\gamma_a, A_a)$, $\mathbf{b} = (\gamma_b, A_b)$ both A-branes (or both B-branes)



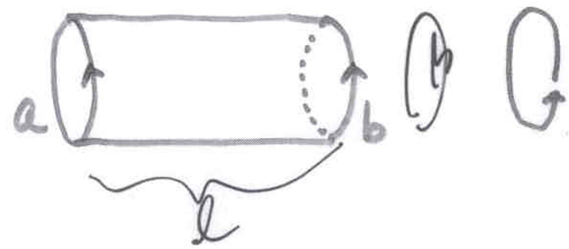
open string

Hilbert space $\mathcal{H}_{a,b}$

$\{ Q, Q^\dagger \} = 2H \qquad (Q = Q_A \text{ (or } Q_B))$

Witten index $\mathbf{I(a,b)} = \mathrm{Tr}_{\mathcal{H}_{a,b(\ell)}} (-1)^F e^{-\beta H(\ell)}$

=



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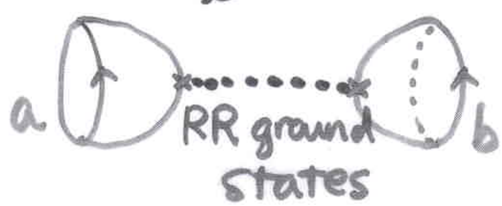
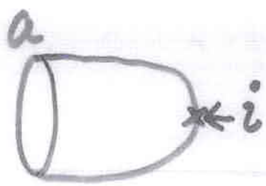
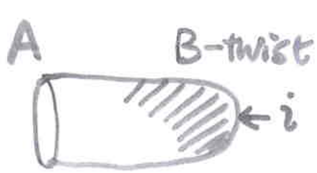


$\Pi_i^a =$

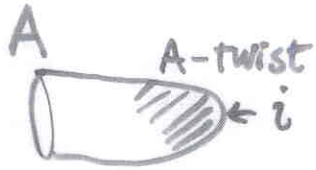


.... <u>RR charge</u> of the brane

($\propto$ mass if BPS in space-time)



- Independent of Kähler class
- $D_i \Pi_j^a = \underbrace{C_{ij}{}^k}_{\text{chiral ring}} \Pi_k^a$ for cplx deformation



.... topological disc amplitudes

★ $L \subset X$ Lagrangian

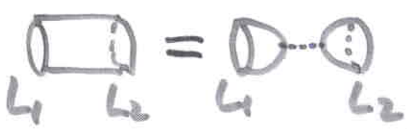
$$I(L_1, L_2) = \#(L_1 \cap L_2)$$

$X = CY$: $\Pi_i^L =$  $\leftarrow \omega_i = \pm i^* \int_L \omega_i$ Period

$\tilde{\Pi}_i^L = \omega_i \rightarrow$  $= \pm i^* \int_L \omega_i$

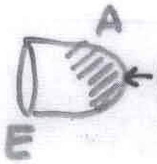
$$\eta_{ij} = \int_X \omega_i \wedge \omega_j$$

inverse

 : $\#(L_1 \cap L_2) = \int_{L_1} \omega_i \, \eta^{ij} \int_{L_2} \omega_j$

★ $E \subset X$ holomorphic bundle

$$I(E_1, E_2) = \chi(E_1, E_2) = \int_X ch(E_1^\vee) ch(E_2) \, Td(X)$$

$\Pi_i^E =$  $\leftarrow \omega_i = \int_X e^{B + i\omega} \omega_i \, ch(E^\vee) \sqrt{Td(X)} + \cdots$

$\tilde{\Pi}_i^E = \omega_i \rightarrow$  $= \int_X e^{-B - i\omega} \omega_i \, ch(E) \sqrt{Td(X)} + \cdots$

 :

$$\chi(E_1, E_2) = \int_X e^{B + i\omega} \omega_i \, ch(E_1^\vee) \sqrt{Td(X)} \; \eta^{ij} \int_X e^{-B - i\omega} \omega_j \, ch(E_2) \sqrt{Td(X)}$$

MIRROR SYMMETRY

non-linear σ-model on X^n_{toric} $\longleftrightarrow$ Landau-Ginzburg model $W : (\mathbb{C}^\times)^n \to \mathbb{C}$

e.g. $X = \mathbb{C}P^{N-1}$ $\longleftrightarrow$ $W = e^{-Y_1} + \dots + e^{-Y_{N-1}} + e^{-t + Y_1 + \dots + Y_{N-1}}$

We will study the maps

B-branes in X $\longleftrightarrow$ A-branes in LG

holomorphic bundles — Lagrangians with $\mathrm{Im}\,W = \mathrm{const}$

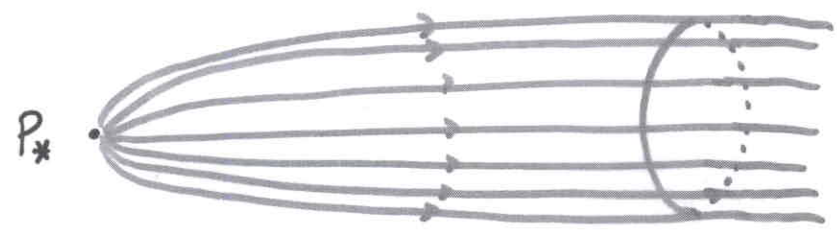
A-branes in X $\longleftrightarrow$ B-branes in LG

