A. Schwarz

Maximally supersymmetric gauge theories.

10-dim SUSY YM

YM. $A_\mu(x) \in \mathfrak{g}$, $\mu = 0, \dots, n-1$.

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + [A_\mu, A_\nu]$$

$$\nabla_\mu = \partial_\mu + A_\mu, \quad [\nabla_\mu, \nabla_\nu] = F_{\mu\nu}.$$

$$S = \int \langle F_{\mu\nu}, F^{\mu\nu} \rangle dx$$

Can one add additional terms to S so that it is supersymmetric?

Possible in dimensions $\underline{\underline{10}}$, 6, 4, 3

(A_μ, ψ)

Can reduce dimension

$A_\mu(x^0, x^1, x^2, x^3)$ & only depends on $x^0, \dots, x^3$.

$\mu = 0, \dots, 9$ $\qquad$ $N=4$ supersymmetry

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further reduction

$A_\mu \quad A_\mu(x^0)$. $\longrightarrow$ BFSS matrix model.

$\downarrow$

IKKT matrix model. $\qquad$ Conjectured to give M-theory.

Compactification

limit $n \to \infty$

Connes, Douglas, Sch. (1997).

- ~~D-branes are described by maximally~~ supersymmetric YM, to some approximation.

  More precise theory: $\underbrace{\text{Born-Infeld}}_{\text{SUSY}}$ theory.

  → this theory should be <u>deformation</u> of maximally SUSY YM.

  Pioline - Schw. (1999).

Work in progress w/ M. Movshev.

1. Geometric formulation of max SUSY YM. th.
   - Pure spinors (Howe, Berkovits).
2. Deformation

   A_∞-algebras. with inner product.

Infinitesimal deformation governed by
HH (Hochsch.), HC (cyclic) — Penkava
Schwarz
(1994)

Superspace

∇_α, ∇_n $\quad \mathbb{R}^{10|16} \quad f(x_0, \dots, x_9, \xi_1, \dots, \xi_{16})$.
16 10

Super YM fields — connection on the superspace
with constraint

$$[\nabla_\alpha, \nabla_\beta]_+ = \Gamma^n_{\alpha\beta} \nabla_n$$

Can rewrite with pure spinors

$$u^\alpha \Gamma^n_{\alpha\beta} u^\beta = 0. \quad \leftarrow O(10).$$

$$\{u\} = SO(10)/U(5)$$

$$u^\alpha \nabla_\alpha = \nabla(u)$$

$$[\nabla(u), \nabla(u)]_+ = 0.$$

$$(\nabla(u))^2 = 0.$$ $\nabla(u)$ is a differential.

Remark: 11-dim supergravity

$u \Gamma u = 0 \quad \leftarrow SO(11, \mathbb{C}) / $ something
good spinors

$\{S, S\} = 0$ odd symplectic

QP-manifold. P-mfld. Schwarz

AKSZ.

Q-mfld

$[\hat{Q}, \hat{Q}] = 0$

$Q^a(x) = \sum m^a_{a_1, \cdots, a_n} x^{a_1} \cdots x^{a_n}$

a : coordinate

algebraic structure appears

$\to L_\infty$-algebra.

$m^a_{a_1, a_2}$ super Lie algebra

$m^a_{a_1}$ $m^a_{a_1, a_2}$ differential Lie algebra.

higher. L_∞.

formal Q-manifold = L_∞-algebra.

formal QP-manifold = L_∞-algebra w/ inner prod.

formal noncomm. Q-mfld — A_∞-alg

formal noncomm QP-mfld — A_∞-alg w/ inner prod.

In the case of maximally supersymmetric gauge theory, one can construct A_∞-alg with inner product.

dif. comm. assoc. algebra.

$A \otimes Mat_N \rightarrow$ <u>L_∞ algebra</u>

A_∞

$\rightarrow$ Can deform

SUSY YM theory.