Quantum Teleportation and Entanglement Purification

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Additional Credit

- Charles H. Bennett
- Samuel L. Braunstein
- Richard Cleve
- Claude Crépeau
- David Deutsch
- David P. DiVincenzo
- Artur K. Ekert
- Richard Jozsa
- Chiara Macchiavello
- Asher Peres
- Sandu Popescu
- Anna Sanpera
- Ben Schumacher
- John A. Smolin
- William K. Wootters


Stand by. I'll
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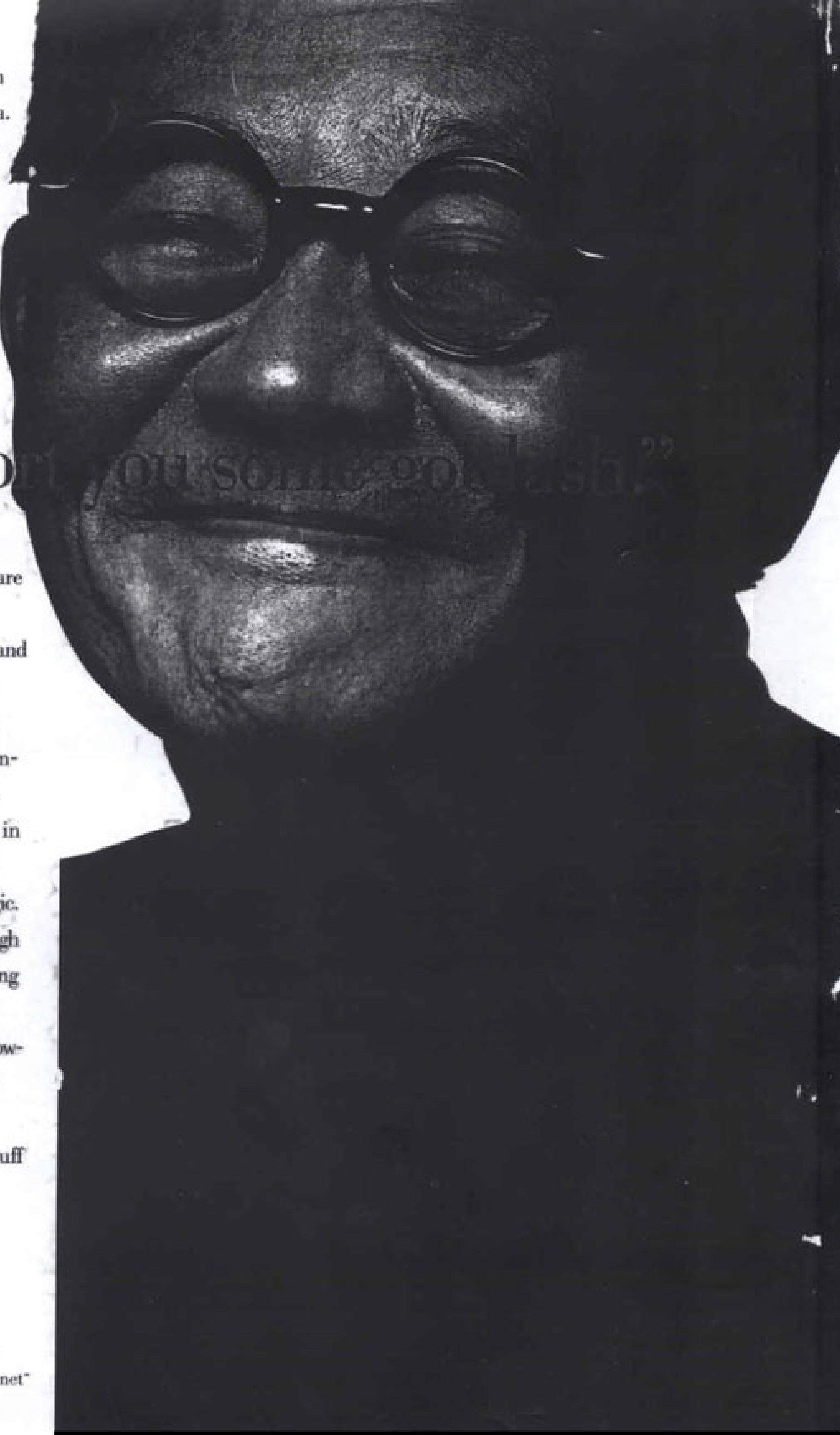
For years, she shared recipes with her friend in Osaka.

She showed him hundreds of ways to use paprika.

He shared his secret recipe for sukiyaki.

One day Margit e-mailed Seiji,

teleport you some goulash.”



Margit is a little premature, but we are working on it.

An IBM scientist and his colleagues have discovered a way to make an object disintegrate in one place and reappear intact in another.

It sounds like magic. But their breakthrough could affect everything from the future of computers to our knowledge of the cosmos.

Smart guys. But none of them can stuff a cabbage.

Yet.

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Wall Street Journal

IBM Advertisements Boldly Go Where No Study Has Gone Before

By THOMAS E. WEBER
Staff Reporter of THE WALL STREET JOURNAL

Somebody at International Business Machines Corp. may have been watching too much "Star Trek."

IBM ads are touting an impressive new research project: "An IBM scientist and his colleagues have discovered a way to make an object disintegrate in one place and reappear intact in another," the ads read, evoking the sci-fi "transporter" used by Captain Kirk and his friends.

Then there's the elderly lady pictured in the ad, who proclaims to a gourmet-pen pal, "Stand by. I'll teleport you some goulash." Her promise may be "a little premature," the ad says, but IBM is working on it.

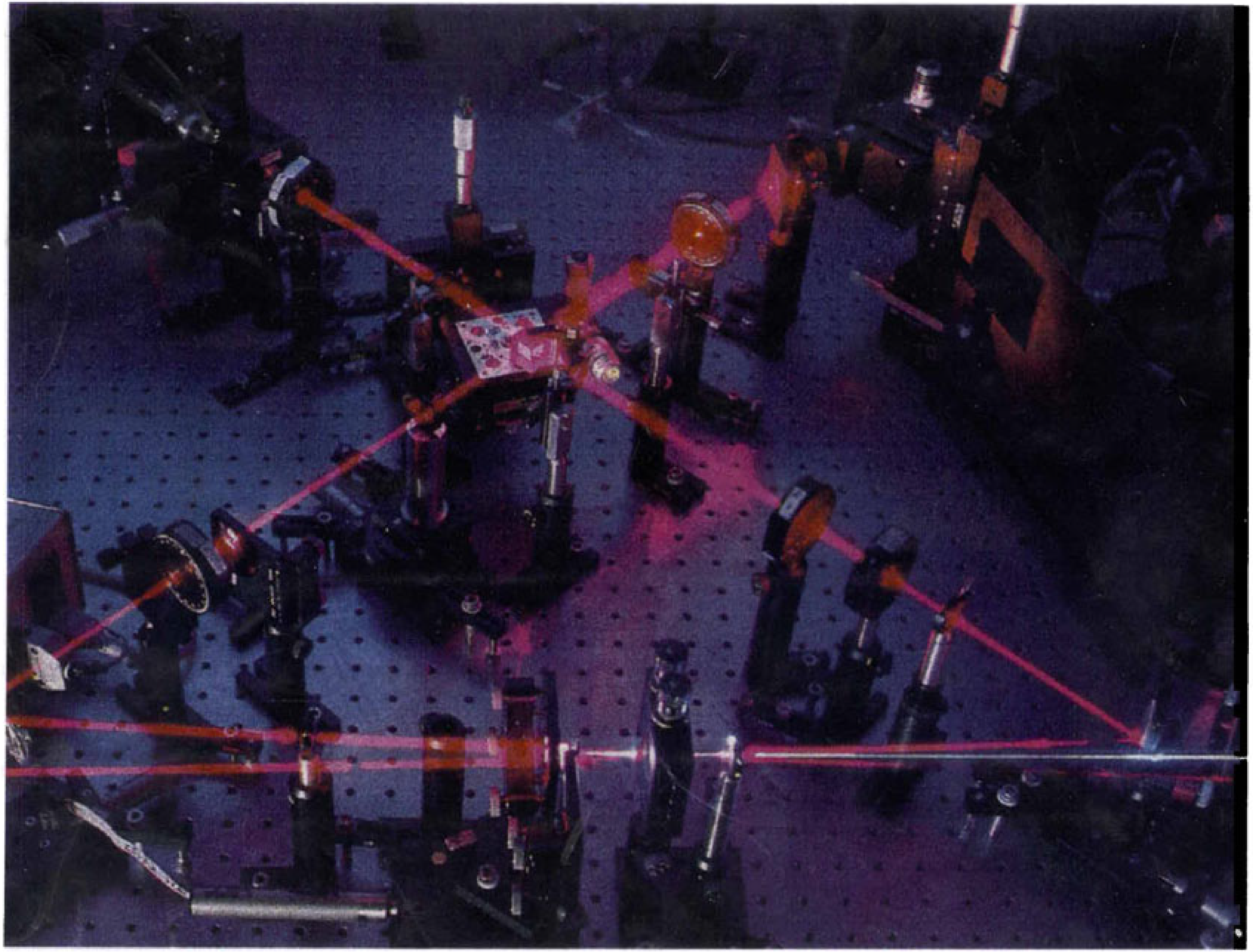
Really? Robert L. Park, a physics professor at the University of Maryland, was so intrigued by the ad, which ran in Rolling Stone, Scientific American and other magazines, that he e-mailed IBM for more information. "This is still under development and no further information is currently available," came the curt reply.

It turns out that an IBM scientist hasn't really "discovered" anything of the sort. Mr. Park is still stewing about it. "It's sort of a bait-and-switch," he complains. "It leaves a taste of bad goulash in my mouth." The IBM scientist in question, one Charles H. Bennett, is a physicist at IBM's vaunted research center in Yorktown Heights, N.Y. But he concentrates more on photons – "a quantum of electromagnetic energy having both particle and wave properties," the dictionary says – than on Hungarian dishes.

"This doesn't really have anything to do with teleporting goulash," he says. It does have to do with an area of physics known as quantum teleportation, on which Mr. Bennett is happy to expound at length. Never mind what he said, exactly; Mr. Spock would have trouble following it.

At ad agency Ogilvy & Mather, executive group director Matt Ross describes the goulash gimmick as "a kind of dramatization" meant to emphasize IBM's research and development capabilities. "It's really just a fun way to evoke a sense of what this could mean."

But it hasn't been all fun for Mr. Bennett. He says he's more than a little embarrassed about the ad's unequivocal claims. "In any organization there's a certain tension between the research end and the advertising end," he says. "I struggled with them over it, but perhaps I didn't struggle hard enough."



