Decoherence in quantum walks

– is it useful? –

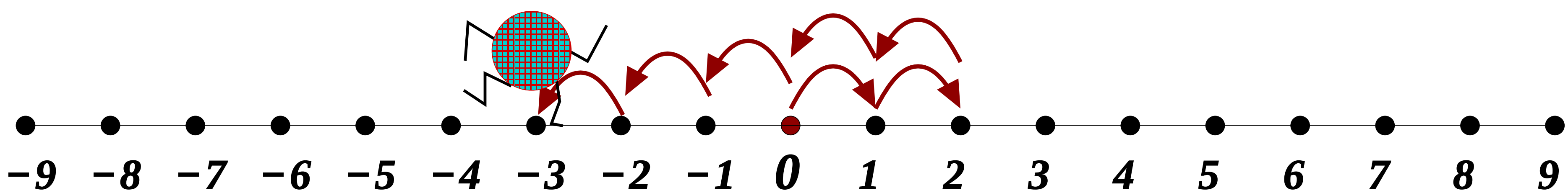
Viv Kendon

Ben Tregenna, Will Flanagan, Rik Maile

thanks to Julia Kempe, Peter Knight

`quant-ph/0209005`

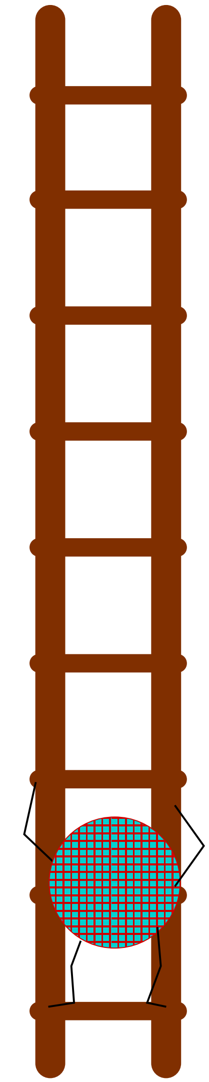
QOLS
Blackett Laboratory
Imperial College
London SW7 2BW
Viv.Kendon@ic.ac.uk



VK and BT funded by the UK Engineering and Physical Sciences Research Council.

Overview

1. classical random walk on a line
2. quantum "random" walk on a line
3. physical implementations of quantum walks
4. decoherence in quantum walk on a line
5. quantum walk on the N-cycle and mixing times
6. quantum walk on a hypercube and hitting times
7. summary



Classical random walk on a line

1. Start at the origin. *(discrete space and time)*

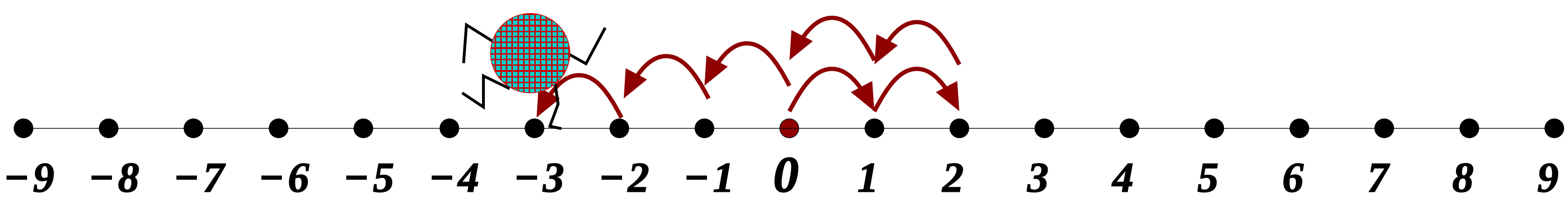
2. Toss a coin, move one unit right for heads, left for tails.

3. Repeat step 2. T times.

4. Record current position, $-T \leq x \leq T$.

Repeat steps 1. to 4. many times $\longrightarrow$ prob. dist. $P(x, T)$, binomial

standard deviation $\langle x^2 \rangle^{1/2} = \sqrt{T}$



Quantum "random" walk on a line

No straightforward quantum analogue of classical random walk.

- Pure quantum system evolves unitarily $\longrightarrow$ not random
- Quantum particle on lattice undergoing unitary evolution $\longrightarrow$ uninteresting translational motion [Meyer 1996]
- Several ways round this, here we choose a "quantum coin"

Quantum coin $\equiv$ 2-state quantum system (qubit), $|R\rangle$ and $|L\rangle$

Particle states $|x\rangle$ (with $x \in \mathbb{Z}$) position on line.

