RANDERS SPACE FORMS AND THE YASUDA-SHIMADA THEOREM

BY HIDEO SHIMADA

[1] History of Randers metric

(1) G. Randers (1941) introduced an asymmetric metric

$$L(x, y) = \sqrt{a_{ij}(x)y^i y^j} + b_i(x)y^i$$
$$:= \alpha + \beta$$

where the first term of the right-hand side is a quasi-Riemannian metric showing the gravitation field and the second term is the electromagnetic field.

R. S. Ingarden (1957) was the first one, who called it a Randers metric.

(2) Many physicists have studied the unified field theory based on Randers metric.

S. Heskia (1971), C. W. Kilmister and G. Stephenson (1954), A. M. Mosharrafa (1948), J. Schaer (1960), etc.

(3) Randers metrics are examples of Finsler metrics.

M. Matsumoto (1972, 1974),

S. Kitamura (1960, Thesis, Kyoto, 36p.)

C. Shibata, H. Shimada, M. Azuma and H. Yasuda (1977)

Purpose

(1) To show the corrected version of the paper [M1].

See [MS].

(2) This new version [MS] coincide with [BR].

(3) In the paper [YS], the so called Yasuda-Shimada theorem hold good under the restricted condition.

(4) Space forms (Sasakian, Kenmotsu, cosymplectic)

[2] Randers spaces of constant curvature

H.Yasuda and H. Shimada (1977) [YS]

M. Matsumoto (1989) [M1]

In the paper [YS], we do not assume $n > 2$ but, in the paper [M1] the restriction $n \geqslant 3$ is assumed.

[YS], [M1], [AIM] ... incorrect.

[BR], [MS]

[3] Counter example and the corrected version

D. Bao and C. Robles [BR] (to appear in Rep. on Math. Phys. 2002)

M. Matsumoto and H. Shimada [MS] (to appear in Tensor, N. S.)

These resul ts are the same. In the papers [BR], [MS] there is no $n > 2$ assumption.