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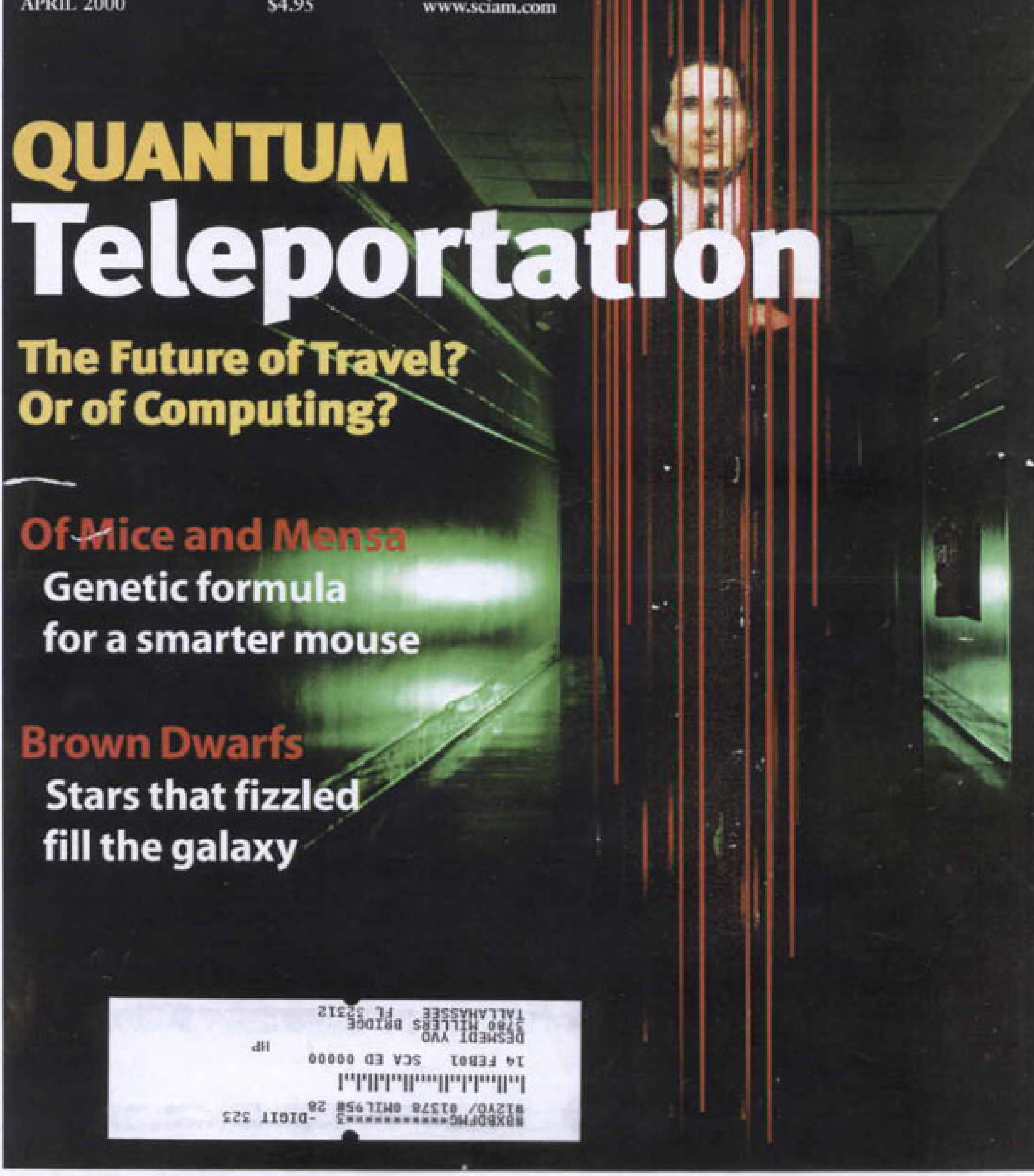


QUARK SOUP

CERN cooks up a new state of matter

see page 16

QUANTUM Teleportation

The Future of Travel? Or of Computing?

Of Mice and Mensa

Genetic formula for a smarter mouse

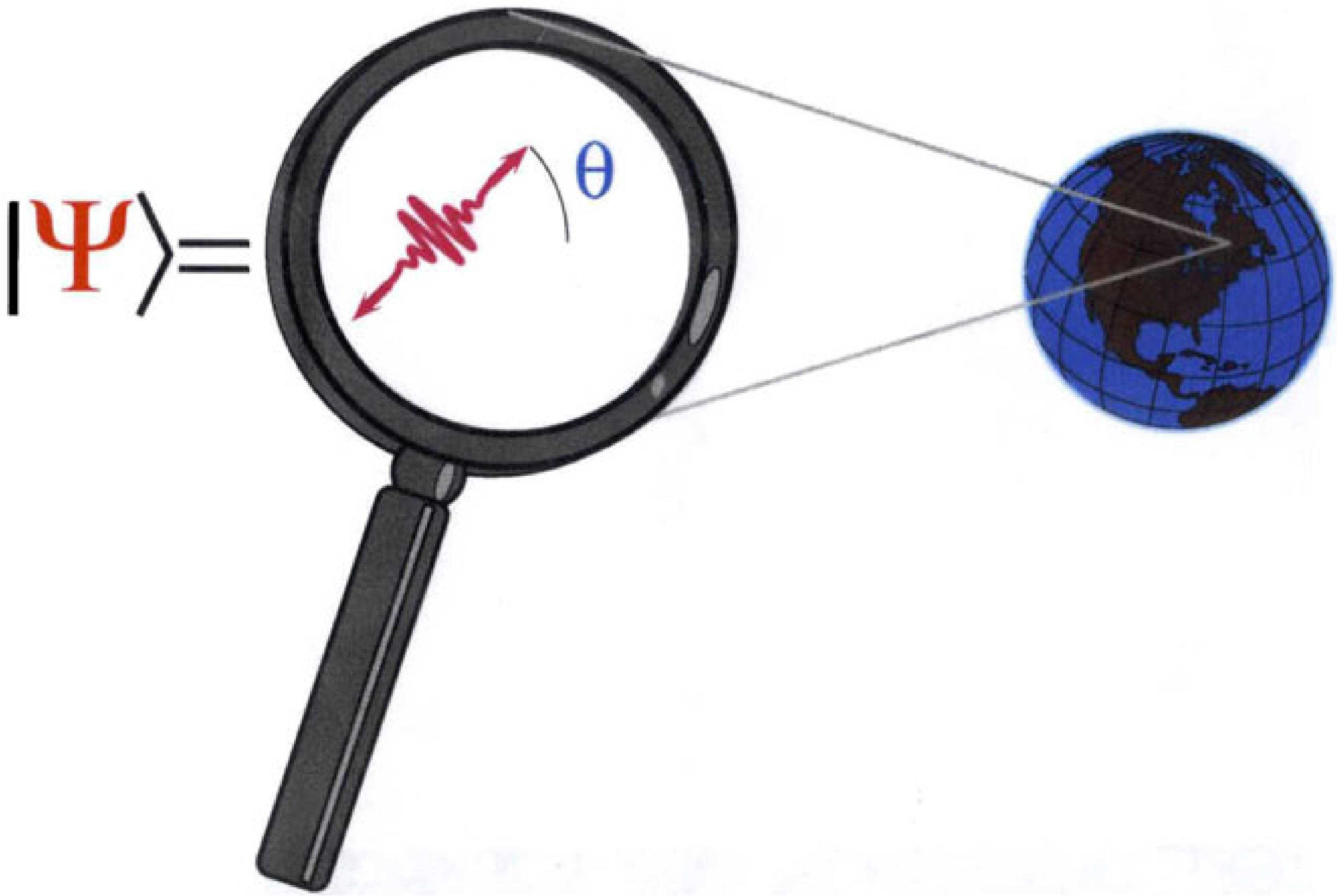
Brown Dwarfs

Stars that fizzled fill the galaxy

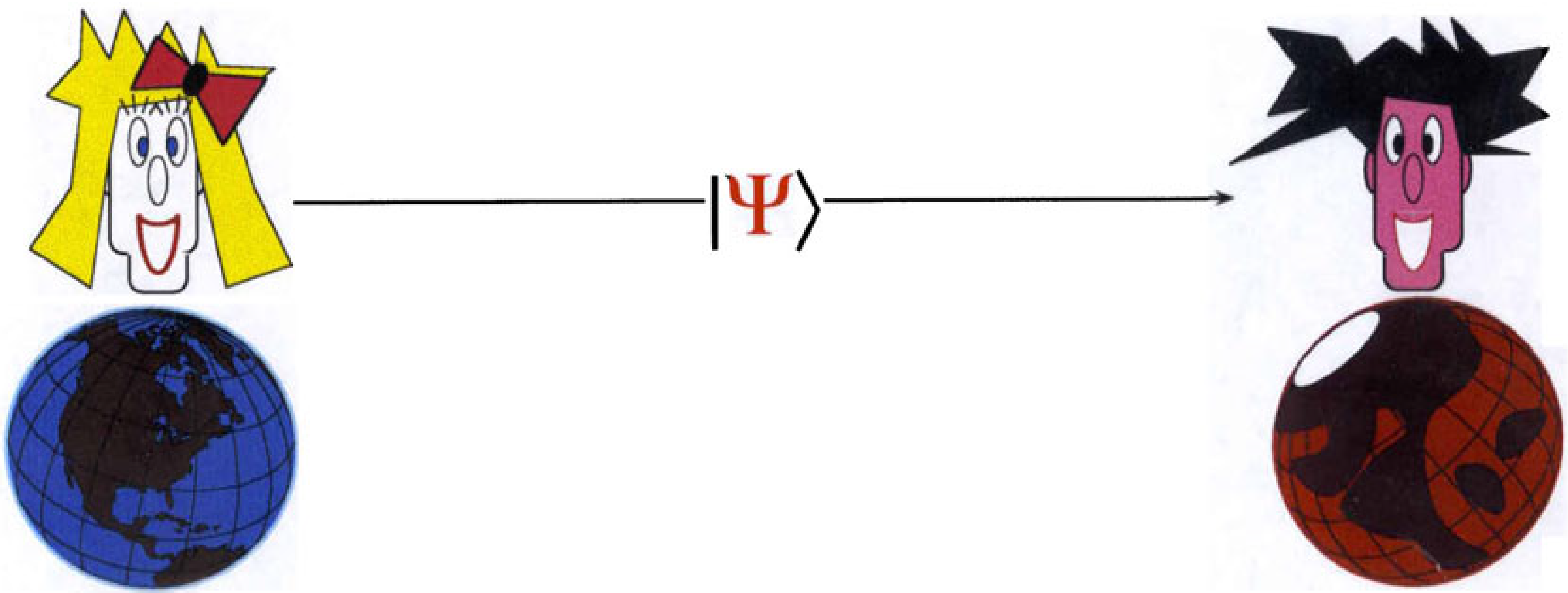
Quantum Information

- Cannot be cloned or copied
- Cannot be broadcast
- Cannot be measured reliably
- Is disturbed by observation
- Sometimes appears to propagate instantaneously
- Can exist in superposition of classical states