Lagrangian submflds — cplx submflds with $W = \mathrm{const}$

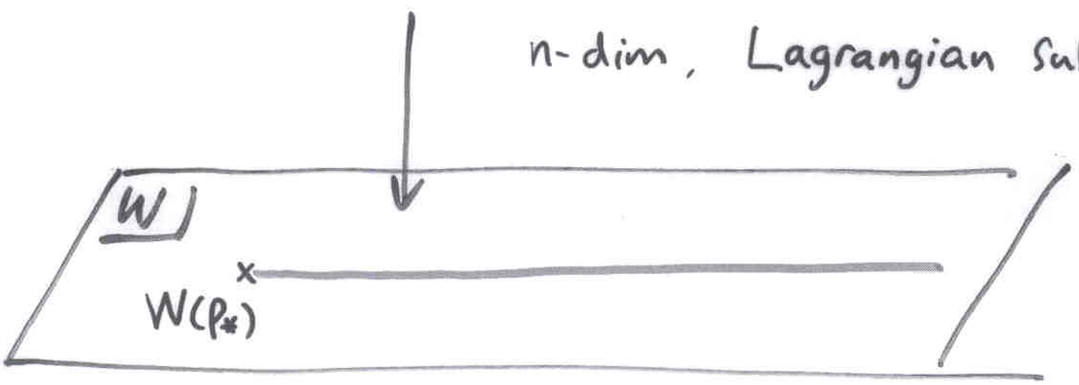
A-branes in LG $W : Y^n \to \mathbb{C}$

p_* a non-degenerate critical point of W.

Gradient flow lines of $\underline{\mathrm{Re}(W)}$ starting from p_*

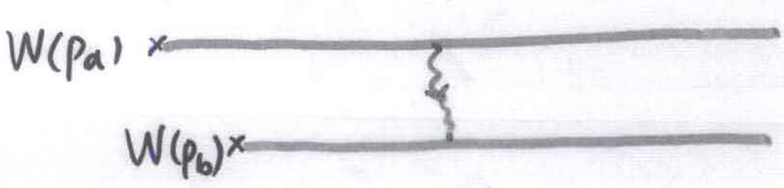


n-dim, Lagrangian submfd γ_{p_*}



$\therefore$ A-brane

two such branes

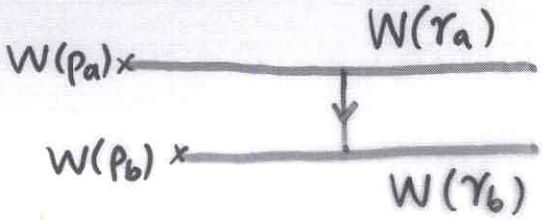


Open string quantum mechanics

$$Q = Q_A = \int \overline{\psi}_+ (\partial_t \phi + \partial_\sigma \phi + i\overline{W}') + \psi_- (\partial_t \overline{\phi} - \partial_\sigma \overline{\phi} + iW')$$

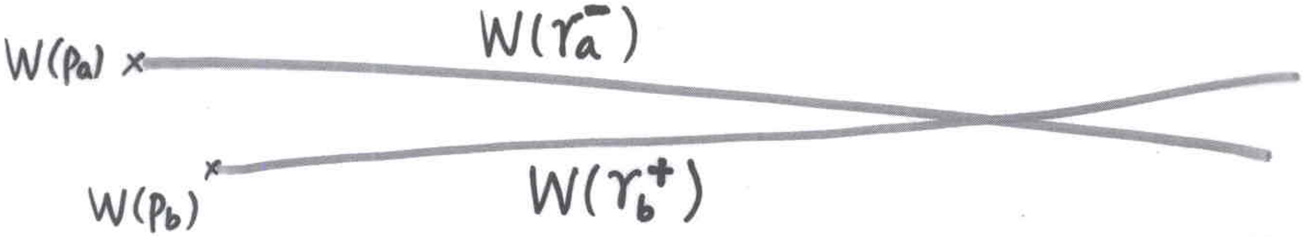
$$Q = Q^\dagger = 0 \quad \Rightarrow \quad \begin{aligned} \partial_t \phi &= 0 \\ \partial_\sigma \phi &= -\,\mathrm{grad}\ \underline{\mathrm{Im}(W)} \end{aligned}$$

SUSY ground states : Grad flows of $-\mathrm{Im}(W)$



$\left(\nexists \text{ if } \mathrm{Im}\, W(p_a) < \mathrm{Im}\, W(p_b) \right)$

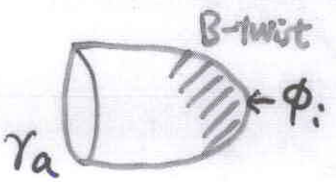
How many ?



