$$a_\mu = \frac{\partial}{\partial x^\mu}. \tag{1}$$

$$\eta_{\mu\nu} = a_\mu a_\nu. \tag{2}$$

$$z^2 = \eta_{\mu\nu} z^\mu z^\nu \tag{3}$$

$$ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu \tag{4}$$

$$\begin{aligned} ds^2 &> 0 \text{ timelike} \\ ds^2 &= 0 \text{ null} \\ ds^2 &< 0 \text{ spacelike} \end{aligned} \tag{5}$$

$$y^\alpha = \frac{dx^\alpha}{ds} \tag{6}$$

$$\eta_{\alpha\beta} y^\alpha y^\beta = 1 \tag{7}$$

$$ds^2 = c^2 d\tau^2 \tag{8}$$

$$v^\alpha = \frac{dx^\alpha}{d\tau}, \quad \eta_{\alpha\beta} v^\alpha v^\beta = c^2 \tag{9}$$

$$(\omega/c; \vec{k}), \quad \vec{k} \cdot \vec{k} = \omega^2/c^2 \tag{10}$$

$$b_\mu(\tau) = L^\nu_\mu(\tau) a_\nu \tag{11}$$

$$\left(\frac{\partial}{\partial x^\mu}, \frac{\partial}{\partial y^\alpha}\right) \tag{12}$$

$$(dx^\mu, dy^\alpha) \tag{13}$$

$$dS^2 = \eta_{\mu\nu} dx^\mu dx^\nu + \eta_{\alpha\beta} dy^\alpha dy^\beta \tag{14}$$

$$\eta_{\mu\nu} = \eta_{\alpha\beta} e^\alpha_\mu e^\beta_\nu \tag{15}$$

$$dx^\mu = y^\mu ds \tag{16}$$

$$\bar{y}^\mu = Y^{\mu *}_\nu y^\nu \tag{17}$$

$$Y^{\mu *}_\nu Y^\nu_\lambda = \delta^\mu_\lambda \tag{18}$$

$$\frac{\partial}{\partial \overline{y}^\alpha} = Y^\gamma_\alpha \frac{\partial}{\partial y^\gamma} \tag{19}$$

$$h_{\alpha\beta} = \eta_{\gamma\delta} Y^\gamma_\alpha Y^\delta_\beta \tag{20}$$

$$F^2 = \eta_{\mu\nu} y^\mu y^\nu \tag{21}$$

$$F^2 = g_{\mu\nu} \bar{y}^\mu \bar{y}^\nu \tag{22}$$

$$g_{\mu\nu} = h_{\alpha\beta} e^\alpha_\mu e^\beta_\nu \tag{23}$$

$$g_{\mu\nu} = \eta_{\gamma\delta} b^\gamma_\mu b^\delta_\nu \tag{24}$$

$$b^\alpha_\mu = Y^\alpha_\beta e^\beta_\mu \tag{25}$$

$$Y^\mu_\nu = Y^\mu_\nu (x^\mu, y^\alpha) \tag{26}$$

$$g_{\mu\nu} = Y_\mu^\sigma Y_\nu^\rho \eta_{\sigma\rho} \tag{27}$$

$$\tilde{F}^2 = g_{\mu\nu} y^\mu y^\nu \tag{28}$$

$$f_{\mu\nu} = \frac{1}{2}\frac{\partial^2 \tilde{F}^2}{\partial y^\mu \partial y^\nu} \tag{29}$$

$$\frac{\partial Y_\nu^\mu}{\partial y^\alpha} y^\mu = 0 \tag{30}$$

$$\tilde{F}^2 = f_{\mu\nu} y^\mu y^\nu \tag{31}$$

$$E = \frac{\partial L}{\partial y^\mu} y^\mu - L \tag{32}$$

$$L = \tilde{F}^2 \tag{33}$$

$$\frac{\partial Y_\nu^\mu}{\partial y^\alpha} y^\alpha = 0 \tag{34}$$

$$g_\mu = a_\sigma Y_\mu^\sigma \tag{35}$$

$$g_{\mu\nu} = g_\mu g_\nu = a_\sigma a_\rho Y_\mu^\sigma Y_\nu^\rho = \eta_{\sigma\rho} Y_\mu^\sigma Y_\nu^\rho \tag{36}$$