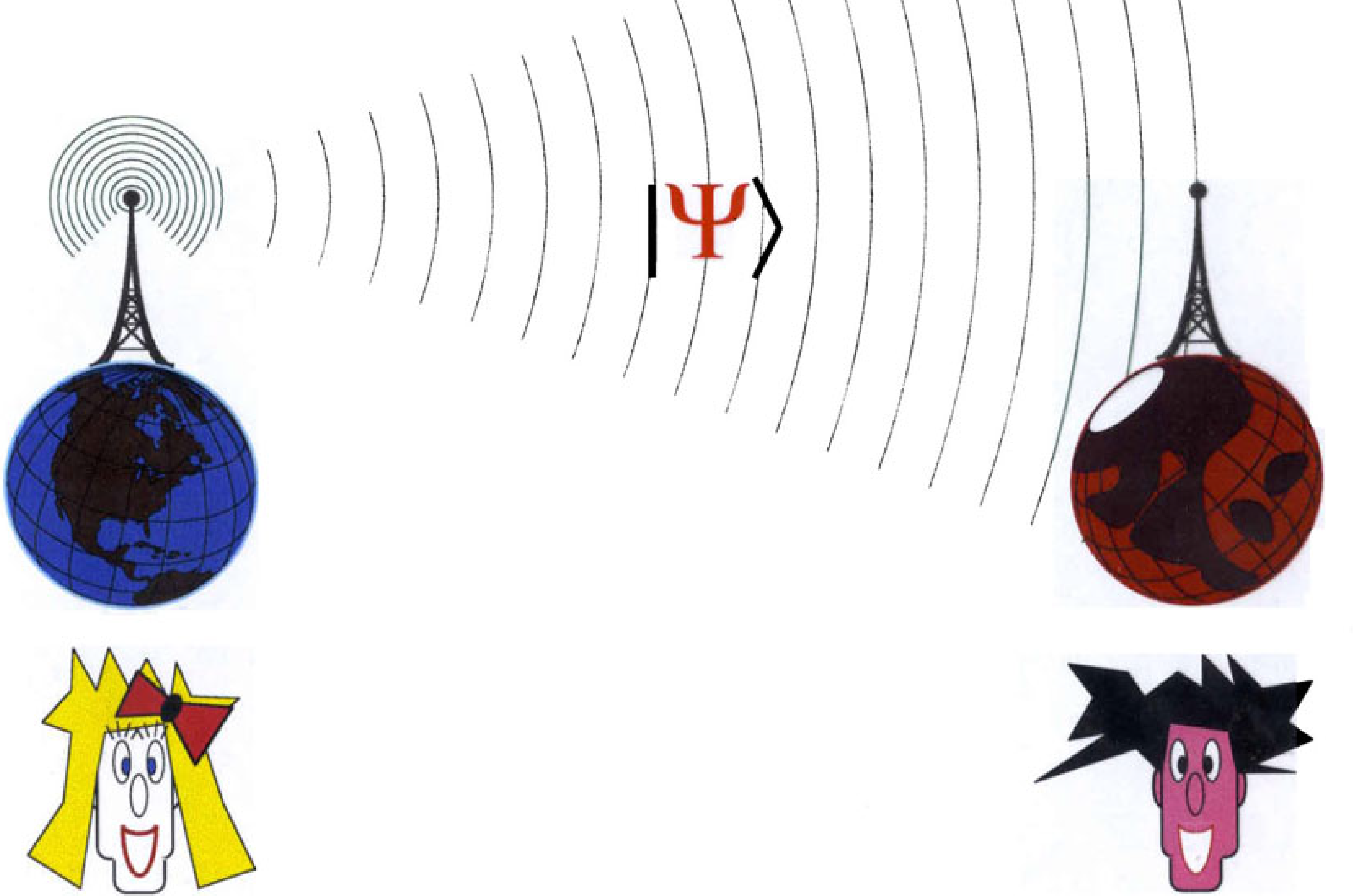
Unknown Quantum State $|\Psi\rangle$



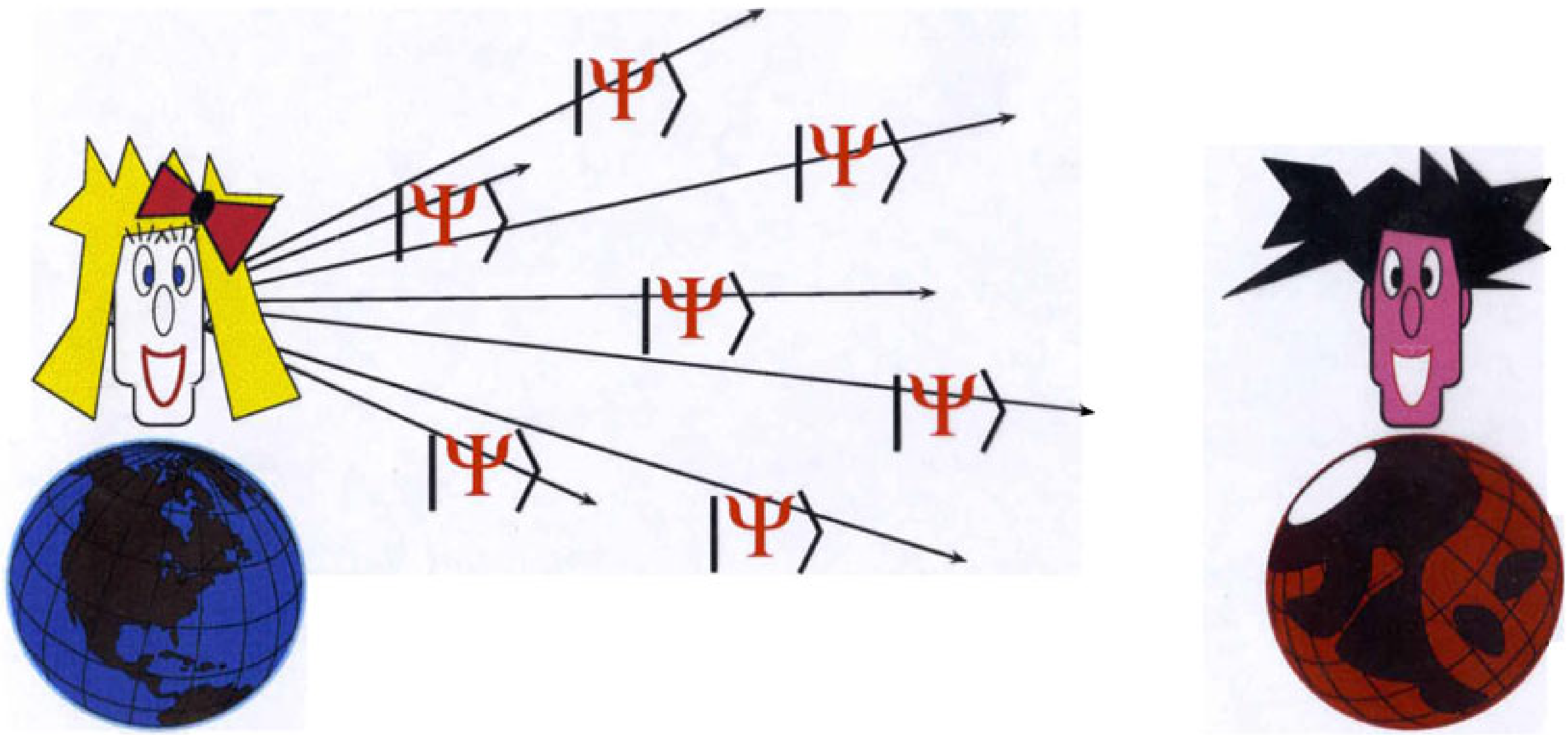
Unknown Quantum State $|\Psi\rangle$



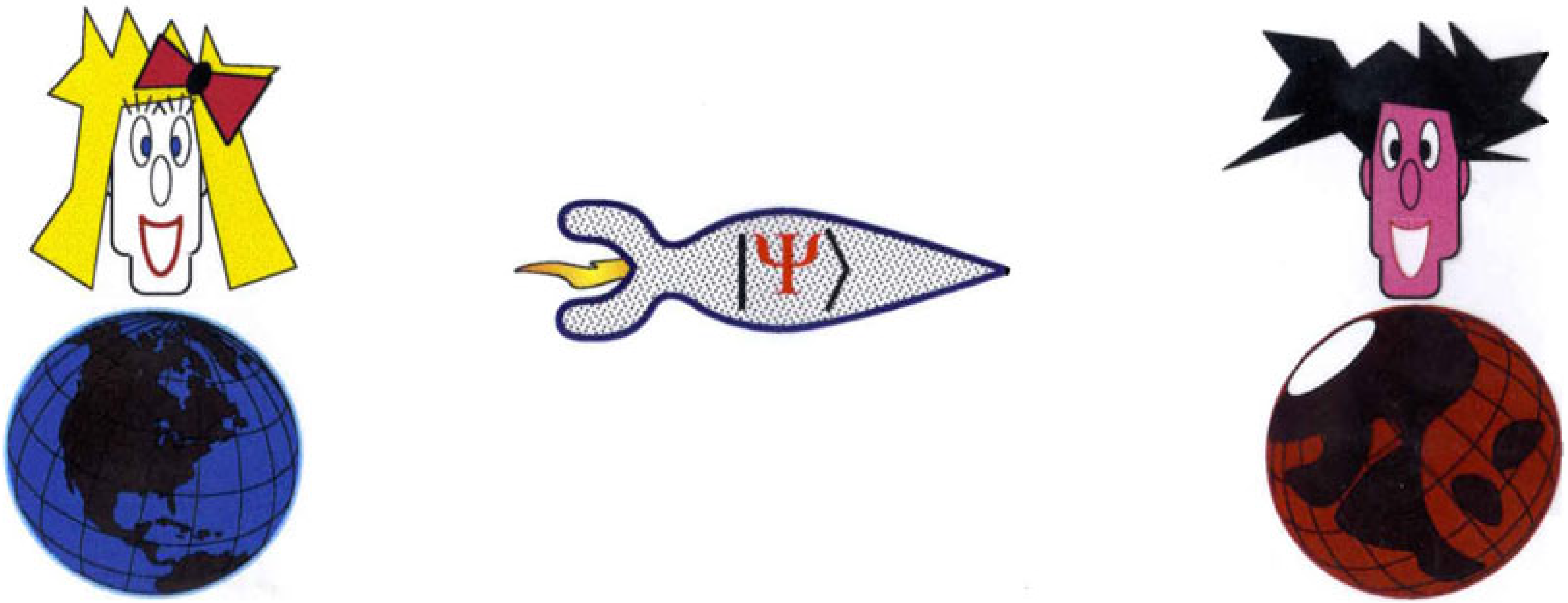
Impossibility of Measuring $|\Psi\rangle$