- $\boxed{I(a,b) = \#(\gamma_a^- \cap \gamma_b^+)}$ = #(BPS solitons in a-b sector)

- $\mathcal{H}_{SUSY} = HF^{\bullet}_{W}(\gamma_a, \gamma_b)$ dimension $|I(a,b)|$

Overlap with RR ground states

$\Pi_i^a =$  $= \int_{\gamma_a^-} e^{-iW} \phi_i \Omega$

$\widetilde{\Pi}_{\bar{i}}^a =$  $= \int_{\gamma_a^+} e^{-i\overline{W}} * \phi_{\bar{i}} \overline{\Omega}$

$I(a,b) = \Pi_i^a \, g^{i\bar{j}} \, \widetilde{\Pi}_{\bar{j}}^b$: Riemann's bilinear identity

B-branes in X $\leftrightarrow$ A-branes in LG

$$X = \mathbb{CP}^{N-1} \quad \leftrightarrow \quad W = e^{-Y_1} + \cdots + e^{-Y_{N-1}} + e^{-t+Y_1+\cdots+Y_{N-1}}$$

$\dim H^\bullet(X) = N$ $\qquad$ N : crit pts $e^{-Y_1} = \cdots = e^{-Y_{N-1}} = e^{-t/N} e^{2\pi i l/N}$

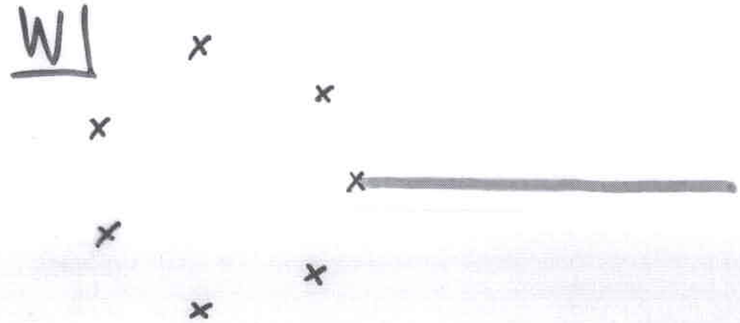
$l = 0, 1, \cdots, N-1$

set $B = 0$ $\qquad$ $\theta = 0 \quad (t \in \mathbb{R}_+)$

$D(2(N-1))$ brane wrapped on X

trivial gauge field

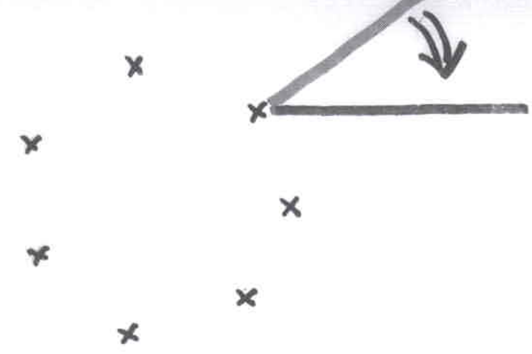
$\cdots$ pure Neumann B.C. $\leftrightarrow$ $Y_1, \cdots, Y_{N-1}$ all real



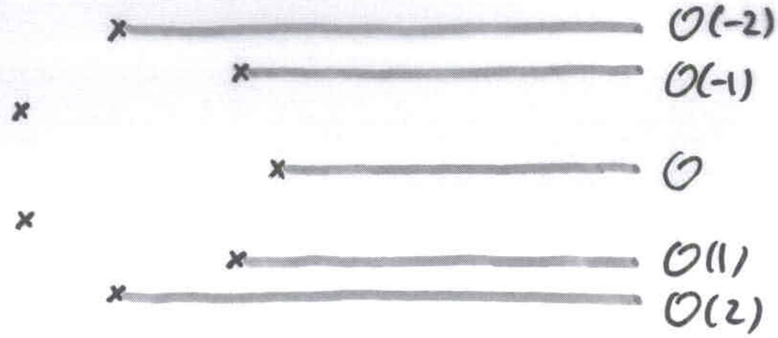
axial R-rotation

$$B \to B + N r \quad \leftrightarrow \quad \theta \to \theta + N r, \quad W \to e^{ir} W$$

$\left[\begin{array}{l} B = 2\pi, \text{ D-brane on } X \\ \quad \text{trivial gauge field} \\ \equiv B = 0, \text{ D-brane on } X \\ \quad \text{supporting } \mathcal{O}(-1) \end{array} \right] \leftrightarrow$



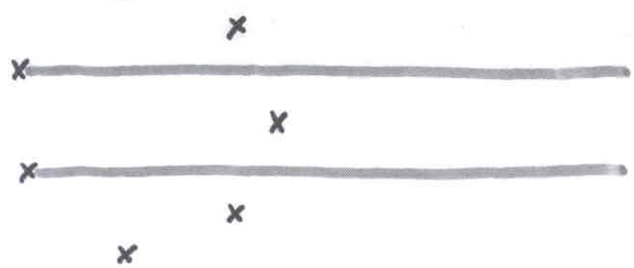
preserves $\bar{Q}_+ + e^{\frac{2\pi i}{N}} \bar{Q}_-$



indeed

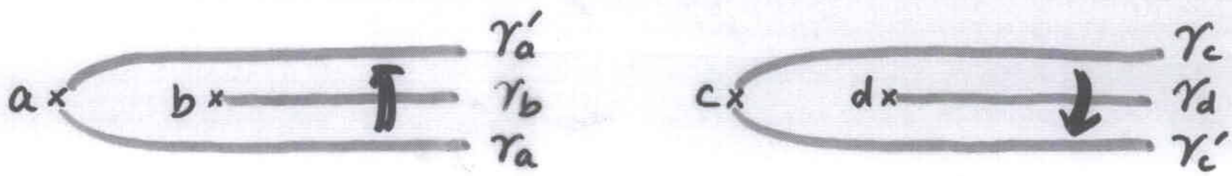
$$\chi(\mathcal{O}(i),\mathcal{O}(j)) = \delta_{i<j}\binom{N+j-i-1}{j-i} = I(\gamma_i,\gamma_j)$$

- What about $\mathcal{O}(\pm 3)$, $\mathcal{O}(\pm 4)$, ... ?
- What are



mirror to ?