$$\omega^\sigma = Y_\mu^\sigma dx^\mu \tag{37}$$

$$d\tilde{P} = a_\sigma Y_\mu^\sigma dx^\mu \tag{38}$$

$$d\tilde{P} = a_\sigma \omega^\sigma \tag{39}$$

$$d\tilde{P} = g_\mu dx^\mu \tag{40}$$

$$d(d\tilde{P}) = a_\sigma d\omega^\sigma = a_\sigma dY^\sigma_\mu \wedge dx^\mu \tag{41}$$

$$d(d\tilde{P}) = a_\sigma dY^\sigma_\mu Y^{\mu *}_\rho \wedge \omega^\rho \tag{42}$$

$$\omega^\sigma_\rho = dY^\sigma_\mu Y^{\mu *}_\rho \tag{43}$$

$$d(d\tilde{P}) = a_\sigma \omega^\sigma_\rho \wedge \omega^\rho \tag{44}$$

$$d(dg_\mu) = 0 \tag{45}$$

$$\begin{aligned} \omega^\sigma_\rho &= \frac{\partial Y^\sigma_\mu}{\partial x^\kappa} Y^{\mu *}_\rho dx^\kappa + \frac{\partial Y^\sigma_\mu}{\partial y^\lambda} Y^{\mu *}_\rho dy^\lambda \\ &= L^\sigma_{\rho\kappa} dx^\kappa + B^\sigma_{\rho\lambda} dy^\lambda \end{aligned} \tag{46}$$

$$f_{\mu\nu} = \eta_{\alpha\beta} Y^\alpha_\mu Y^\beta_\nu \tag{47}$$

$$\begin{aligned} df_{\mu\nu} &= \eta_{\alpha\beta}(dY^\alpha_\mu Y^\beta_\nu + Y^\alpha_\mu dY^\beta_\nu) \\ &= f_{\sigma\rho} Y^{\sigma *}_\alpha Y^{\rho *}_\beta (dY^\alpha_\mu Y^\beta_\nu + Y^\alpha_\mu dY^\beta_\nu) \\ &+ f_{\sigma\rho} Y^{\sigma *}_\alpha dY^\alpha_\mu \delta^\rho_\nu + f_{\sigma\rho} Y^{\rho *}_\beta dY^\beta_\nu \delta^\sigma_\mu \end{aligned} \tag{48}$$

$$\begin{aligned} &= f_{\sigma\nu}\omega^\sigma_\mu + f_{\mu\rho}\omega^\rho_\nu \\ &= f_{\sigma\nu} L^\sigma_{\mu\kappa} dx^\kappa + f_{\sigma\nu} B^\sigma_{\nu\lambda} dy^\lambda \\ &+ f_{\mu\rho} L^\rho_{\nu\kappa} dx^\kappa + f_{\mu\rho} B^\rho_{\nu\lambda} dy^\lambda \end{aligned} \tag{49}$$

$$df_{\mu\nu} = \frac{\partial f_{\mu\nu}}{\partial x^\kappa} dx^\kappa + \frac{\partial f_{\mu\nu}}{\partial y^\lambda} dy^\lambda \tag{50}$$

$$
\begin{aligned}
\frac{\partial f_{\mu\nu}}{\partial x^{\kappa}} &= f_{\sigma\nu} L^{\sigma}_{\mu\kappa} + f_{\mu\rho} L^{\rho}_{\nu\kappa} = L_{\nu\mu\kappa} + L_{\mu\nu\kappa} \\
\frac{\partial f_{\mu\nu}}{\partial y^{\lambda}} &= f_{\sigma\nu} B^{\sigma}_{\mu\lambda} + f_{\mu\rho} B^{\rho}_{\nu\lambda} = B_{\nu\mu\lambda} + B_{\mu\nu\lambda}
\end{aligned} \tag{51}
$$

$$\frac{\delta}{\delta x^{\mu}} = \frac{\partial}{\partial x^{\mu}} - N^{\nu}_{\mu} \frac{\partial}{\partial y^{\nu}} \tag{52}$$

$$\left(\frac{\delta}{\delta x^{\mu}}, \frac{\partial}{\partial y^{\mu}} \right) \tag{53}$$

$$(dx^{\mu}, \delta y^{\mu} = dy^{\mu} + N^{\mu}_{\nu} dx^{\nu}) \tag{54}$$

$$df_{\mu\nu} = \frac{\delta f_{\mu\nu}}{\delta x^{\kappa}} dx^{\kappa} + \frac{\partial f_{\mu\nu}}{\partial y^{\lambda}} \delta y^{\lambda} \tag{55}$$