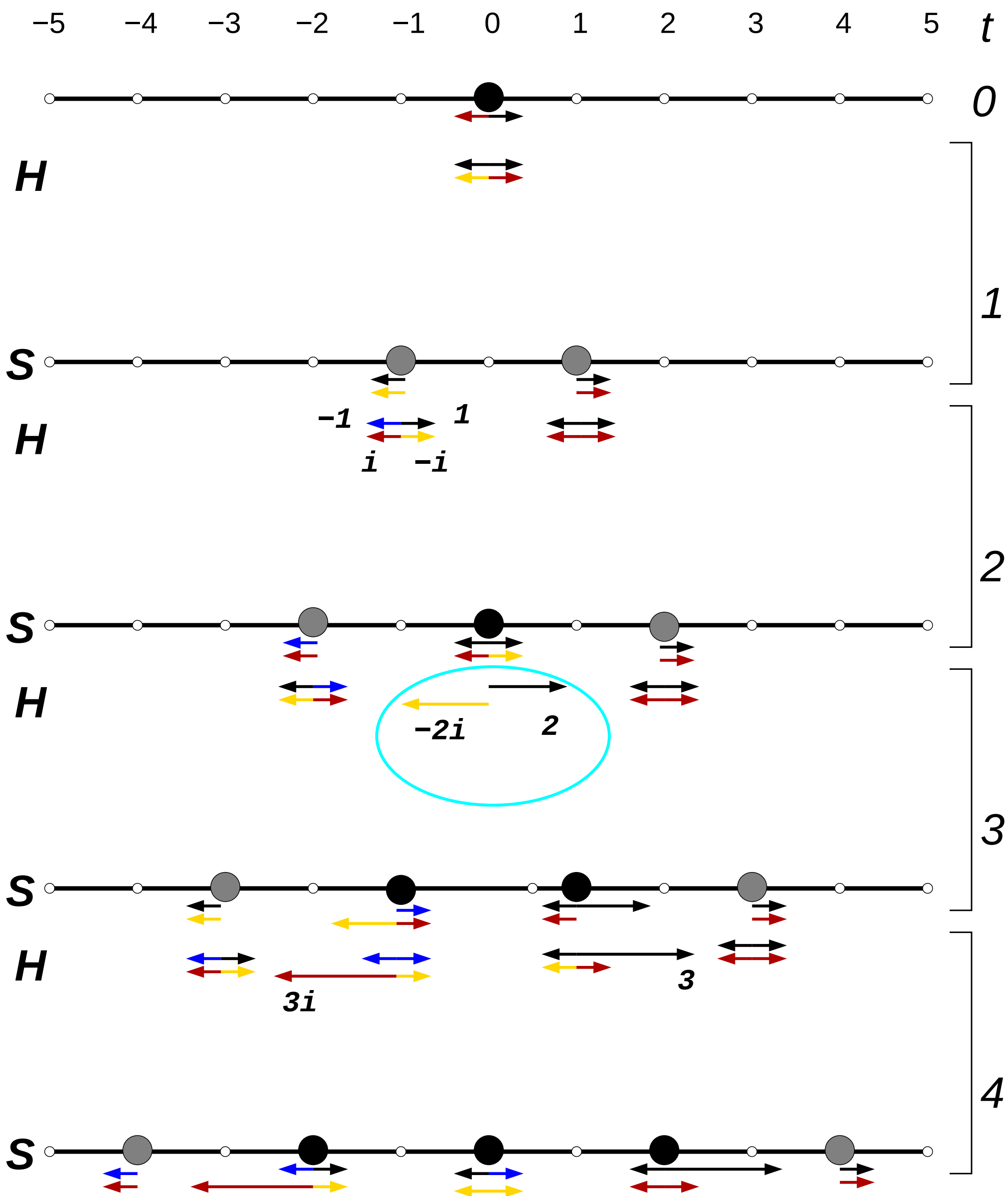
Flip coin $\equiv$ Hadamard rotation:

$$H|x, R\rangle = (|x, R\rangle + |x, L\rangle)/\sqrt{2}$$

$$H|x, L\rangle = (|x, R\rangle - |x, L\rangle)/\sqrt{2}$$

particle then shifts (S) one unit left/right conditioned on coin state:

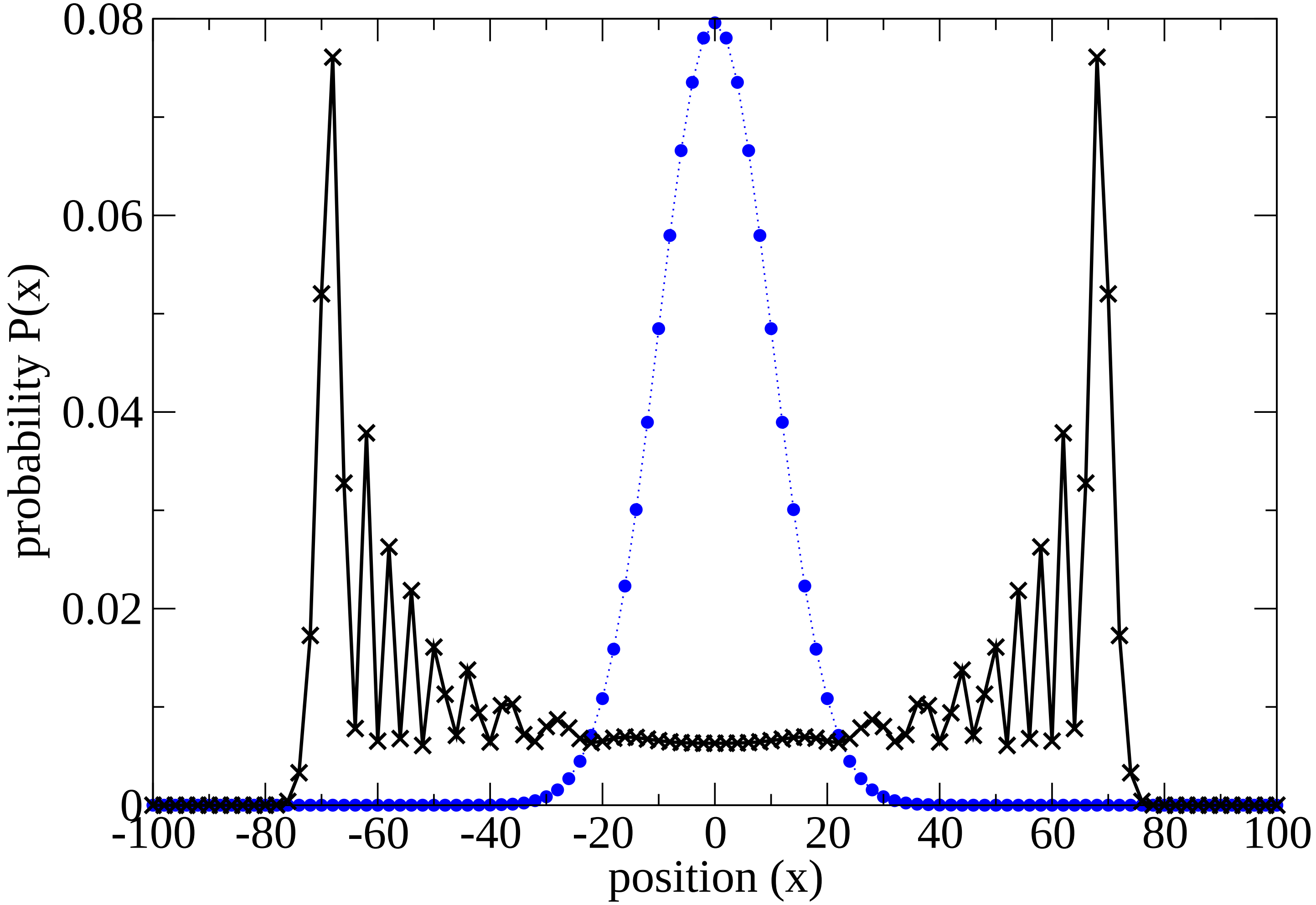
$$S|x, L\rangle = |x - 1, L\rangle \qquad S|x, R\rangle = |x + 1, R\rangle$$

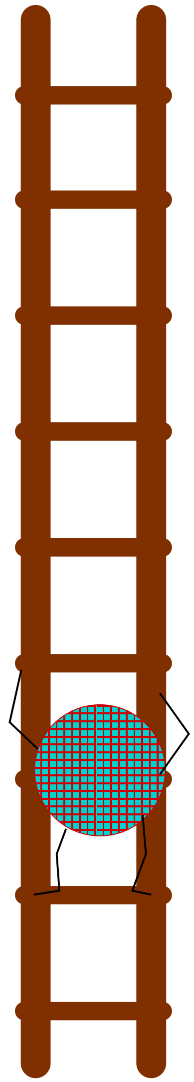


First 4 steps

interference starts step 3

Quantum walk on a line – 100 steps





Standard deviation

Discrete quantum walk on a line solved analytically by
`Ambainis, Bach, Nayak, Vishwanath, Watrous, STOC'01 (2001).`

Solutions are complicated, mainly due to "parity" property:
– support only on even (odd) points for even (odd) time steps

Measure progress of walk by standard deviation (from the origin)

$$\sigma^2(T) \equiv \sum_x \sum_a x^2 P(x, a, T)$$

Asymptotic (large T): $\sigma(T) = (1 - 1/\sqrt{2})^{1/2} T$

Contrast with $\sqrt{T}$ for classical random walk

quadratic speed up

Physical implementations

Three proposals that are within reach of current experimental capabilities

Trapped ion:

`Travaglione, Milburn, PRA 65:032310 (2002) quant-ph/0109076`

Propose this as severe test of decoherence in system

Phase space of cavity field:

`Sanders, Bartlett, Tregenna, Knight, quant-ph/0207028`

Haroche group? atom plays role of coin as it passes through cavity

Atom in optical lattice:

`Dür, Raussendorf, Briegel, Kendon, quant-ph/0207137`

Munich optical lattice experimentalists?