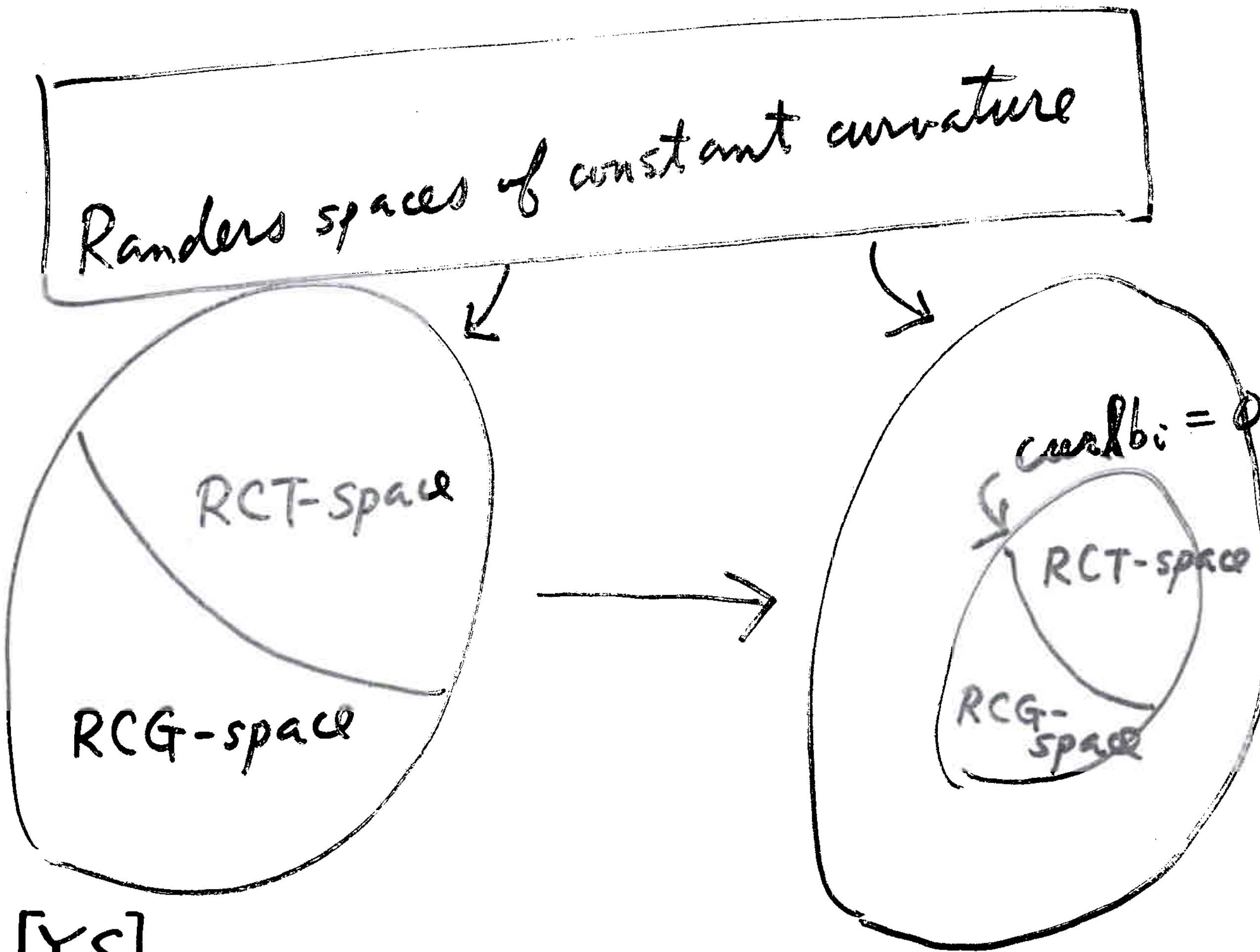
Additional Remark

A Randers space of constant curvature is an RCG-space if and only if it is locally projectively flat, provided that $\alpha^2 \not\equiv 0 \pmod{\beta}$ and $b^2 \neq 1$.

Proof. → the associated Riemannian space is of constant curvature. Consequently the Randers space is locally projectively flat.

← Suppose Randers space of const. curvature is locally projectively flat. Then the space is Douglas space. Consequently b_i is gradient.

QED.



[YS]
old version

corrected version

References

[AIM] Antonelli, P.L. Ingarden, R.S. Matsumoto, M., The theory of Sprays and Finsler spaces with application..., Kluwer (1993).

[BR] Bao, D., Robles, C., On Randers metrics of constant curvature, (to appear in Rep. on Math. Phys. (2002)

[M1] Matsumoto, M., Randers spaces of constant curvature, Rep. on Math. Phys. 28 (1989).

[MS] Matsumoto, M., Shimada, H., The corrected fundamental theorem on the Randers spaces of constant curvature, to appear Tensor, N.S.

[YS] Yasuda, H., Shimada, H., On Randers spaces of scalar curvature, Rep. on Math. Phys. (1977).

[BF] Bejancu, A., Farran, H.R., Finsler metrics of positive constant flag curvature on Sasakian space forms, to appear.

[BS] Finsler metrics of constant positive curvature on the Lie group S^3, J. London Math. Society, to appear.

If F^n is a Randers space of constant flag curvature K and $\operatorname{curl} b_i = 0$, then

(1) for $K > 0$, we have RCT-space [BS]; [BF]

(2) for $K = 0$, we have locally Minkowski

(3) for $K < 0$, we have RCG-space.

Bejancu and Farran have shown that a Sasakian space form induces the structure of a Randers space of positive constant flag curvature.

Inspired by [BF], we have found out that a cosymplectic space form $M(c)$ of constant φ-sectional curvature $c = 0$ induces a natural structure of Randers space of constant flag curvature $K=0$ (actually, this is a locally Minkowski space), and that a Kenmotsu space induces a structure of Randers space of negative constant flag curvature.

But there is an one problem that we must solve. Namely this Randers space is not a positive definite one.

[Hasegawa, I, Sabau Sorin, Shimada]

cosymplectic mfd

If the Levi-Civita connection of the Riemannian metric a of an almost contact metric structure (φ, ξ, η, a) on M satisfies the relation

$$\nabla_X \varphi = 0, \quad \nabla_X \eta = 0$$

Kenmotsu mfd

$$(\nabla_X \varphi) Y = \eta(Y) \varphi X - a(\varphi X, Y) \xi$$

Kenmotsu	Sasakian
$\nabla_X \xi = -X + \eta(X) \xi$	$\nabla_X \xi = \varphi X$
$(\nabla_X \eta) Y = -(a(X, Y) - \eta(X) \eta(Y))$	$(\nabla_X \eta) Y = a(\varphi X, Y)$
$(\nabla_X \eta) Y - (\nabla_Y \eta) X = 0$	$(\nabla_X \eta) Y + (\nabla_Y \eta) X = 0$