$$df_{\mu\nu} = \frac{\partial f_{\mu\nu}}{\partial x^{\kappa}} dx^{\kappa} + \frac{\partial f_{\mu\nu}}{\partial y^{\lambda}} dy^{\lambda} \tag{56}$$

$$F_{\mu\nu\kappa} = \frac{1}{2} \left(\frac{\delta f_{\mu\nu}}{\delta x^{\kappa}} + \frac{\delta f_{\kappa\mu}}{\delta x^{\nu}} - \frac{\delta f_{\kappa\nu}}{\delta x^{\mu}} \right) \tag{57}$$

$$C_{\mu\nu\lambda} = \frac{1}{2} \left(\frac{\partial f_{\mu\nu}}{\partial y^{\lambda}} + \frac{\partial f_{\lambda\mu}}{\partial y^{\nu}} - \frac{\partial f_{\lambda\nu}}{\partial y^{\nu}} \right) = \frac{1}{2} \frac{\partial f_{\mu\nu}}{\partial y^{\lambda}} \tag{58}$$

$$C_{\mu\nu\lambda} = \frac{1}{2} (B_{\mu\nu\lambda} + B_{\nu\mu\lambda}) \tag{59}$$

$$\gamma_{\mu\nu\kappa} = \frac{1}{2} \left(\frac{\partial f_{\mu\nu}}{\partial x^{\kappa}} + \frac{\partial f_{\kappa\mu}}{\partial x^{\nu}} - \frac{\partial f_{\kappa\nu}}{\partial x^{\mu}} \right) \tag{60}$$

$$\gamma_{\mu\nu\kappa} = \frac{1}{2} \left(L_{\mu\nu\kappa} + L_{\nu\mu\kappa} + L_{\kappa\mu\nu} + L_{\mu\kappa\nu} - L_{\kappa\nu\mu} - L_{\nu\kappa\mu} \right) \tag{61}$$

$$\frac{dv_\lambda}{d\tau} + \gamma_{\lambda\mu\nu} v^\mu v^\nu = 0 \tag{62}$$

$$v^\mu = \frac{dx^\mu}{d\tau} \tag{63}$$

$$\eta_{\mu\nu} v^\mu v^\nu = c^2 \tag{64}$$

$$\frac{d}{d\tau}\left(\frac{\partial \tilde{F}^2}{\partial v^\mu}\right) - \frac{\partial \tilde{F}^2}{\partial x^\mu} \tag{65}$$

$$Y^\mu_\nu = \delta^\mu_\nu - B^{-2}\left[1 - (1 + kB^2)^{\frac{1}{2}}\right] B^\mu B_\nu \tag{66}$$

$$Y^{\nu *}_\lambda = \delta^\nu_\lambda - B^{-2}\left[1 - (1 + kB^2)^{-\frac{1}{2}}\right] B^\nu B_\lambda \tag{67}$$

$$B^\mu = A^\mu + \frac{\partial \Lambda}{\partial x^\nu}\eta^{\mu\nu} \tag{68}$$

$$A_\mu = -\frac{mc}{e} v_\mu + \frac{\partial \phi}{\partial x^\mu} \tag{69}$$

$$g_{\mu\nu} = \eta_{\sigma\rho} Y^\sigma_\mu Y^\rho_\nu = \eta_{\mu\nu} + kB_\mu B_\nu \tag{70}$$

$$\tilde{F}^2_K = \alpha^2 + \beta^2, \quad \alpha = (\eta_{\mu\nu} v^\mu v^\nu)^{\frac{1}{2}}, \quad \beta = k^{\frac{1}{2}} B_\mu v^\mu \tag{71}$$

$$L_{\lambda\mu\nu} = B^{-2}\left[1 - (1 + kB^2)^{\frac{1}{2}}\right]\left[B_\lambda \frac{\partial B_\mu}{\partial x^\nu} - \frac{\partial B_\lambda}{\partial x^\nu} B_\mu\right] \tag{72}$$