Decoherence

How to model decoherence?

$$\rho(t+1) = (1-p)S \cdot H \cdot \rho(t) \cdot H^\dagger \cdot S^\dagger$$
$$+p\,\mathbb{P} \cdot S \cdot H \cdot \rho(t) \cdot H^\dagger \cdot S^\dagger \cdot \mathbb{P}^\dagger$$

$\mathbb{P}$ is a projection that represents the action of the decoherence and p is the probability of a decoherence event happening per time step.

Possible $\mathbb{P}$

– coin state only

– particle state only

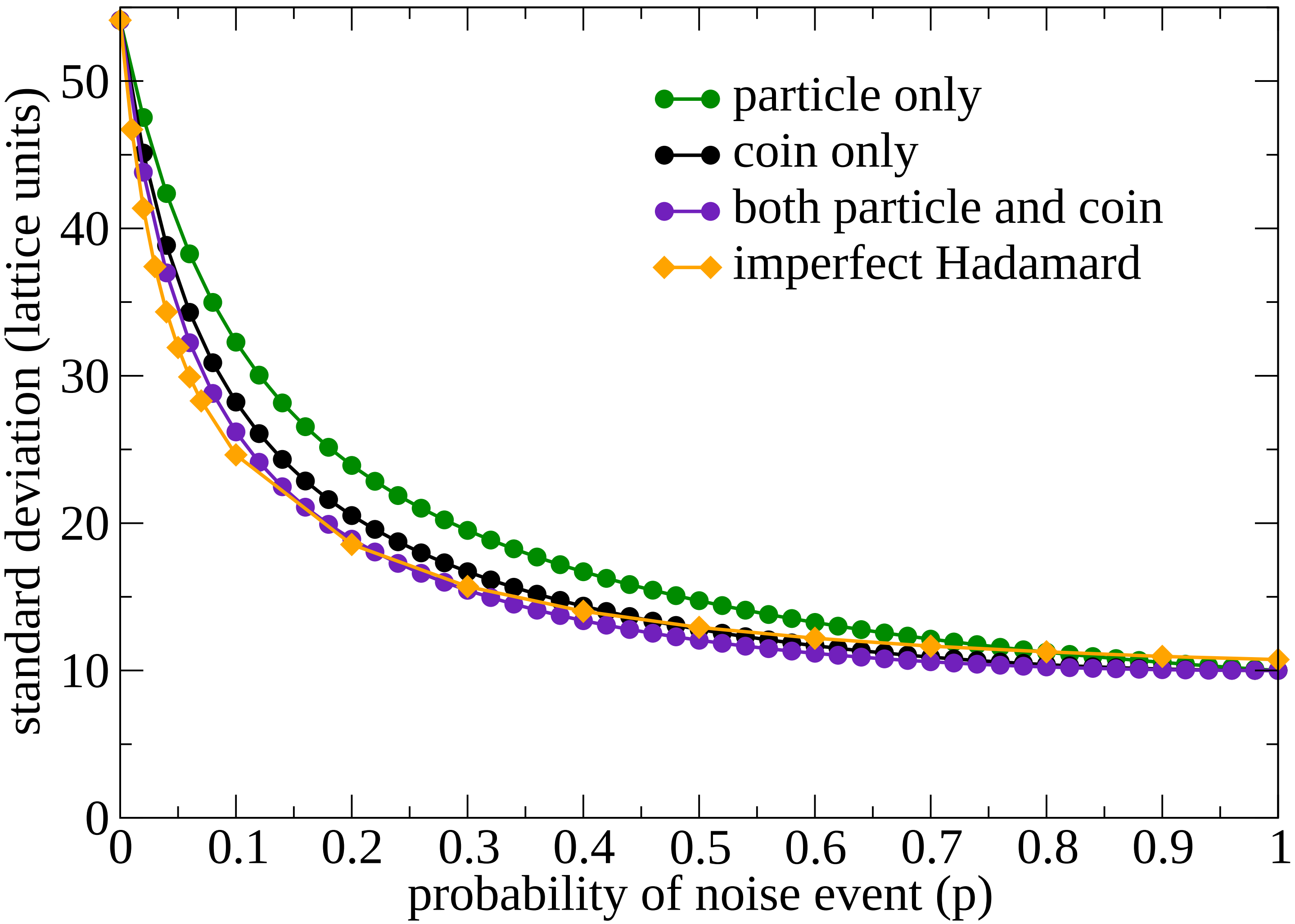
– both at once, projection into the prefered basis