<u>Picard-Lefshetz</u> :



$$\gamma_a \to \gamma_a' + \gamma_b I(b,a) \qquad \gamma_c \to \gamma_c' + \gamma_d I(c,d)$$

$$\updownarrow \qquad\qquad \updownarrow$$

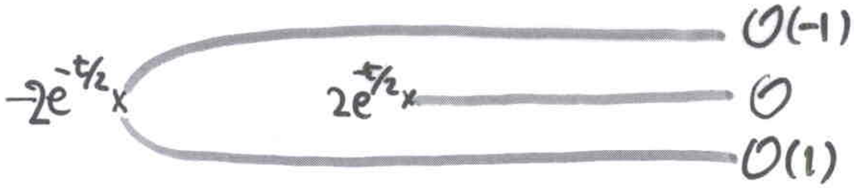
$$E_a \to E_a' + E_b\chi(E_b,E_a) \qquad E_c \to E_c' + E_d\chi(E_c,E_d)$$

$$\uparrow \qquad\qquad \uparrow$$

$$\pm L_{E_b}E_a \qquad \underline{\text{Mutation}} \qquad \pm R_{E_d}E_c$$

e.g. $X = \mathbb{CP}^1 \longleftrightarrow W = e^{-Y} + e^{-t+Y}$

crit. pts: $e^{-Y} = \pm e^{-t/2}$



$$0 \to \mathcal{O}(-1) \to \mathrm{Ext}^0(\mathcal{O},\mathcal{O}(1)) \otimes \mathcal{O} \to \mathcal{O}(1) \to 0$$

$$(f,g) \longmapsto fX_0 + gX_1$$

$$\mathcal{O}(-1) = L_{\mathcal{O}}\mathcal{O}(1) \qquad \sigma \longmapsto (X_1\sigma, -X_0\sigma)$$

$$0 \to \mathcal{O}(-1) \to \mathrm{Ext}^0(\mathcal{O}(-1),\mathcal{O})^* \otimes \mathcal{O} \to \mathcal{O}(1) \to 0$$

$$\mathcal{O}(1) = R_{\mathcal{O}}\mathcal{O}(-1)$$

$$\begin{tikzcd} \cdots \mathcal{O}(-2) & \mathcal{O}(-1) \arrow[rr, bend right, "R"'] & \mathcal{O} & \mathcal{O}(1) \arrow[ll, bend right, "L"'] & \mathcal{O}(2) \cdots \end{tikzcd}$$

<u>Helix of Period 2</u>

$X = \mathbb{CP}^{N-1}$

$$\begin{tikzcd} \cdots \mathcal{O} \arrow[rr, bend right] & \mathcal{O}(1) \cdots & \mathcal{O}(N) \arrow[ll, bend right] \cdots \end{tikzcd}$$

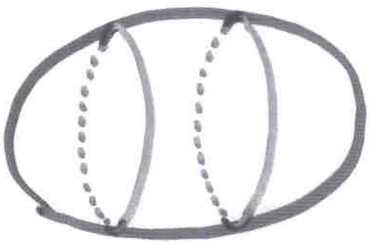
Helix of Period N

A-branes in X $\longleftrightarrow$ B-branes in LG, $W : Y \to \mathbb{C}$

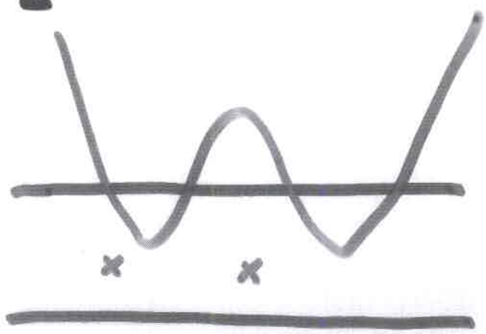
$L \subset X$ Lag $\qquad$ $Z \subset Y$ cplx

$\qquad\qquad\qquad\quad$ $W|_Z = \text{const}$

①



$\longleftrightarrow$



torus fibers $\qquad\qquad$ points

$\sigma : X \to X$ $\longleftrightarrow$ $\hat{\sigma} : Y \to Y$ holo. inv.

anti-holo. involution $\qquad$ s.t. $W(\hat{\sigma} y) = -W(y)$

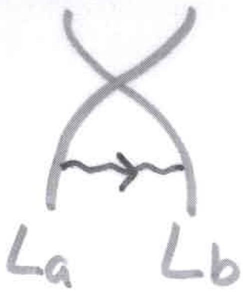
$\left[\begin{array}{c} \exists \text{ Parity anomaly} \\ \text{if } w_2(X) \neq 0 \end{array}\right]$ I. Brunner & KH $\qquad$ [$\nexists$ such map]