Impossibility of Cloning/Broadcasting $|\Psi\rangle$



Transmitting $|\Psi\rangle$ Physically



Noisy Channel

The Quantum Bit: Qubit

Classical bits: $|0\rangle$ and $|1\rangle$

Quantum bit: $|\Psi\rangle = \alpha|0\rangle + \beta|1\rangle$

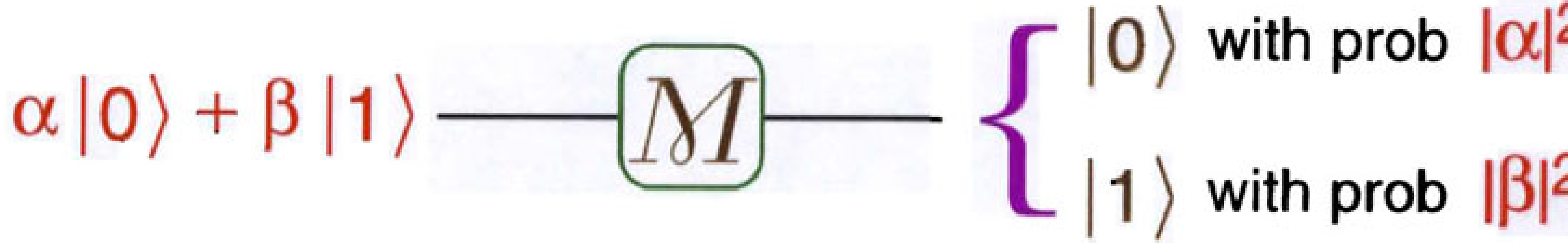
α and β are called *amplitudes*

$+$ denotes the *quantum superposition*

α and β are complex numbers

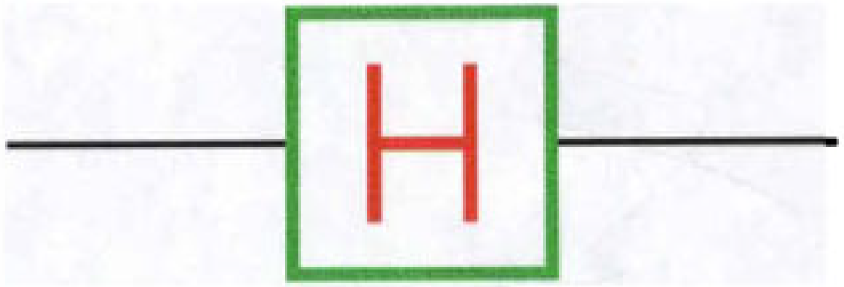
$$|\alpha|^2 + |\beta|^2 = 1$$

Measurement Gate



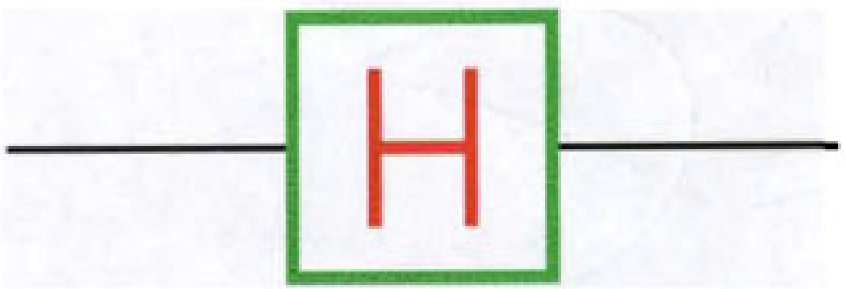
Walsh–Hadamard Transform

$|0\rangle$



$\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$

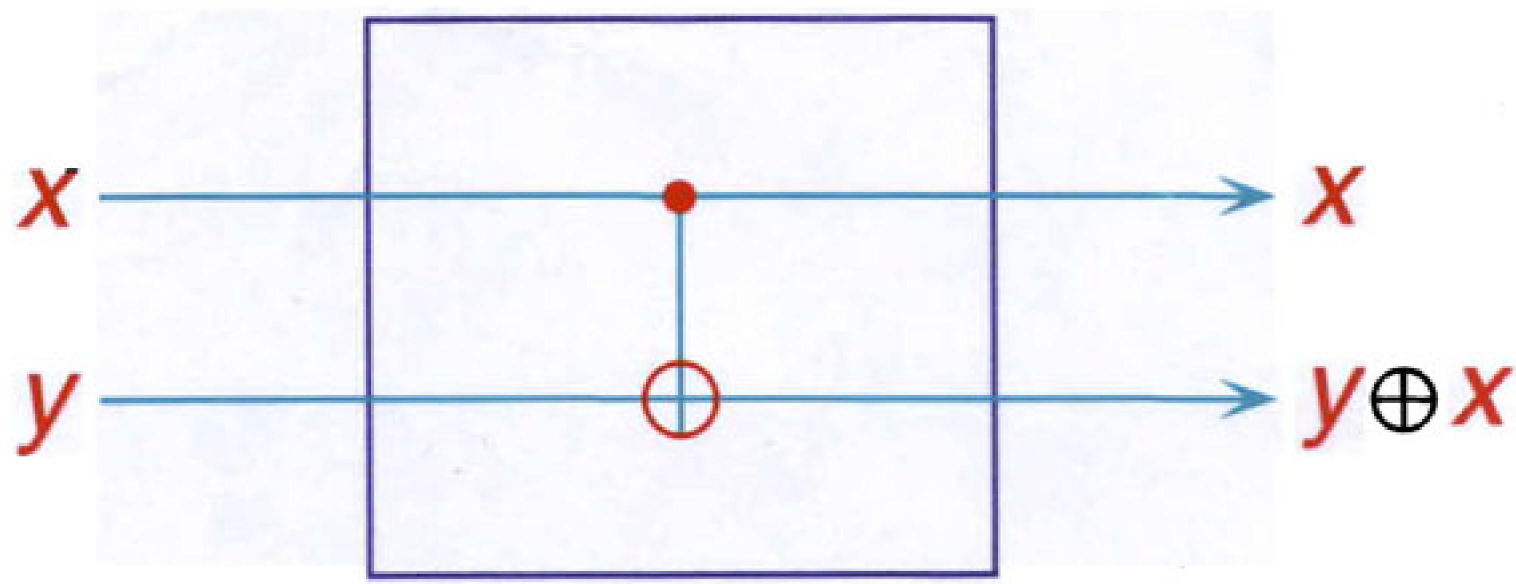
$|1\rangle$



$\frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle$

Quantum XOR

(Controlled NOT)

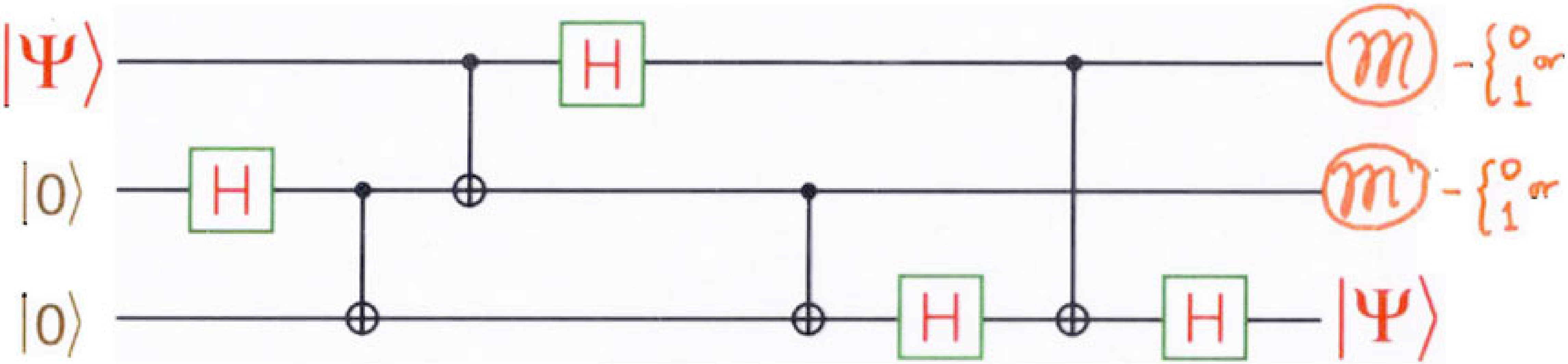


$$y \oplus x = \begin{cases} y & \text{if } x = 0 \\ 1-y & \text{if } x = 1 \end{cases}$$

$$|00\rangle \longrightarrow |00\rangle \qquad |10\rangle \longrightarrow |11\rangle$$

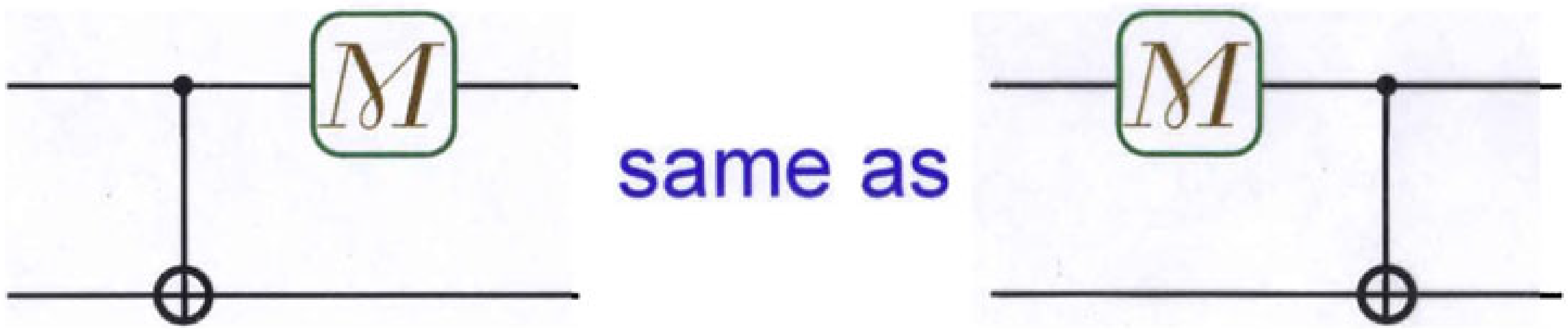
$$|01\rangle \longrightarrow |01\rangle \qquad |11\rangle \longrightarrow |10\rangle$$