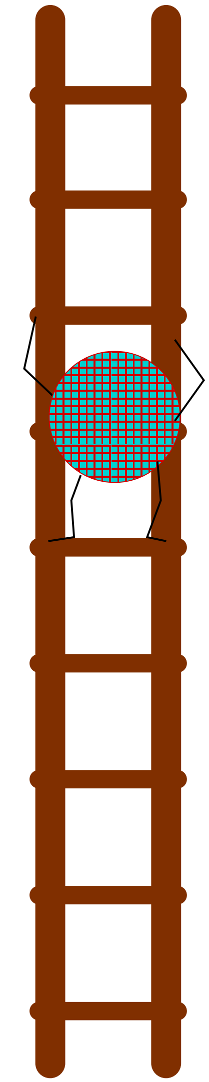
Other possibilities:

– imperfect Hadamard operation

– imperfect shift

Standard deviation with decoherence

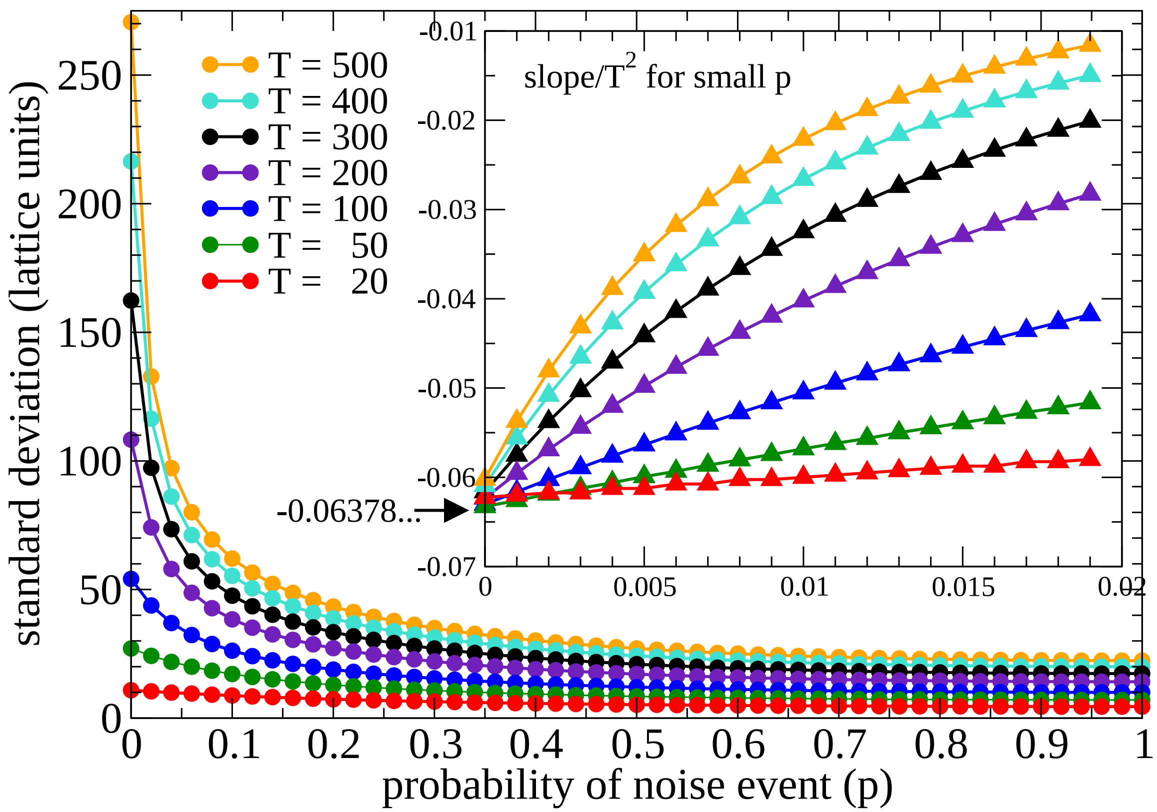


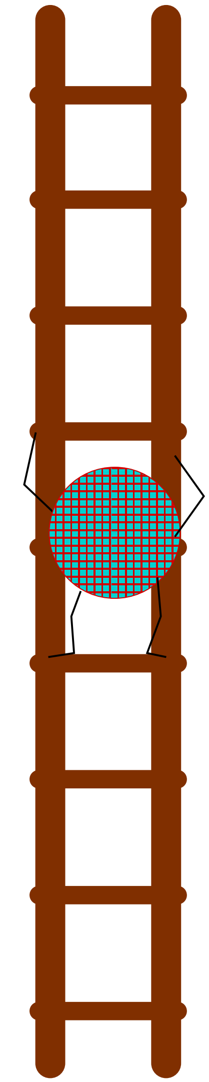


Slope at $p \to 0$