② $\{(x, \sigma x)\} \in X \times X$ $\longleftrightarrow$ $\{(y, \hat{\sigma} y)\} \in Y \times Y$

extra B-field if $w_2(X) \neq 0$ $\qquad$ $W = W(y) + W_B(\hat{\sigma} y) = 0$

③ $X^{\sigma} \subset X$ $\longleftrightarrow$ $Y^{\hat{\sigma}} \subset Y$

A-branes in X … Lagrangian Submfds of (X, ω)



$$\mathcal{H}_{SUSY} = Q = Q_A\text{-“cohomology”}$$

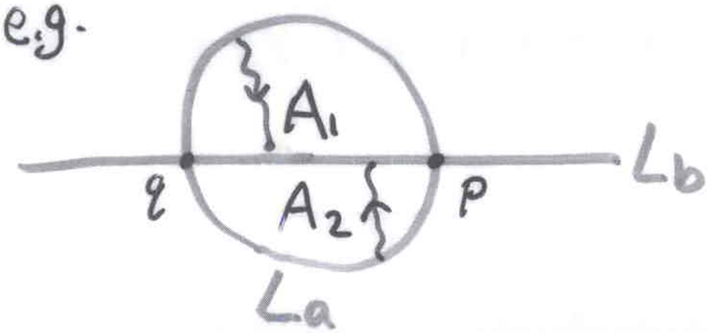
$$= HF(L_a, L_b) \text{ Floer “homology”}$$

Fukaya-Oh-Ohta-Ono:

$Q^2 = 0$ not-always true,

“$Q^2 = 0$ anomaly”

e.g.



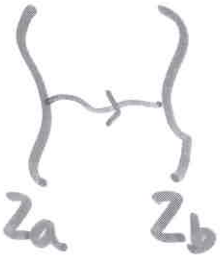
$$\left.\begin{aligned} Qq &= e^{-A_1} p \\ Qp &= e^{-A_2} q \end{aligned}\right\} \quad \underline{Q^2 q = e^{-(A_1+A_2)} q \neq 0 \,!}$$

$\Longleftrightarrow$ Bubbing off of a Holomorphic Disc.

<u>$q \to q$ family</u>

B-branes in LG $\quad W : Y \to \mathbb{C}$

$$Q = Q_B = \frac{1}{2\pi}\int_0^{\pi} dx^1 \left\{ (\bar\psi_- + \bar\psi_+)\,\partial_0\phi - (\bar\psi_- - \bar\psi_+)\,\partial_1\phi + \frac{i}{2}(\psi_- - \psi_+)W' \right\}$$



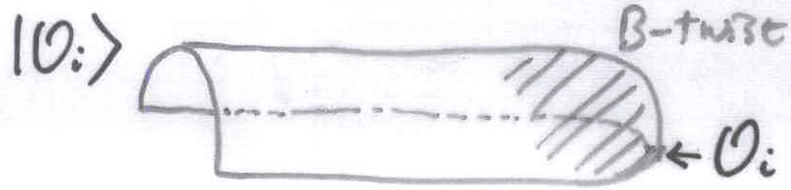
$$Q^2 = \frac{1}{2\pi i}\left(W|_{z_b} - W|_{z_a} \right)$$

$$Q^2 = 0 \quad \text{iff} \quad W|_{z_b} = W|_{z_a}$$

This is the mirror statement of FOOO's "$Q^2 = 0$ anomaly"

$$z_a = z_b =: z \Rightarrow Q^2 = 0$$

What is the $\mathcal{H}_{SUSY}$?



B-twist $\quad \eta^{\bar i} = -(\bar\psi_-^{\bar i} + \bar\psi_+^{\bar i}),\ \theta_i = g_{i\bar j}(\bar\psi_-^{\bar j} - \bar\psi_+^{\bar j}),\ \rho_z^i = \psi_-^i \quad \rho_{\bar z}^i = \psi_+^i$

$\delta = \bar\epsilon Q$

$$\boxed{\begin{aligned} &\delta\phi^i = 0 & &\delta\bar\phi^{\bar i} = \bar\epsilon\eta^{\bar i} \\ &\delta\theta^i = \bar\epsilon\,\partial_i W & &\delta\eta^{\bar i} = 0 \\ &\delta\rho_\mu^i = -2\bar\epsilon J^\nu{}_\mu \partial_\nu\phi^i & & \end{aligned}}$$

fixed pt : Constant map to Crit(W)

Boundary Condition

$$\left.\begin{array}{ll} \phi \in Z & \\ \eta^{\bar{i}} : \text{tan to } Z & \theta_{i} : \text{normal to } Z \\ \rho_n^{i} : \text{tan to } Z & \rho_t^{i} : \text{normal to } Z \end{array}\right\} \text{ on } \partial\Sigma$$

$$\eta \text{ in } \overline{T^*_Z} \qquad \theta \text{ in } N_{Z/X} = T_X / T_Z$$

$$Q = \bar{\partial} + \partial W \cdot \qquad ; \quad \partial W \cdot : \Lambda^q N \to \Lambda^{q-1} N \quad \text{contraction}$$

$$\underline{\mathcal{H}_{SUSY} = H\left(\Omega^{0,\cdot}(Z, \Lambda^{\cdot} N_{Z/X}),\ \bar{\partial} + \partial W \cdot\right)}$$

Examples

(i) $Z \cap \mathrm{Crit}(W) = \phi$

$$\mathcal{H}_{SUSY} = 0 \qquad I(Z, Z) = 0$$

(ii) $Z = \{p_*\}$ a critical pt of W

$$\mathcal{H}_{SUSY} = \Lambda^{\cdot} N_{Z/X} = \Lambda^{\cdot} \mathbb{C}^n \cong \mathbb{C}^{2^n}$$