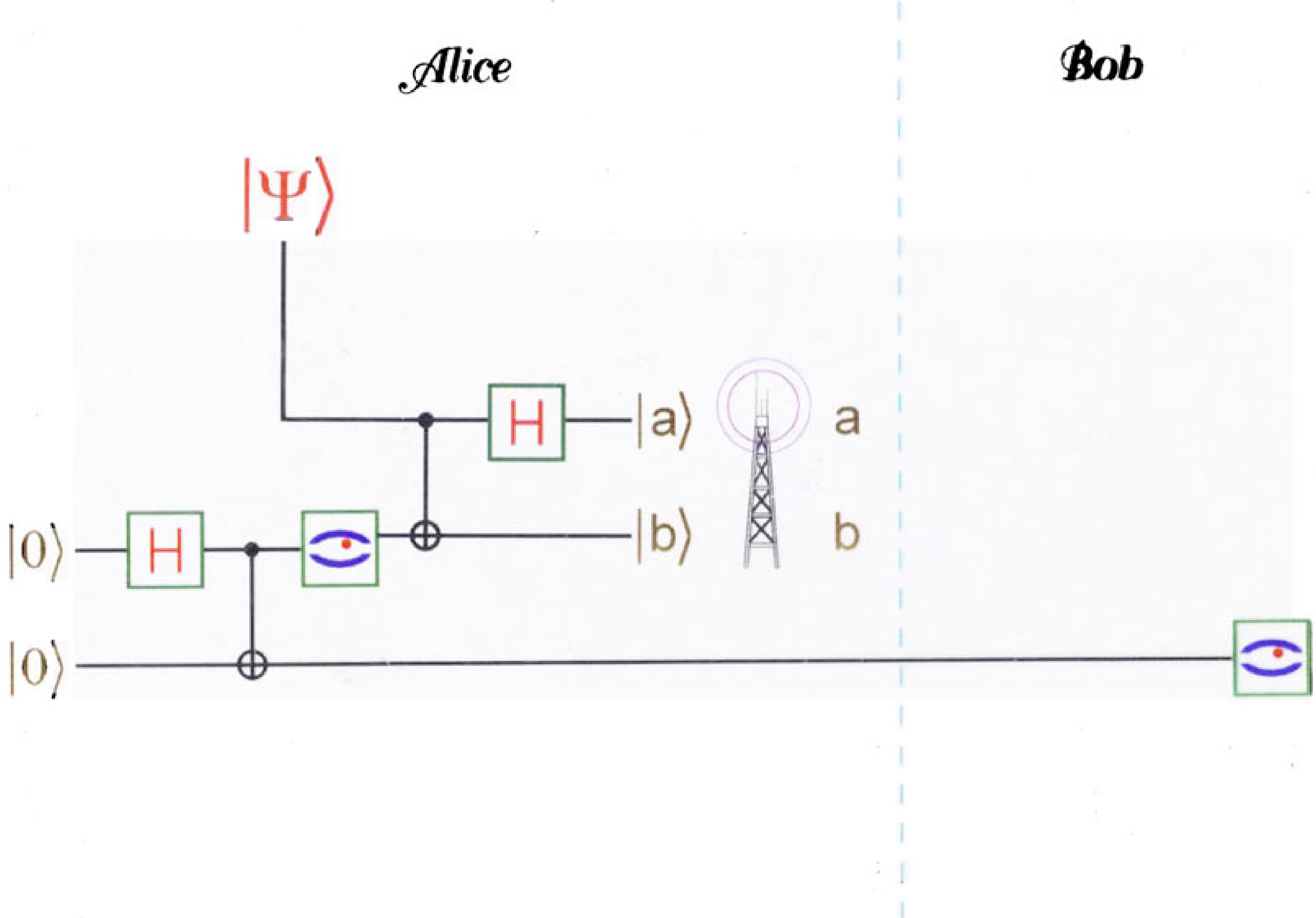
Quantum Circuit



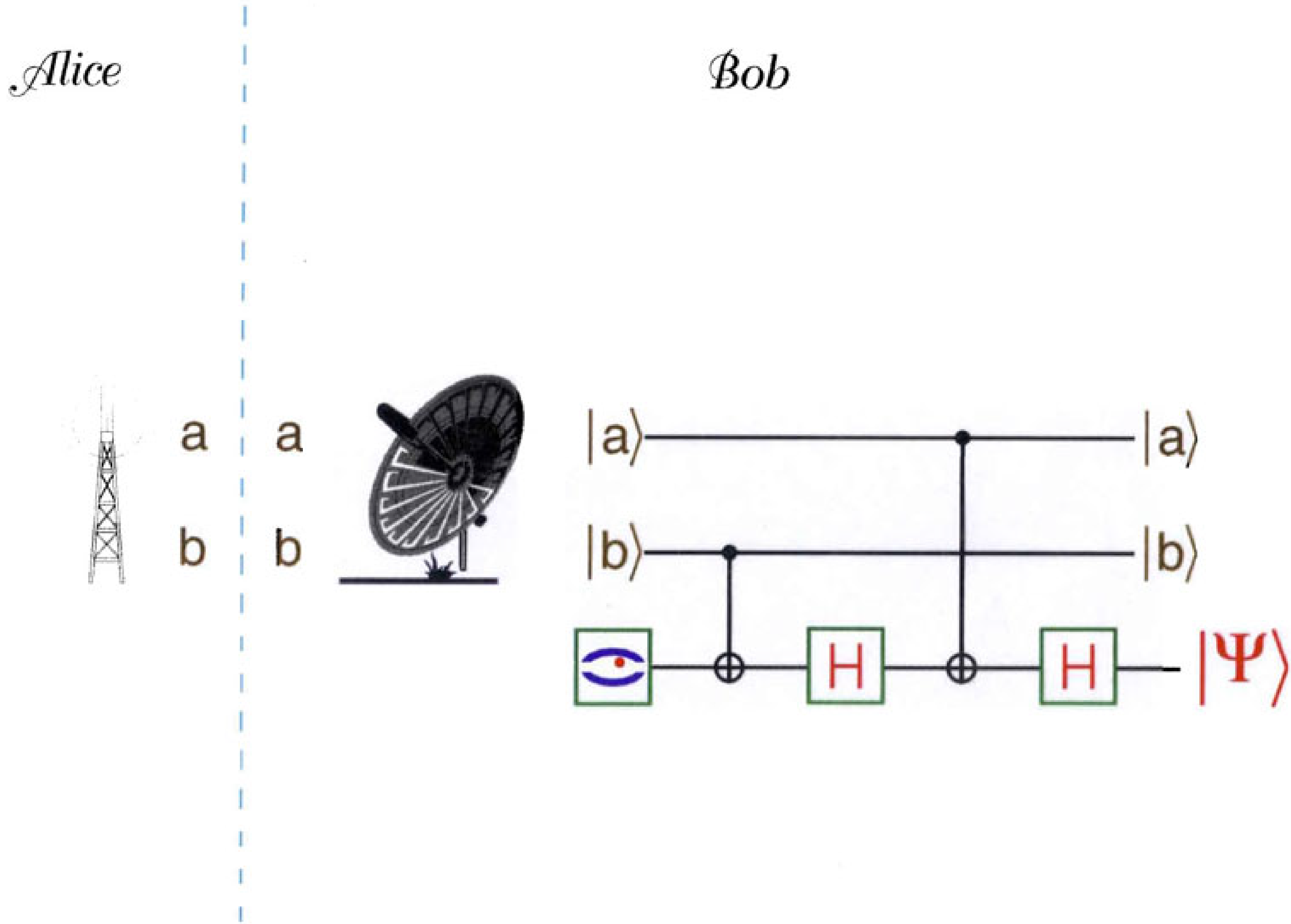
To Measure or not to Measure

Robert Griffiths and Chi–Sheng Niu

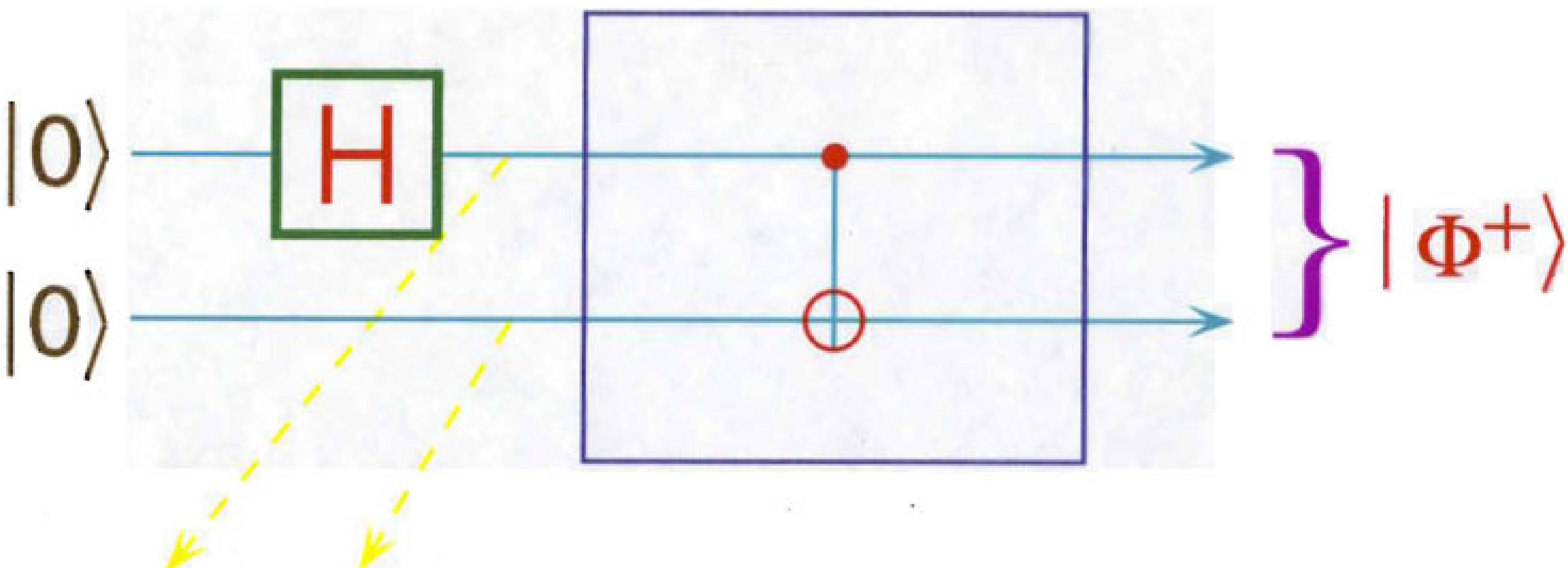




Alice
Bob
|Ψ⟩
H
|a⟩
a
|0⟩
H
|b⟩
b
|0⟩

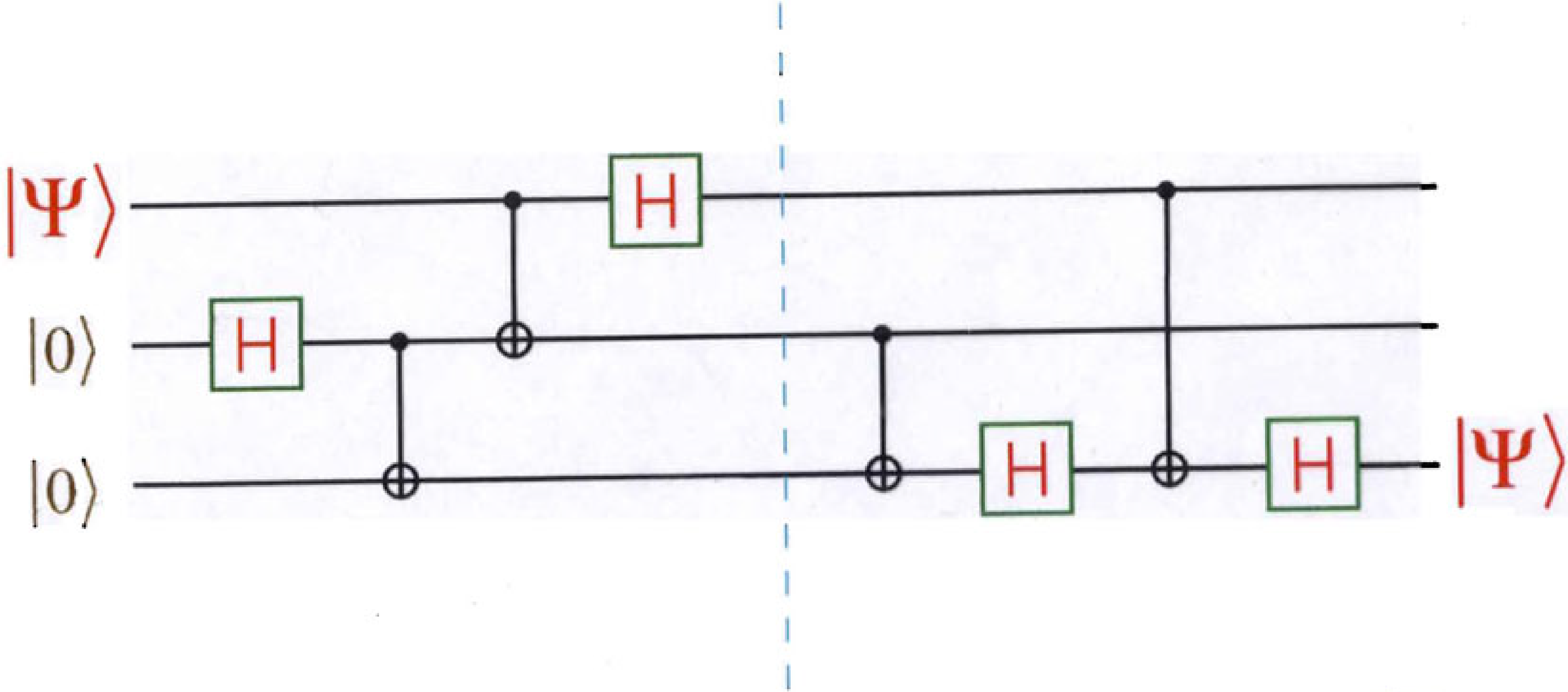

Alice
Bob
a
b
a
b
|a⟩
|b⟩
H
H
|a⟩
|b⟩
|Ψ⟩

Creation of Entanglement



$(|0\rangle + |1\rangle)\,|0\rangle$
$= |00\rangle + |10\rangle \longrightarrow |00\rangle + |11\rangle$

(up to normalization)



$$\left(|00\rangle|\Psi\rangle + |01\rangle N|\Psi\rangle + |10\rangle F|\Psi\rangle + |11\rangle NF|\Psi\rangle \right) / 2$$

$$N \begin{cases} |0\rangle \rightarrow |1\rangle \\ |1\rangle \rightarrow |0\rangle \end{cases} \qquad F \begin{cases} |0\rangle \rightarrow |0\rangle \\ |1\rangle \rightarrow -|1\rangle \end{cases}$$

Measurement Gate

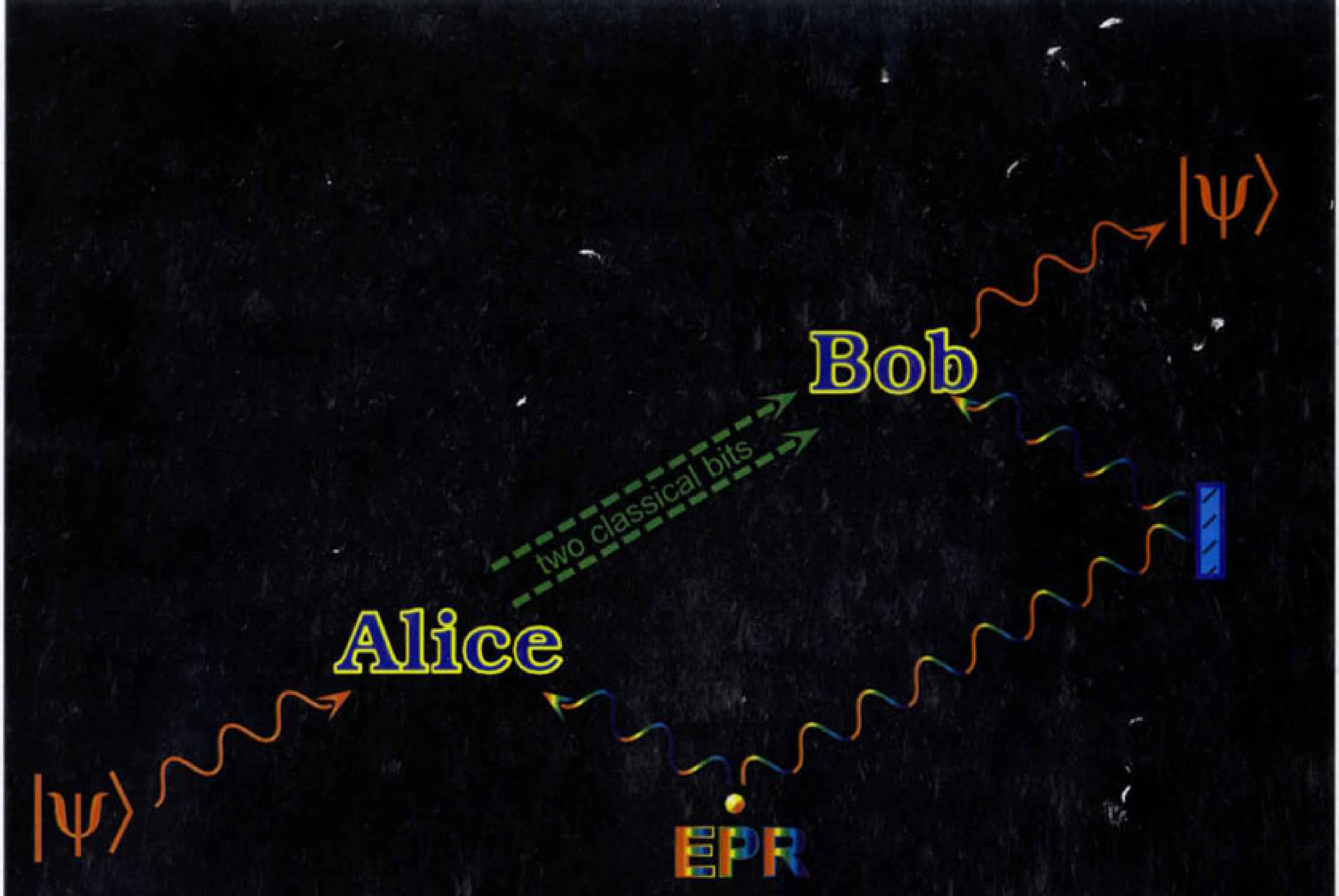
$\alpha|0\rangle + \beta|1\rangle$

$$\begin{cases} |0\rangle & \text{with prob } |\alpha|^2 \\ |1\rangle & \text{with prob } |\beta|^2 \end{cases}$$