$$\sigma_p(T) \simeq \sigma(T)\left\{1 - \frac{pT}{6\sqrt{2}} + O(p)\right\}.$$

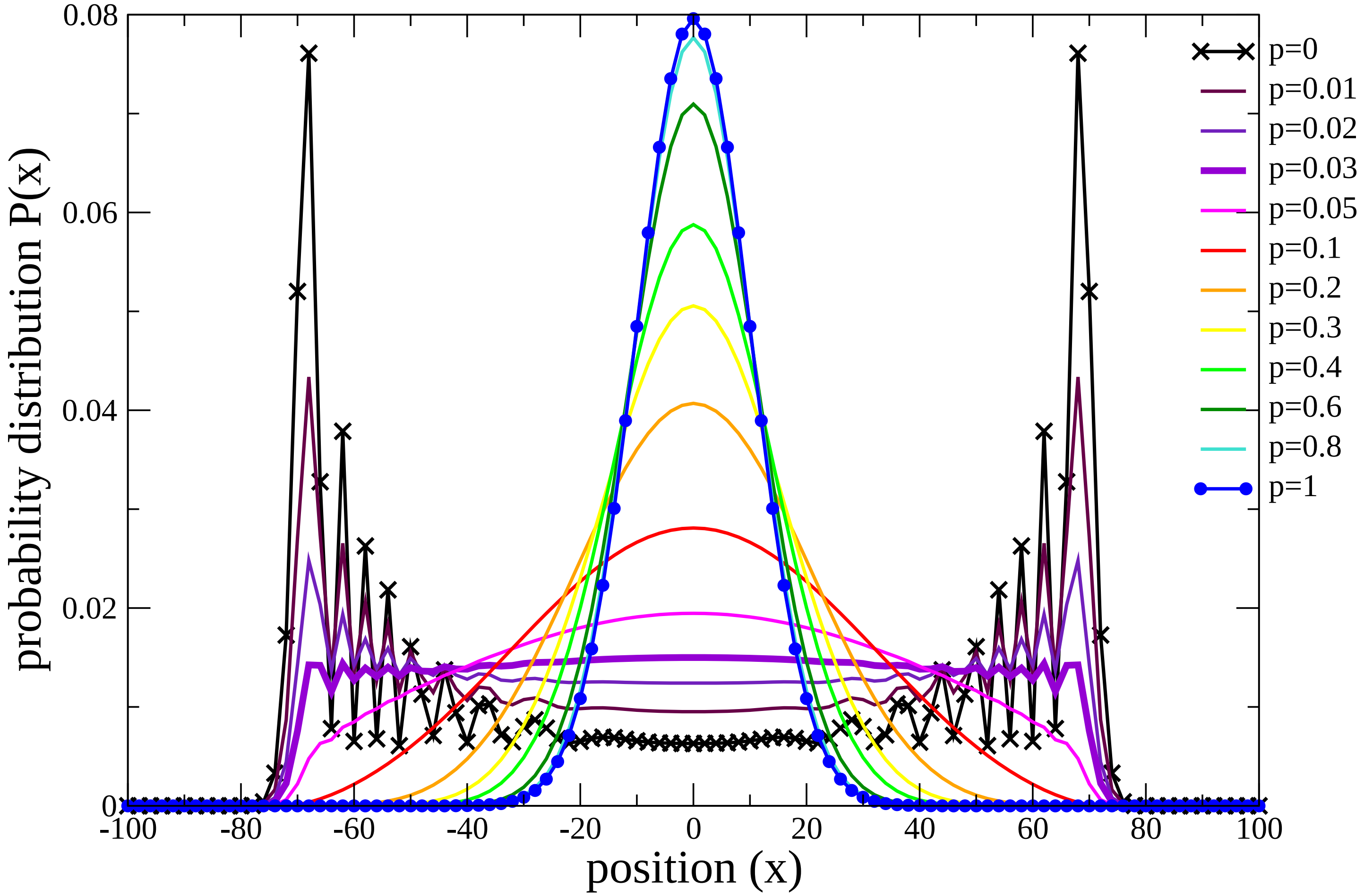
Compares well with simulation data, with second order correction for $\sigma(T) = (1 - 1/\sqrt{2})^{1/2}(T - 1/T)$