$$+kB_\lambda \frac{\partial B_\mu}{\partial x^\nu}$$

$$
\begin{aligned}
\gamma_{\lambda\mu\nu} &= \frac{1}{2}k\left[B_\lambda\left(\frac{\partial B_\mu}{\partial x^\mu}+\frac{\partial B_\nu}{\partial x^\mu}\right)\right] \\
&+ \frac{1}{2}k\left[B_\mu\left(\frac{\partial B_\lambda}{\partial x^\nu}-\frac{\partial B_\nu}{\partial x^\lambda}\right)+B_\nu\left(\frac{\partial B_\lambda}{\partial x^\mu}-\frac{\partial B_\mu}{\partial x^\lambda}\right)\right]
\end{aligned} \tag{73}
$$

$$
B_\lambda v^\lambda = \frac{e}{mck} \tag{74}
$$

$$
\frac{dv_\lambda}{d\tau}+\frac{e}{mc}\left(\frac{\partial B_\lambda}{\partial x^\mu}-\frac{\partial B_\mu}{\partial x^\lambda}\right)v^\mu = 0 \tag{75}
$$

$$
F_{\mu\lambda} = \frac{\partial B_\lambda}{\partial x^\mu}-\frac{\partial B_\mu}{\partial x^\lambda} \tag{76}
$$

$$
k = 4\kappa c^{-4} \tag{77}
$$

$$
Y^\mu_\nu = \Theta^{\frac{1}{2}}\delta^\mu_\nu+\left[1-\Theta^{\frac{1}{2}}\right]\frac{v_\nu v^\mu}{\alpha^2}+\frac{k^{\frac{1}{2}}B_\nu v^\mu}{\alpha} \tag{78}
$$

$$
Y^{\nu *}_\lambda = \Theta^{-\frac{1}{2}}\delta^\nu_\lambda+\Theta^{-\frac{3}{2}}\left[\Theta^{\frac{1}{2}}-1\right]\frac{v_\lambda v^\nu}{\alpha^3}-\Theta^{-\frac{3}{2}}\frac{k^{\frac{1}{2}}B_\lambda v^\nu}{\alpha} \tag{79}
$$

$$
\Theta = 1+\frac{\beta}{\alpha} \tag{80}
$$

$$
g_{\mu\nu} = \Theta\left(\eta_{\mu\nu}-\frac{v_\mu v_\nu}{\alpha^2}\right)+\left(\frac{v_\mu}{\alpha}+k^{\frac{1}{2}}B_\mu\right)\left(\frac{v_\nu}{\alpha}+k^{\frac{1}{2}}B_\nu\right) \tag{81}
$$

$$
\tilde{F}^2_R = (\alpha+\beta)^2 \tag{82}
$$

$$
\Theta\left(\frac{dv_\lambda}{d\tau}+k^{\frac{1}{2}}\alpha F_{\mu\lambda}v^\mu\right) = 0 \tag{83}
$$

$$k^{\frac{1}{2}} = \frac{e}{mc^2} \tag{84}$$

$$\begin{aligned} Y^{\mu}_{\nu} &= \Theta^{\frac{1}{2}}\delta^{\mu}_{\nu} - \Theta^{\frac{1}{2}}\frac{v_\nu v^\mu}{\alpha^2} \\ &+ \Theta^{-\frac{1}{2}}\left[1 + \left(3 - \frac{\alpha^2 k B^2}{\beta^2}\right)^{\frac{1}{2}}\right]\frac{k^{\frac{1}{2}} B_\nu v^\mu}{\alpha} - k\left(\frac{\beta}{\alpha}\right)^{-\frac{3}{2}} B_\nu v^\mu \end{aligned} \tag{85}$$

$$g_{\mu\nu} = \frac{\beta}{\alpha}\eta_{\mu\nu} - \frac{\beta}{\alpha^3}v_\mu v_\nu + \frac{k^{\frac{1}{2}}}{\alpha}(v_\mu B_\nu + B_\mu v_\nu) \tag{86}$$

$$\tilde{F}^2_W = 2\alpha\beta \tag{87}$$

$$\frac{dv_\lambda}{d\tau} + \frac{\alpha^2 k^{\frac{1}{2}}}{\beta} F_{\mu\lambda} v^\mu = 0 \tag{88}$$

$$\beta = \frac{k^{\frac{1}{2}} mc^3}{e} \tag{89}$$

$$\tilde{F}^2_R = \tilde{F}^2_K + \tilde{F}^2_W \tag{90}$$

	Randers	Kaluza-Klein	Weyl
1. Electromagnetic gauge relation implied	No	Yes	Yes
2. k can be gravitational	No	Yes	Yes
3. e/m not in metric	No	Yes	No