Same if input is entangled within a system in state

$$\alpha|0\rangle|\Phi\rangle + \beta|1\rangle|\Psi\rangle$$

for arbitrary normalized $|\Phi\rangle$ and $|\Psi\rangle$


|ψ⟩
Alice
two classical bits
Bob
|ψ⟩
EPR

Quantum Entanglement Purification

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Purification of Noisy Entanglement and Faithful Teleportation via Noisy Channels

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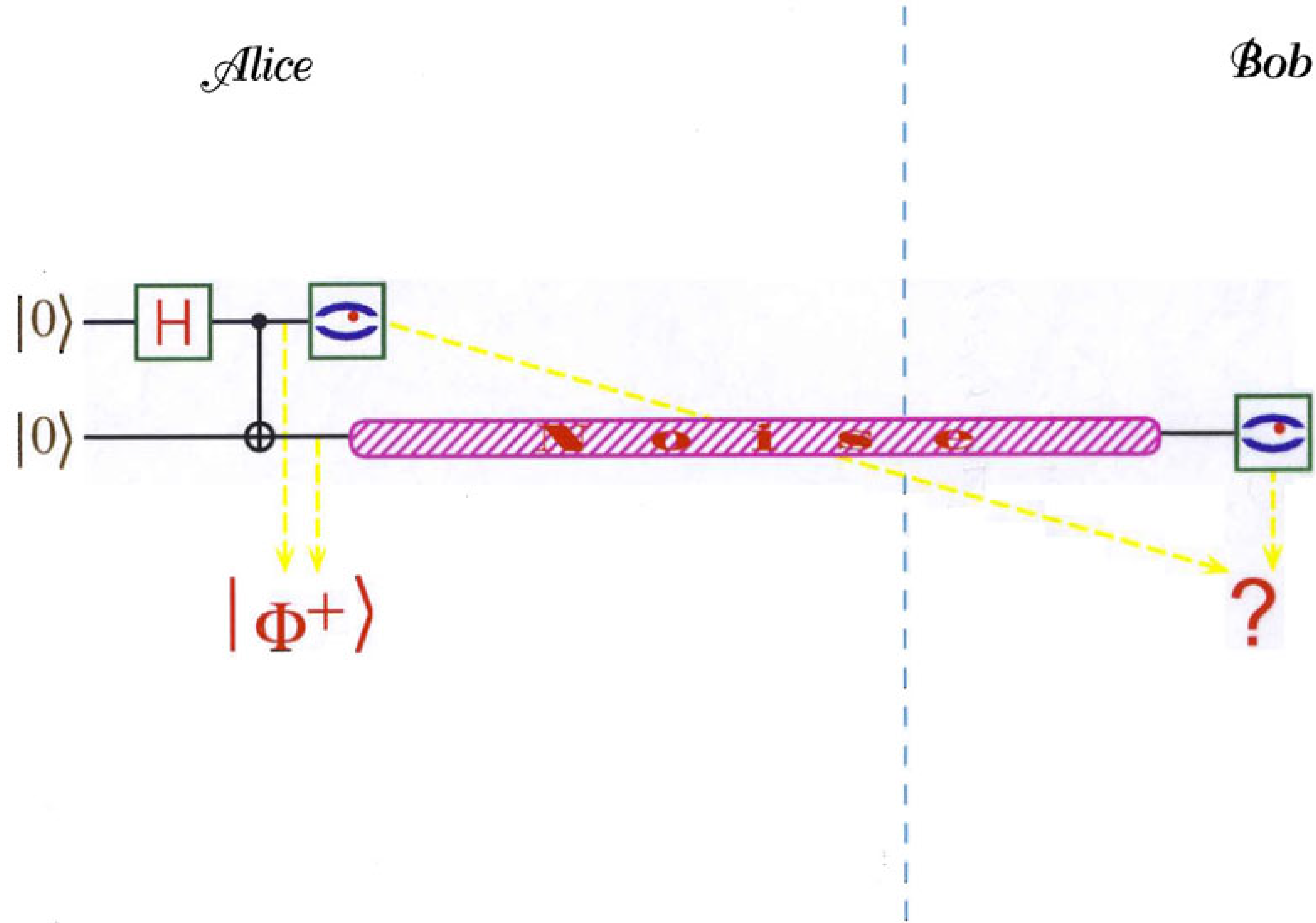
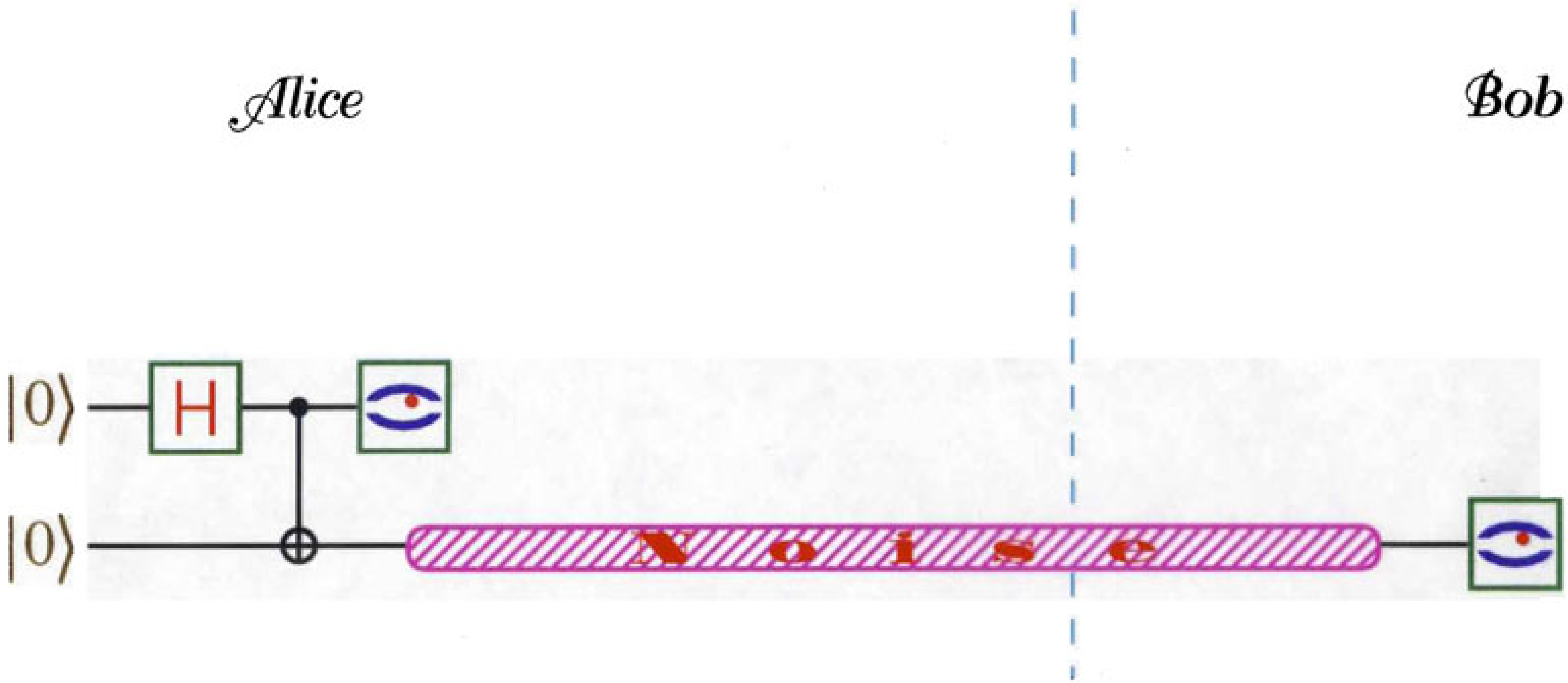
(Received 24 April 1995)

Two separated observers, by applying local operations to a supply of not-too-impure entangled states (e.g., singlets shared through a noisy channel), can prepare a smaller number of entangled pairs of arbitrarily high purity (e.g., near-perfect singlets). These can then be used to faithfully teleport unknown quantum states from one observer to the other, thereby achieving faithful transmission of quantum information through a noisy channel. We give upper and lower bounds on the yield $D(M)$ of pure singlets ($|\Psi^-\rangle$) distillable from mixed states M, showing $D(M) > 0$ if $\langle\Psi^-|M|\Psi^-\rangle > \frac{1}{2}$.

PACS numbers: 03.65.Bz, 42.50.Dv, 89.70.+c



Alice
Bob
|0⟩
H
|0⟩
Noise
|Φ+⟩
?



Perfect with probability 1/2

Randomized with prob 1/2

Bell States

$$|\Psi^-\rangle = |10\rangle - |01\rangle$$

$$|\Psi^+\rangle = |10\rangle + |01\rangle$$

$$|\Phi^-\rangle = |11\rangle - |00\rangle$$

$$|\Phi^+\rangle = |11\rangle + |00\rangle$$

(up to normalization)

Note: only $|\Psi^-\rangle$ is basis independent

Werner State

$$\frac{1}{2}|\Phi^+\rangle\langle\Phi^+| + \frac{1}{2}\Big[\frac{1}{4}|00\rangle\langle 00| + \frac{1}{4}|01\rangle\langle 10|$$

$$+\frac{1}{4}|10\rangle\langle 10| + \frac{1}{4}|11\rangle\langle 11|\Big]$$

$$\frac{1}{4}|00\rangle\langle 00| + \frac{1}{4}|01\rangle\langle 10|$$

$$+\frac{1}{4}|10\rangle\langle 10| + \frac{1}{4}|11\rangle\langle 11|$$

$$=$$

$$\frac{1}{4}|\Phi^{+}\rangle\langle \Phi^{+}| + \frac{1}{4}|\Phi^{-}\rangle\langle \Phi^{-}|$$

$$+\frac{1}{4}|\Psi^{-}\rangle\langle \Psi^{-}| + \frac{1}{4}|\Psi^{+}\rangle\langle \Psi^{+}|$$

Density Matrices

$$|W_{5/8}\rangle = \frac{5}{8}|\Phi^+\rangle\langle\Phi^+| + \frac{1}{8}|\Phi^-\rangle\langle\Phi^-|$$
$$+ \frac{1}{8}|\Psi^-\rangle\langle\Psi^-| + \frac{1}{8}|\Psi^+\rangle\langle\Psi^+|$$

$$=$$

$$\frac{1}{2}|\Phi^+\rangle\langle\Phi^+| + \frac{1}{2}\Big[\frac{1}{4}|\Phi^+\rangle\langle\Phi^+| + \frac{1}{4}|\Phi^-\rangle\langle\Phi^-|$$
$$+ \frac{1}{4}|\Psi^-\rangle\langle\Psi^-| + \frac{1}{4}|\Psi^+\rangle\langle\Psi^+|\Big]$$

