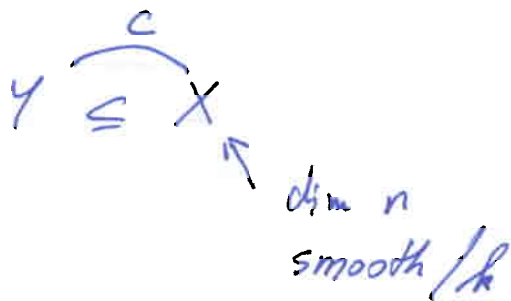


Manuel Blickle

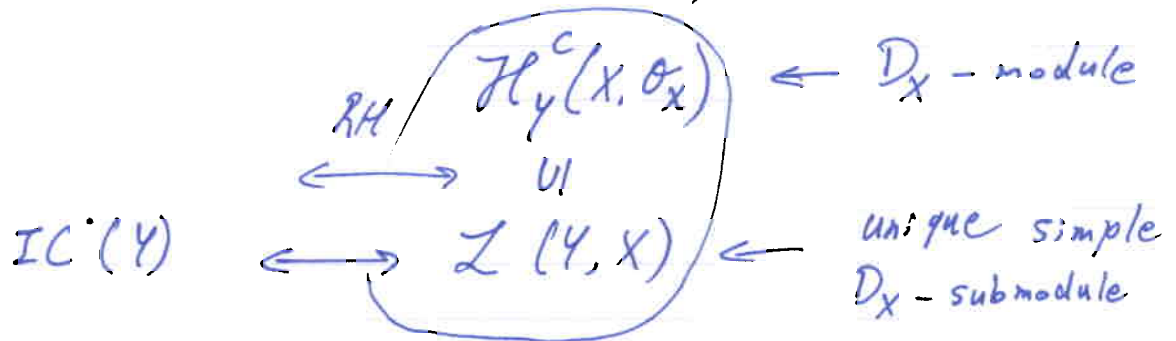
Dec. 4, '02

"Intersection homology D-module in pos. char."



constructible \mathbb{C} - \mathcal{V} s

(holonomic D -modules)



$$\mathcal{H}_Y^c(X, \mathcal{O}_X) \Big|_{X-\text{Sing } Y} = \mathcal{I}(Y, X) \Big|_{X-\text{Sing } Y}$$

Thm. 1

$Y \subseteq X$ as before (char(k) arbitrary)

$\exists!$ simple D_X -submodule

$$\mathcal{I}(Y, X) \stackrel{*}{\subseteq} \mathcal{H}_Y^c(X, \mathcal{O}_X)$$

finite length
(in char. p
Lyubeznik)

Moreover, away from $\text{Sing } Y$ $*$ is $\stackrel{*}{=}$

(A) proof

(C) Corollaries

(B) construction of Z

(D) R-H like correspondence

(A) $i: X-Z \hookrightarrow X, \quad Z = \text{sing } Y$

$$Z \subseteq \mathcal{H}_Y^c(X, \sigma_X) \subseteq i_* \mathcal{H}_{X-Z}(X-Z, \sigma_{X-Z})$$

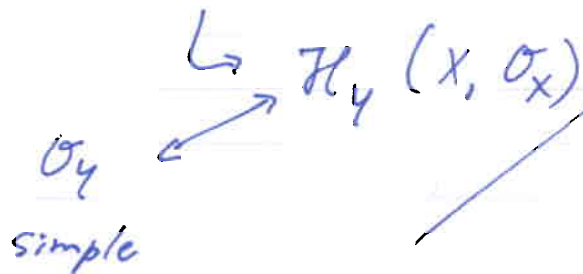
any simple \mathcal{O}_X -submod.

$\mathcal{H}_{Y-Z}(X-Z, \sigma_{X-Z}) \leftarrow$ simple

$$\begin{matrix} Z|_{X-Z} \\ \# \\ 0 \end{matrix} \subseteq \mathcal{H}_Y^c(X, \sigma_X)|_{X-Z} \subseteq i_* \mathcal{H}_{X-Z}(X-Z, \sigma_{X-Z})|_{X-Z}$$

$$\begin{matrix} \text{Hom}(Z, i_* \mathcal{H}_{X-Z}) \\ \# \\ 0 \end{matrix} = \text{Hom}(Z|_{X-Z}, \mathcal{H}_{X-Z})$$

$Y \subseteq X$ both smooth



replace D -module structure by Frobenius: ($\text{char} = p > 0$)

unit $R[F]$ -modules:

Def. $R[F]$ module is an R -module M & R -lin. map

$$\mathcal{F}: F^*M \rightarrow M$$

$X = \text{Spec}(R) \leftarrow \text{regular}$
 $Y = \text{Spec}(\underline{R[z]})$
 (A, m)
 (R, m) is local.

$$F: R \rightarrow R$$

$$r \mapsto r^p$$

$$F: \text{Spec } R$$

$$\downarrow$$

$$\text{Spec } R$$

$R^{(e)}$: R - R bimodule

left action: usual
 right action: via F
 $m \cdot r = r^p m$

note: $F^*M = R^{(1)} \otimes M$

(M, \mathcal{F}) is called unit of \mathcal{F} is " \cong ".