Powers of $\theta_1 \cdots \theta_n$

$$\mathcal{H}^B_{SUSY} = \Lambda^{even} \mathbb{C}^n, \quad \mathcal{H}^F_{SUSY} = \Lambda^{odd} \mathbb{C}^n$$

$$I(p_*, p_*) = 0$$

$$Y = \mathbb{C}^2 = \{(U,V)\}$$

(iii) $W = UV$ crit pt at $U=V=0$

$$Z_0 = \{U=V=0\} \quad \text{done} \checkmark$$

$$Z_2 = \{V=0\} = \{U\}$$

$$Z_2' = \{U=0\} = \{V\}$$

$\underline{Z = Z_2}$ $\quad Q = \bar{\partial} + U\,dV\cdot$

$$f = e^{-|U|^2}\left(1 - d\bar{U} \otimes \frac{\partial}{\partial V}\right) \quad \text{solves } Q = Q^\dagger = 0$$

$$\mathcal{H}_{SUSY} \cong \mathbb{C}$$

$$I(Z_2, Z_2) = 1$$

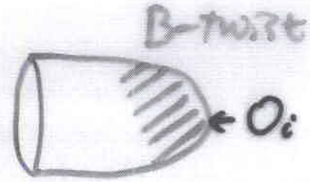
Explicit quantization of other pairs $\Rightarrow$

$$\mathcal{H}_{SUSY}^{Z_0 - Z_2} = \underset{\uparrow B}{\mathbb{C}} \oplus \underset{\uparrow F}{\mathbb{C}} \qquad I(Z_0, Z_2) = 0$$

$$\mathcal{H}_{SUSY}^{Z_2 - Z_2'} = \mathbb{C} \qquad I(Z_2, Z_2') = \pm 1$$

(iv) General Z

$$I(Z,Z) = \begin{cases} 0 & \dim Z \neq \frac{1}{2}\dim Y \\ \#(Z \cap \mathrm{Crit}(W)) & \dim Z = \frac{1}{2}\dim Y \end{cases}$$

$\Pi_i^Z = Z$ 

- sum over crit(W)

- zero modes .... $\phi^{tan}, \eta^{\overline{tan}}, \theta_{normal}$

at $p_* \in \mathrm{Crit}(W)$

$$\int d\phi^t \, d\eta^{\bar t} \, d\theta_n \, \exp\left(-|dW|^2 - \partial_{\bar i}\partial_{\bar j}\overline{W}\, \eta^{\bar i} g^{\bar j i} \theta_i\right) O_i(p_*)$$

$$= \begin{cases} 0 & \text{if } \dim Z \neq \frac{1}{2}\dim Y \\ |\det \partial_t \partial_n W|^{-2} \cdot \det \partial_{\bar t}\partial_{\bar n}\overline{W} \cdot O_i(p_*) & \text{if } = \end{cases}$$

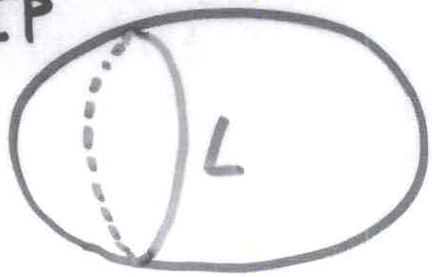
$$= \frac{O(p_*)}{\det \partial_t \partial_n W} = \frac{O(p_*)}{\mathrm{Pf}^Z_{p_*} \partial\partial W}$$

$$\Omega^{t\cdots t\, n\cdots n}\, \partial_t\partial_n W \cdots \partial_t \partial_n W$$

$$\boxed{\begin{array}{l} \Pi_i^Z = \displaystyle\sum_{p_* \in Z \cap \mathrm{Crit}(W)} \frac{O_i(p_*)}{\mathrm{Pf}^Z_{p_*}\partial\partial W} = (-1)^{\frac{n}{2}}\, \widetilde{\Pi}_i^Z \\[2ex] I(Z_1, Z_2) = \Pi_i^{Z_1}\, \eta^{ij}\, \widetilde{\Pi}_j^{Z_2} \qquad \left(\eta_{ij} = \displaystyle\sum_{\mathrm{Crit}(W)} \frac{O_i O_j}{\det \partial\partial W}\right) \end{array}}$$

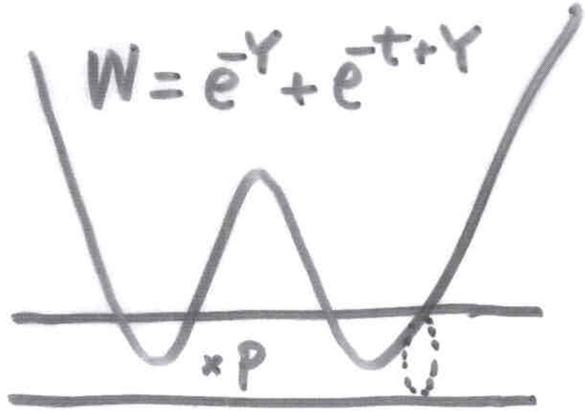
Back to Mirror

$X = \mathbb{CP}^1$



$\Longleftrightarrow$

$W = e^{-Y} + e^{-t+Y}$



$$\left[\begin{array}{l} \mathrm{Area}[\;\;]^{L} = 4\pi c \\ \mathrm{hol}(L) = e^{i(a - \frac{c}{r}\theta)} \end{array}\right. \Longleftrightarrow \quad p = \left\{ e^{-Y} = e^{-c+ia} \right\}$$