Werner State

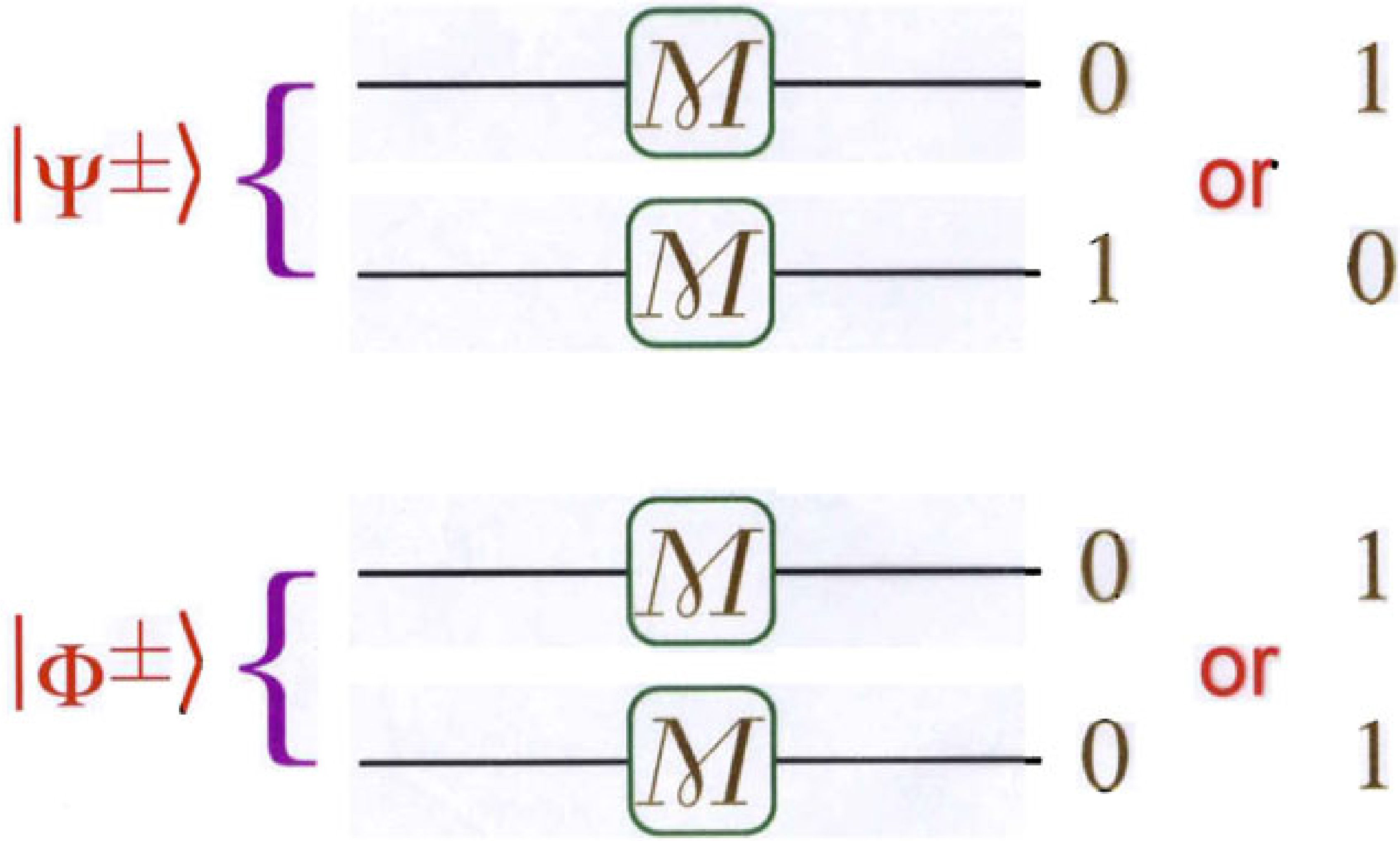
2 qubits jointly in state

$|\Phi^+\rangle$ with probability F

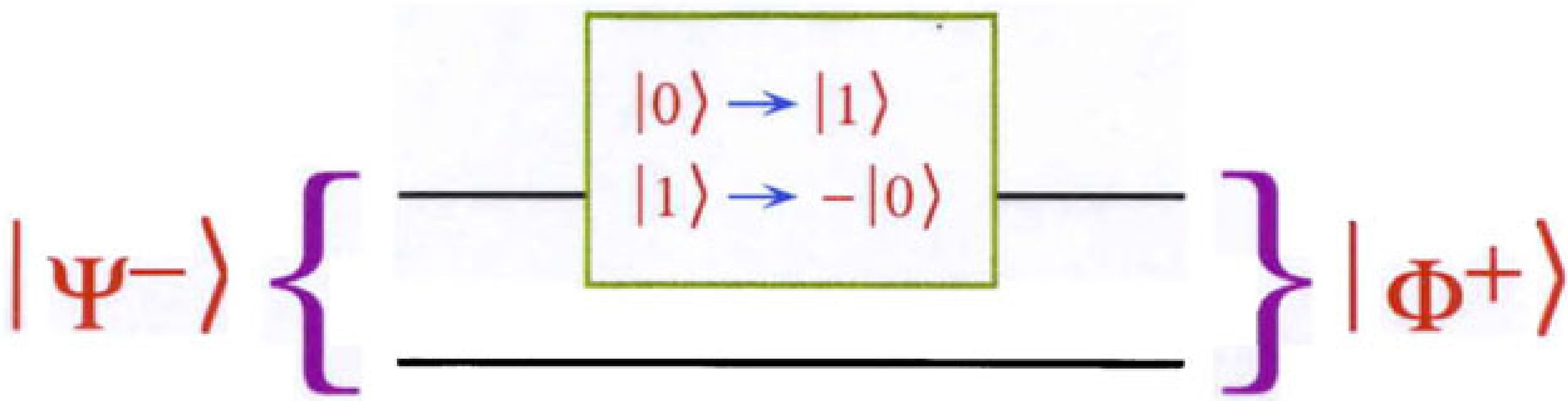
$|\Phi^-\rangle$, $|\Psi^+\rangle$, $|\Psi^-\rangle$ each with probability $\frac{1-F}{3}$

(F = 5/8)

Distinguishing Bell States



Manipulating Bell States

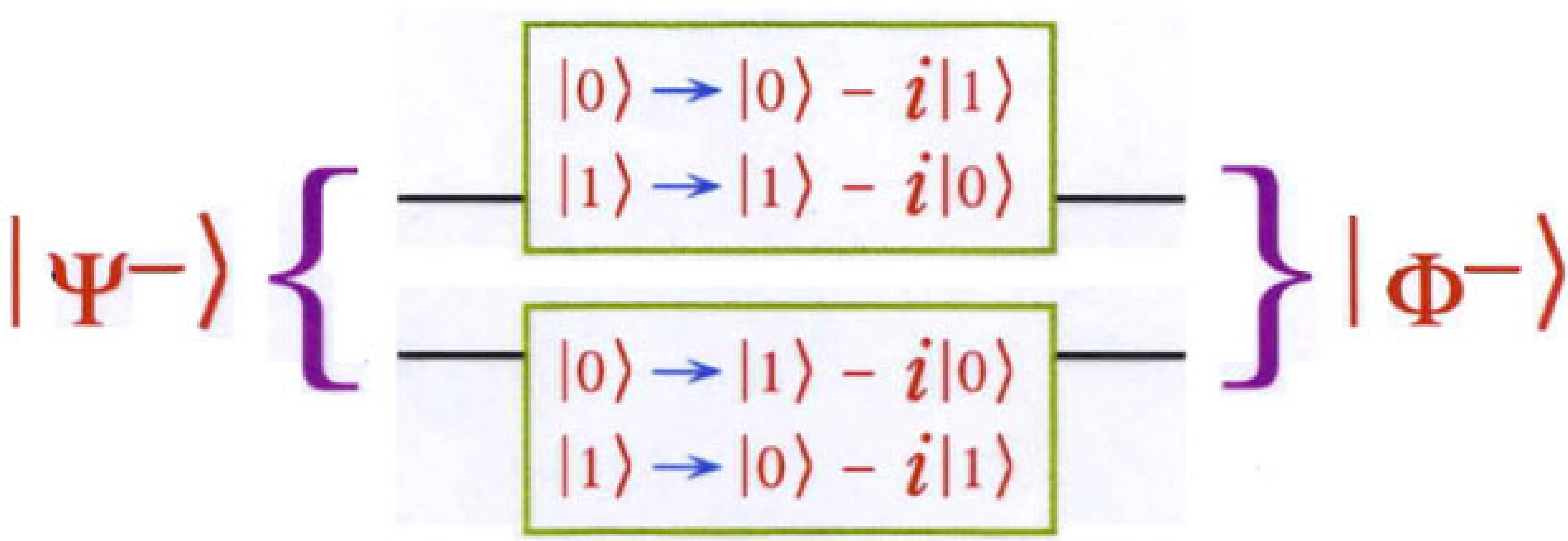


$|01\rangle - |10\rangle$ $\qquad\qquad$ $|11\rangle + |00\rangle$

$$|\Psi^-\rangle \longleftrightarrow |\Phi^+\rangle$$

$$|\Psi^+\rangle \longleftrightarrow |\Phi^-\rangle$$

Manipulating Bell States

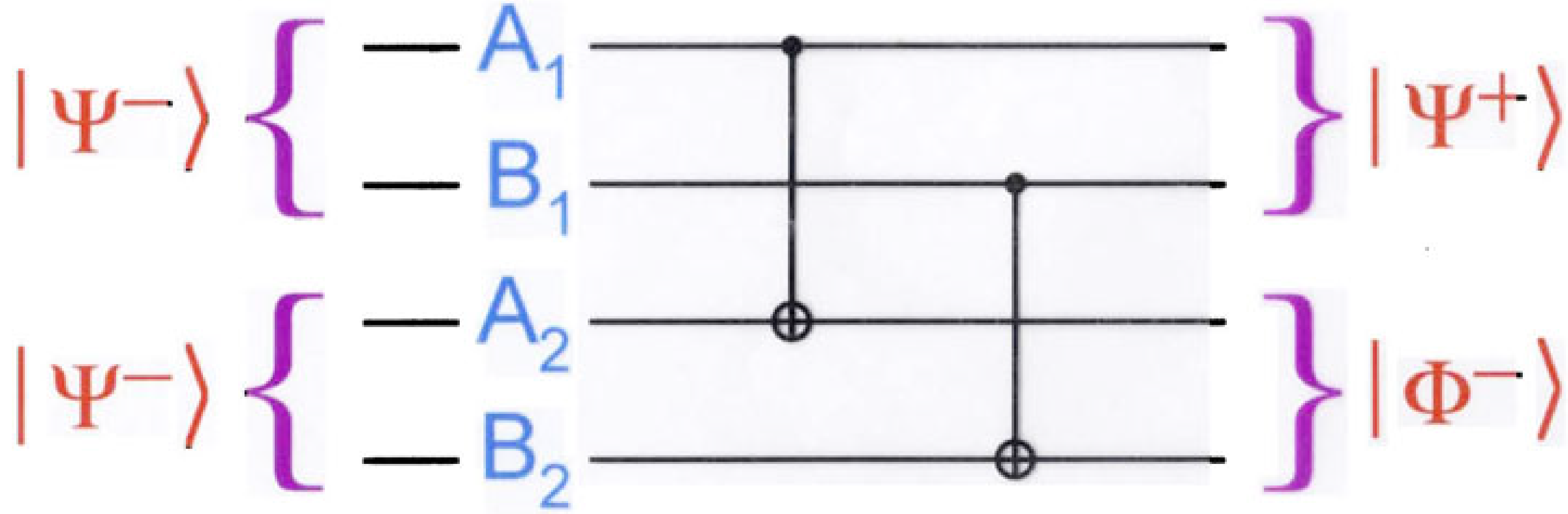


$$|\Psi^-\rangle \longleftrightarrow |\Phi^-\rangle$$

$|\Psi^+\rangle$ and $|\Phi^+\rangle$ are unaffected

(up to phase)