Rmk. $R[F] := \frac{R\langle F \rangle}{\langle r^p F - Fr \mid r \in R \rangle}$

$$\mathcal{F} \in \text{Hom}_R(F^*M, M) \cong \text{Hom}_R(M, F_*M)$$

$$\mathcal{F}_\mathcal{F}(rm) = r^p \mathcal{F}_\mathcal{F}(m)$$

Examples • R is unit $R[F]$ -module:

$$\begin{aligned} R^{(1)} \otimes R &\longrightarrow R \\ r \otimes s &\longmapsto r s^P \\ r \otimes 1 &\longleftarrow r \end{aligned}$$

• $f \in R$ R_f is unit $R[F]$ -module.

$$\begin{aligned} R^{(1)} \otimes R_f &\longrightarrow R_f \\ r \otimes t/s &\longmapsto r \frac{t^P}{s^P} \\ t s^{P-1} \otimes \frac{1}{s} &\longleftarrow t/s \end{aligned}$$

R_f is gen. by $\frac{1}{f}$ as $R[F]$ -module.

Thm. (R regular) Cont. f.g. unit $R[F]$ -modules is

(1) abelian and closed under extensions

(2) all f.g. u. $R[F]$ -modules have finite length as such.

Def. $D_R := \bigcup_e \text{Hom}_{R^{pe}}(R, R)$

$$= \bigcup_e \text{Hom}_{\text{mod-}R}(R^{(e)}, R^{(e)})$$

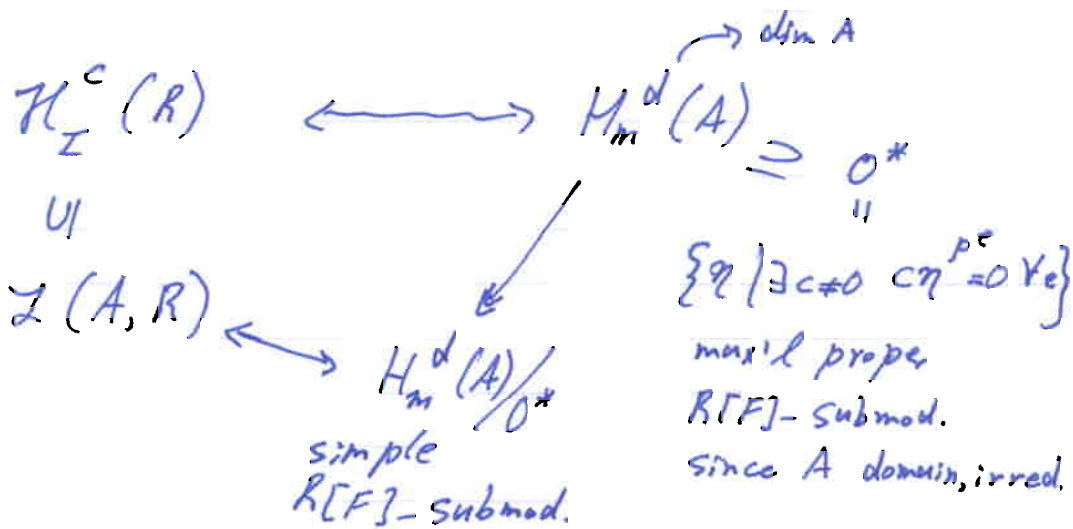
note: $\frac{\partial}{\partial x} : R \rightarrow R$ sends $R^P \rightarrow 0$.

$\frac{\partial^p}{p! \partial x^p}$ con. over R^{p^2}

Thm. (cont'd)

(3) also finite length as a D_R -module
 (length $D_R = \text{length u. } R[F], k = \bar{k}$)

setup:



Thm. $Z(A, R) = \left(H_m^d(A)/O^* \right)^{VF}$

Cor. • if A is F -rational, i.e. $O^* = 0$,
 then $H_I^c(R)$ is D -simple.

• $H_I^c(R)$ is simple $\iff O^*$ is F -nilpotent

$$(A \text{ c-M}) \Leftrightarrow (x_1, \dots, x_d)^* = (x_1, \dots, x_d)^F$$

for any s.o.p.

Cor. ~~A curve~~ Y curve, $\pi_Y^c(X, \sigma_X)$ is simple
 $\Leftrightarrow \tilde{Y} \rightarrow Y$ is injective.

example $A = k[x, y, z, w] / \underbrace{xy - zw}_= f$

char p
 $\Rightarrow H_f^1(R)$ simple

char 0
 $\Rightarrow b_f = (s+1)(s+2)$