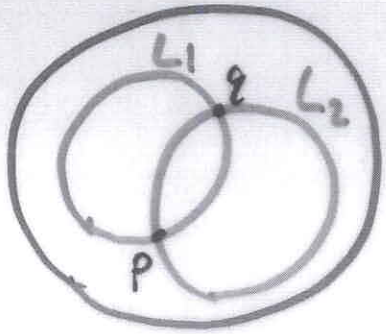
$$\left[\begin{array}{l} \mathrm{Area}_1 = \begin{cases} \mathrm{Area}_2 \\ 4\pi r - \mathrm{Area}_2 \end{cases} \\ \mathrm{hol}(L_1) = \begin{cases} \mathrm{hol}(L_2) \\ \mathrm{hol}(L_2)^{-1} \end{cases} \end{array}\right. \Longleftrightarrow \left[\begin{array}{l} W(p_1) = W(p_2) \\ \Longleftrightarrow c_1 - ia_1 = \begin{cases} c_2 - ia_2 \\ t - c_2 + ia_2 \end{cases} \end{array}\right.$$

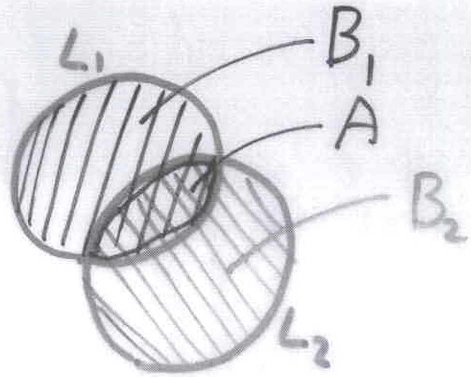
$$L = \left[\begin{array}{l} \mathrm{Area}[\;\;] = 2\pi r \\ \mathrm{hol}(L) = \pm 1 \end{array}\right] =: L_{\pm} \quad \Longleftrightarrow \quad p \in \mathrm{Crit}(W) = \left\{ e^{-Y} = \pm e^{-t/2} \right\}$$

SUSY ground states

$$HF^{\bullet}(L, L) = \begin{cases} 0 & L \neq L_{\pm} \\ \underset{\uparrow_B}{\mathbb{C}} \oplus \underset{\uparrow_F}{\mathbb{C}} & L = L_{\pm} \end{cases}$$

A-model computation FOOO





$$Qq = e^{-A}p - e^{-(4\pi r-(B_1+B_2-A))}p \quad \leftrightarrow$$

$$Qp = e^{-(B_1-A)}q - e^{-(B_2-A)}q \quad \leftrightarrow$$

$$Q^2 q = e^{-B_1}q - e^{-B_2}q - e^{-(4\pi r-B_2)}q + e^{-(4\pi r-B_1)}q$$

$$= 0 \quad \text{iff} \quad B_1 = \begin{cases} B_2 \\ 4\pi r - B_2 \end{cases}$$

Suppose $B_1 = B_2 = B$ $(L_1 \sim L_2)$

$$Qq = e^{-A}(1 - e^{-4\pi+2B})p = 0 \quad \text{only if } B = 2\pi r$$

$$Qp = 0$$

$$\therefore \quad HF^{\bullet}(L,L) = \begin{cases} 0 & B \neq 2\pi r \\ \underset{HF^0}{\mathbb{C}[q]} + \underset{HF^1}{\mathbb{C}[p]} & B = 2\pi r \end{cases}$$

Correlation functions

bulk

$1 \in H^0(\mathbb{CP}^1) \Leftrightarrow 1$

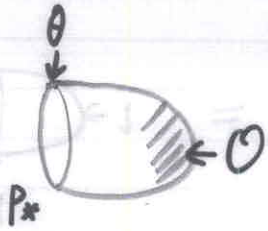
$\omega \in H^2(\mathbb{CP}^1) \Leftrightarrow e^{-Y}$

$$\langle O_1 O_2 O_3 \rangle = \sum_{\mathrm{Crit}\,W} \frac{O_1 O_2 O_3}{\det \partial\partial W}$$

$$\langle 1\,1\,\omega \rangle = 1 = \int_{\mathbb{P}^1} \omega$$

$$\langle \omega\,\omega\,\omega \rangle = e^{-t} \Leftrightarrow \#\left[\begin{matrix} \overset{\Sigma}{\overset{\|}{\mathbb{P}^1}} \to \mathbb{P}^1 \quad \text{degree } 1 \\ 0,1,\infty \to 0,1,\infty \end{matrix}\right] = 1$$

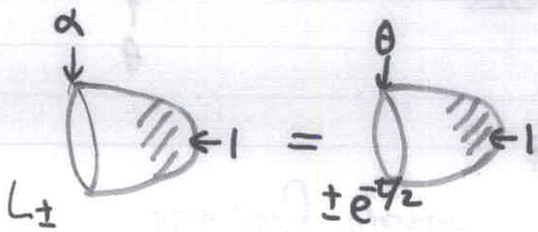
bulk-boundary



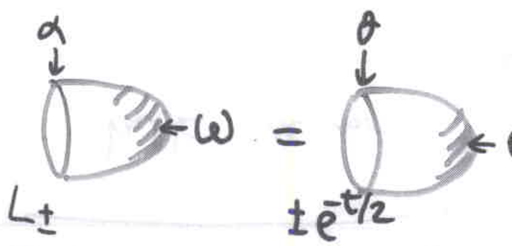
$= \mathcal{O}(p_*)$ [∵ θ is the only zero mode]