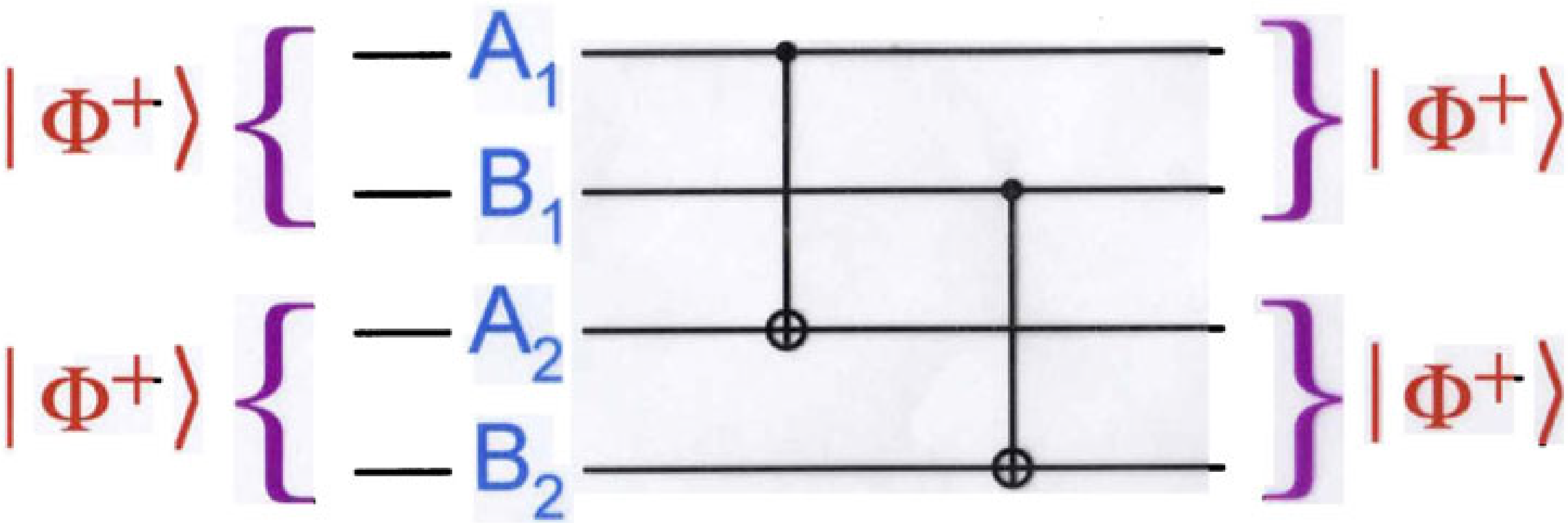
Bilateral XOR



$$(|01\rangle - |10\rangle)(|01\rangle - |10\rangle) =$$
$$|0101\rangle - |0110\rangle - |1001\rangle + |1010\rangle$$

$$\rightarrow |0100\rangle - |0111\rangle - |1011\rangle + |1000\rangle$$
$$= (|01\rangle + |10\rangle)(|00\rangle - |11\rangle)$$

Bilateral XOR



$$(|00\rangle + |11\rangle)(|00\rangle + |11\rangle) =$$
$$|0000\rangle + |0011\rangle + |1100\rangle + |1111\rangle$$

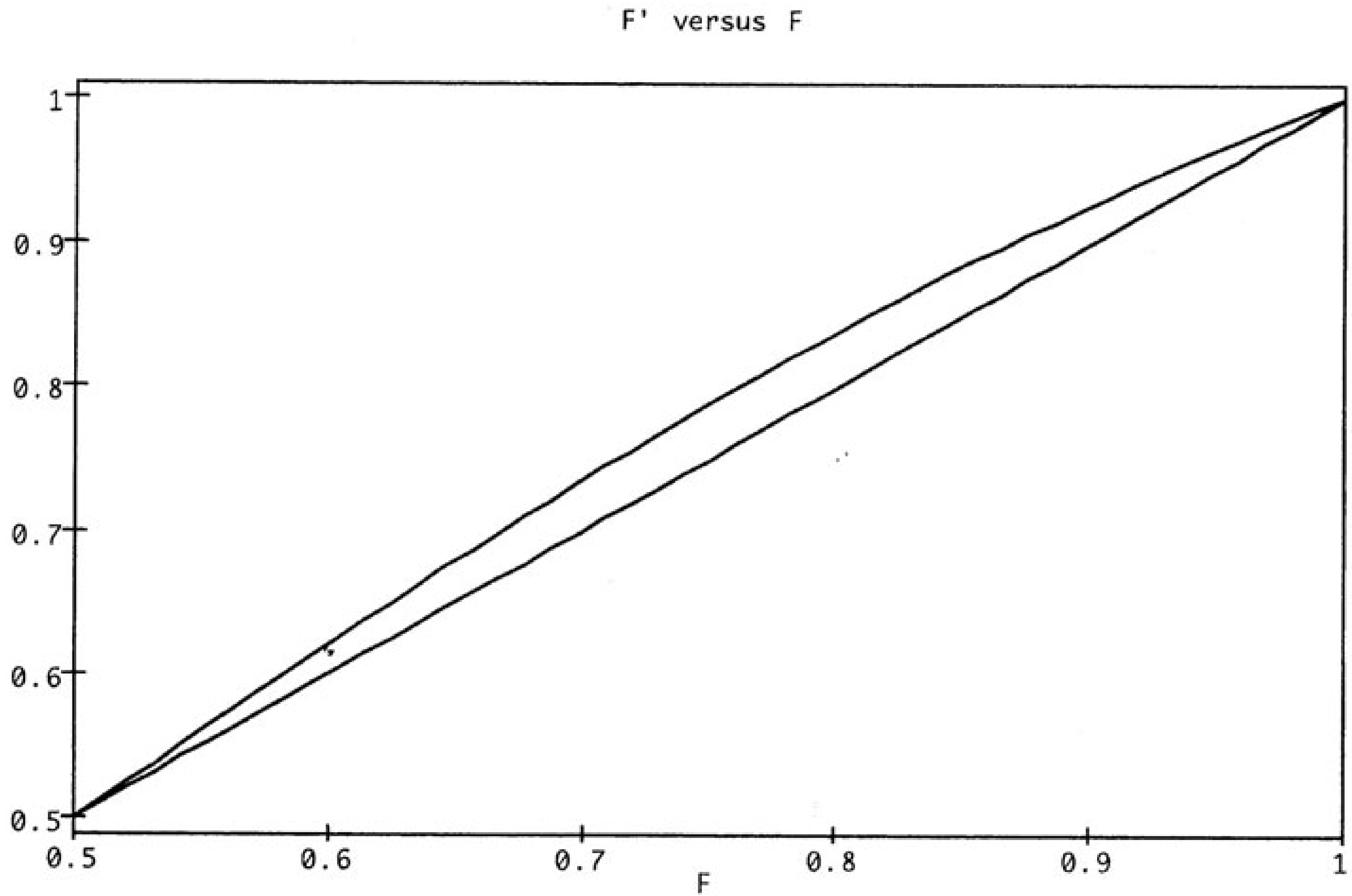
$$|0000\rangle + |0011\rangle + |1111\rangle + |1100\rangle$$
$$= (|00\rangle + |11\rangle)(|00\rangle + |11\rangle)$$

before		probability	after		target	example
source	target		source	target	$= \lvert\Phi\rangle$?	$F = 5/8$
$\lvert\Phi^+\rangle$	$\lvert\Phi^+\rangle$	F^2	$\lvert\Phi^+\rangle$	$\lvert\Phi^+\rangle$	YES	25/64
$\lvert\Phi^+\rangle$	$\lvert\Phi^-\rangle$	$F(1-F)/3$	$\lvert\Phi^-\rangle$	$\lvert\Phi^-\rangle$	YES	5/64
$\lvert\Phi^+\rangle$	$\lvert\Psi^+\rangle$	$F(1-F)/3$	$\lvert\Phi^+\rangle$	$\lvert\Psi^+\rangle$		5/64
$\lvert\Phi^+\rangle$	$\lvert\Psi^-\rangle$	$F(1-F)/3$	$\lvert\Phi^-\rangle$	$\lvert\Psi^-\rangle$		5/64
$\lvert\Phi^-\rangle$	$\lvert\Phi^+\rangle$	$F(1-F)/3$	$\lvert\Phi^-\rangle$	$\lvert\Phi^+\rangle$	YES	5/64
$\lvert\Phi^-\rangle$	$\lvert\Phi^-\rangle$	$(1-F)^2/9$	$\lvert\Phi^+\rangle$	$\lvert\Phi^-\rangle$	YES	1/64
$\lvert\Phi^-\rangle$	$\lvert\Psi^+\rangle$	$(1-F)^2/9$	$\lvert\Phi^-\rangle$	$\lvert\Psi^+\rangle$		1/64
$\lvert\Phi^-\rangle$	$\lvert\Psi^-\rangle$	$(1-F)^2/9$	$\lvert\Phi^+\rangle$	$\lvert\Psi^-\rangle$		1/64
$\lvert\Psi^+\rangle$	$\lvert\Phi^+\rangle$	$F(1-F)/3$	$\lvert\Psi^+\rangle$	$\lvert\Psi^+\rangle$		5/64
$\lvert\Psi^+\rangle$	$\lvert\Phi^-\rangle$	$(1-F)^2/9$	$\lvert\Psi^-\rangle$	$\lvert\Psi^-\rangle$		1/64
$\lvert\Psi^+\rangle$	$\lvert\Psi^+\rangle$	$(1-F)^2/9$	$\lvert\Psi^+\rangle$	$\lvert\Phi^+\rangle$	YES	1/64
$\lvert\Psi^+\rangle$	$\lvert\Psi^-\rangle$	$(1-F)^2/9$	$\lvert\Psi^-\rangle$	$\lvert\Phi^-\rangle$	YES	1/64
$\lvert\Psi^-\rangle$	$\lvert\Phi^+\rangle$	$F(1-F)/3$	$\lvert\Psi^-\rangle$	$\lvert\Psi^+\rangle$		5/64
$\lvert\Psi^-\rangle$	$\lvert\Phi^-\rangle$	$(1-F)^2/9$	$\lvert\Psi^+\rangle$	$\lvert\Psi^-\rangle$		1/64
$\lvert\Psi^-\rangle$	$\lvert\Psi^+\rangle$	$(1-F)^2/9$	$\lvert\Psi^-\rangle$	$\lvert\Phi^+\rangle$	YES	1/64
$\lvert\Psi^-\rangle$	$\lvert\Psi^-\rangle$	$(1-F)^2/9$	$\lvert\Psi^+\rangle$	$\lvert\Phi^-\rangle$	YES	1/64
Prob keep		P				40/64
Keep and $\lvert\Phi^+\rangle$		Q				26/64
$\lvert\Phi^+\rangle$ if kept		$F' = Q/P$				13/20

$$P = F^2 + \frac{2F(1-F)}{3} + \frac{5(1-F)^2}{9} = \frac{8F^2 - 4F + 5}{9}$$

$$Q = F^2 + \frac{(1-F)^2}{9} = \frac{8F^2 - 4F + 5}{9}$$

$$F' = \frac{10F^2 - 2F + 1}{8F^2 - 4F + 5} > F \quad \text{provided } F > 1/2$$



F' versus F
1
0.9
0.8
0.7
0.6
0.5
0.5
0.6
0.7
0.8
0.9
1
F

Repeat Process?

Round	Fidelity	
0	62.5%	(5/8)
1	65%	(13/20)
2	59.1463...%	
3	51.9430...%	

What's Wrong?

After First Round:

$|\Phi^+\rangle$ with probability 13/20

$|\Phi^-\rangle$ with probability 1/4

$|\Psi^+\rangle$ with probability 1/20

$|\Psi^-\rangle$ with probability 1/20

Not a Werner State!

Solution (1)

Wernerize between rounds

$|\Psi^+\rangle \longleftrightarrow |\Psi^-\rangle$ or

$|\Psi^-\rangle \longleftrightarrow |\Phi^-\rangle$

or $|\Phi^-\rangle \longleftrightarrow |\Psi^+\rangle$

each with probability 1/3

After Wernerizing First Round:

$|\Phi^+\rangle$ with probability 13/20

$|\Phi^-\rangle$, $|\Psi^+\rangle$, $|\Psi^-\rangle$ each with probability 7/60

Result of Wernerization

Round	Fidelity	Rate
0	62.5%	100.00%
1	65.0%	31.25%
2	67.9%	10.03%
3	71.2%	3.33%
4	74.8%	1.15%
...	...	...
15	99.1%	1.29×10^{-6}

Solution (2)

Macchiavellize between rounds

$$|\Psi^-\rangle \longleftrightarrow |\Phi^-\rangle$$

Round	Fidelity	Rate
0	62.5%	100.00%
1	65.0%	31.25%
2	73.3%	9.06%
3	82.6%	2.96%
4	89.3%	1.13%
5	96.7%	0.47%
6	99.3%	0.22%

≈1/455