$1 \Leftrightarrow 1$

$\alpha = \frac{dz}{z} \Leftrightarrow \theta$



$= 1 \Leftrightarrow \#\left[\begin{matrix} \text{const} \\ (D^2, \partial D^2) \to (\mathbb{P}^1, L_\pm) \\ \quad\ \rotatebox{90}{$\in$} \qquad\qquad\quad \rotatebox{90}{$\in$} \\ \quad\ p \qquad\qquad\quad p \end{matrix}\right] = 1$



$= \pm e^{-t/2}$ $\quad e^{\oint A} = \pm 1$

$$\Leftrightarrow \#\left[\begin{matrix} \text{area } t/2 \text{ map} \\ (D^2, \partial D^2) \to (\mathbb{P}^1, L_\pm) \\ \rotatebox{90}{$\in$}\quad \rotatebox{90}{$\in$} \qquad\quad \rotatebox{90}{$\in$}\quad \rotatebox{90}{$\in$} \\ 0 \quad\ 1 \qquad\quad 0 \quad\ 1 \end{matrix}\right] = 1$$

D-branes, supersymmetry, and mirror symmetry

K. Hori

Compactification on

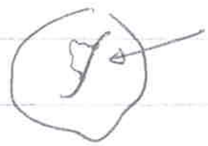
CY^3

$\times$

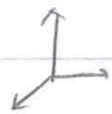
space $\mathbb{R}^3$

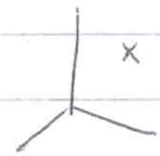
$\times$

time $\mathbb{R}$



D-brane, where open string can end.





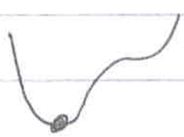
Get $N=2$ SUSY in $\mathbb{R}^{3+1}$ ← does not correspond to real world

↓ introduce D-branes

$\boxed{N=1 \text{ SUSY in } \mathbb{R}^{3+1}}$

↓

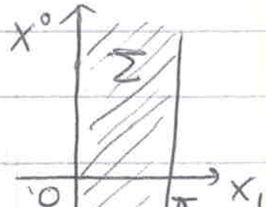
$N=0$



Two motivations to study D-branes

- To obtain theory resembling real world
- To study particles in $N=2$ theory.

Open string worldsheet



- Nonlinear Sigma Model

$$\Sigma \xrightarrow{\phi,\,\psi} X$$

action $S[\phi,\psi] = \int_\Sigma \left(|d\phi|^2 + i\bar{\psi}\not{D}\psi + R\psi^4 \right).$

- LG model. (Landau-Ginzburg)

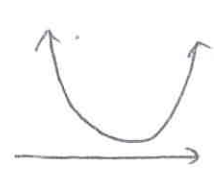
$\Sigma \longrightarrow X$, $W : X \to \mathbb{C}$ holomorphic

action $S[\phi,\psi]$

$$= \int_\Sigma \Big(|d\phi|^2 + i\bar{\psi}\not{D}\psi + R\psi^4 - |\partial W|^2 - \partial\partial W \psi_+\psi_- - \overline{\partial\partial W \psi_+\psi_-} \Big)$$

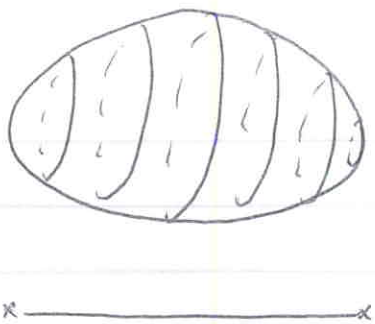
$W : X \to \mathbb{C}$ holomorphic.

(e.g. $X = \mathbb{C}$
$W = \phi^N + \cdots$



potential growth typically compactifies the system.

- Mirror symmetry



T-duality

$\longleftrightarrow$

"size" r (Area $= 4\pi r$)

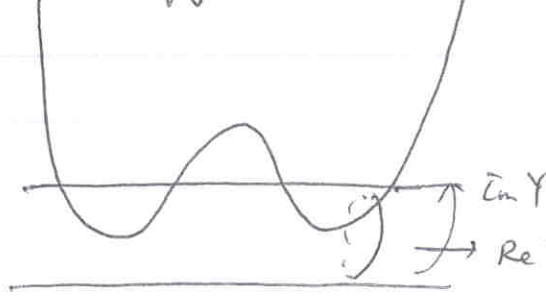
B-field $\theta = \theta + 2\pi$

$$\int D\phi D\psi \, e^{i\int \phi^* B} e^{-S(\phi)}$$

$$B = \theta \cdot H$$

$$W = e^{-Y} + e^{\overset{-t}{\overset{\parallel}{-r+i\theta}}} \cdot e^{Y}$$

- Superpotential is generated



$Y = Y + 2\pi i$

S^2-model $\longleftrightarrow$ sine Gordon model

Generalizes to all toric manifolds.