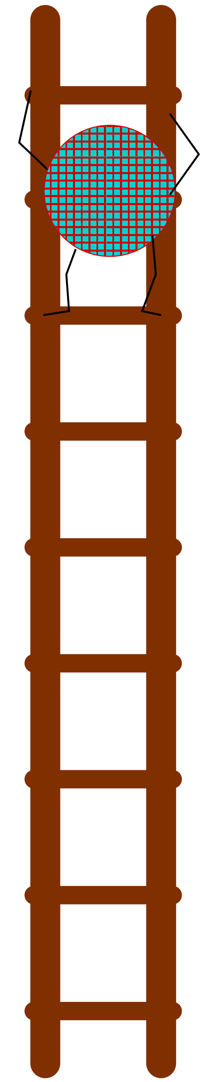




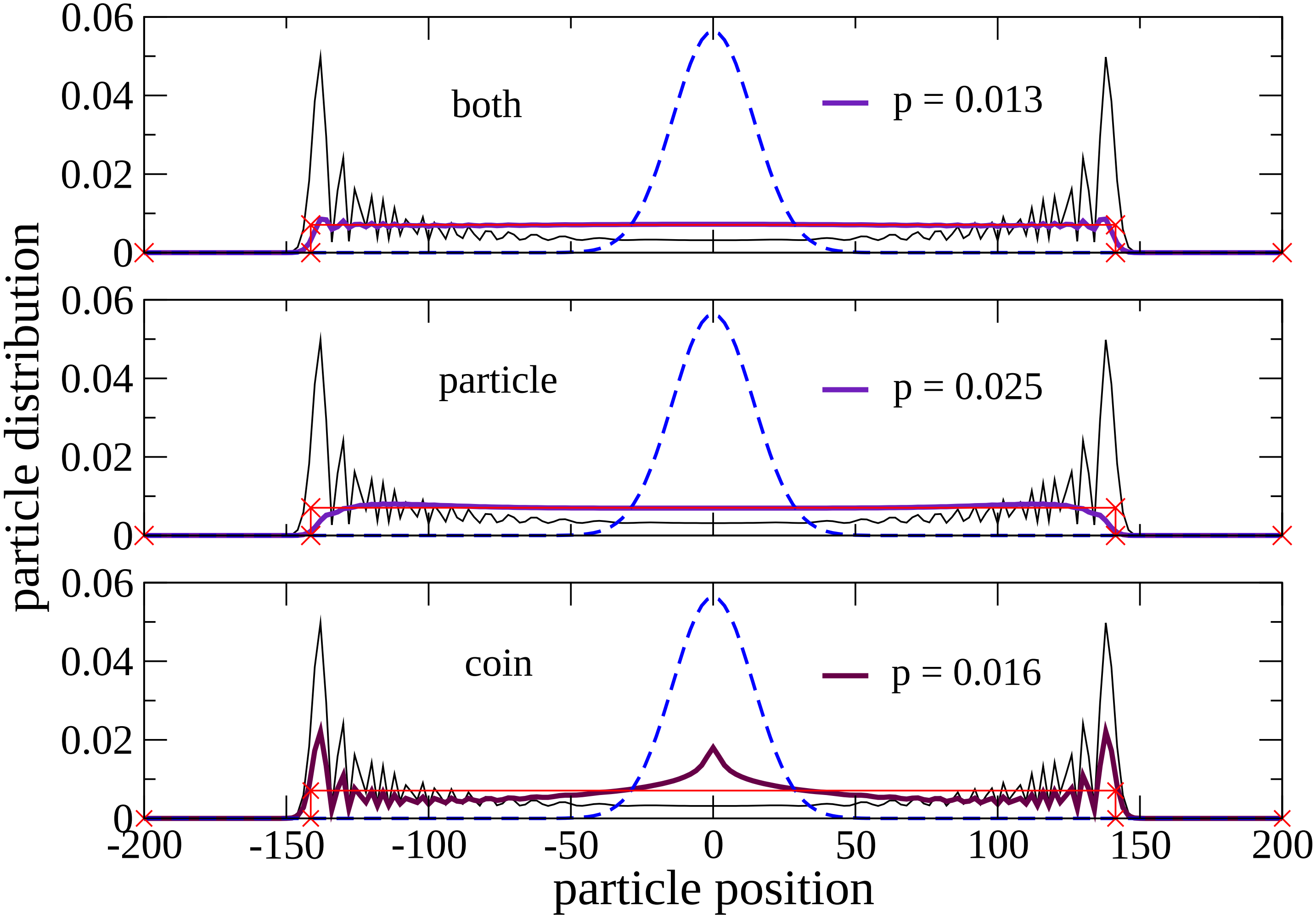
Flat distributions

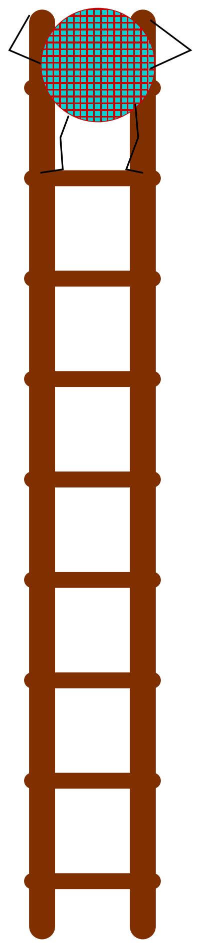
Look how distribution shape changes with p:





Flat distributions





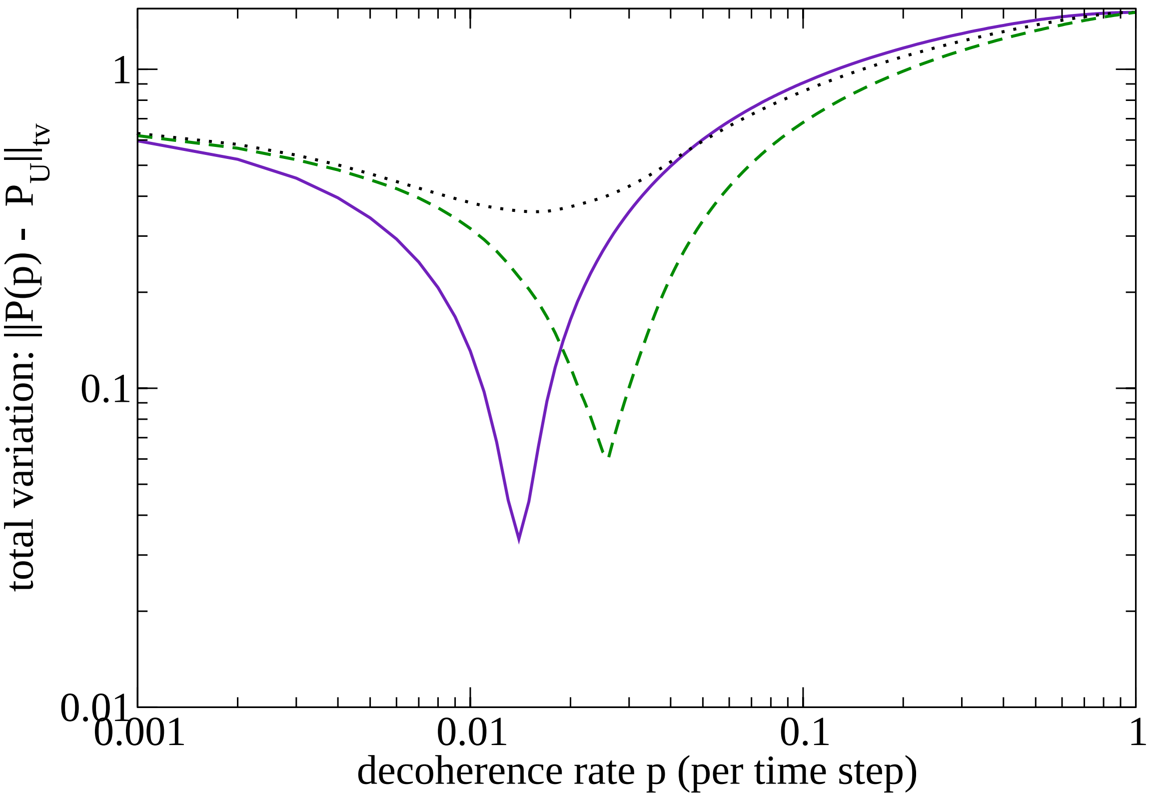
Quantify uniformity of distribution

total variational distance:

$$\nu(p,T) \equiv ||P(x,p,T) - P_u(T)||_{\mathrm{tv}} \equiv \sum_x |P(x,p,T) - P_u(T)|$$

$P(x,p,T)$ is probability particle is at position x
after T time steps (regardless of coin state)

$P_u(T) = \sqrt{2}/T$ for $-T/\sqrt{2} \le x \le T/\sqrt{2}$



$p_u T \simeq 5$ (particle only)

$p_u T \simeq 2.6$ (particle and coin)

Quantum walk on a cycle

`Aharonov, Ambainis, Kempe, Vazirani, STOC'01, quant-ph/0012090`

Mixing time (time averaged)

$$M_\epsilon = \min\left\{T | \forall t > T : ||\overline{P(x,p,t)} - P_u||_{\text{tv}} < \epsilon\right\}$$

where

$$\overline{P(x,p,T)} = \frac{1}{T}\sum_{t=0}^{T} P(x,p,t)$$

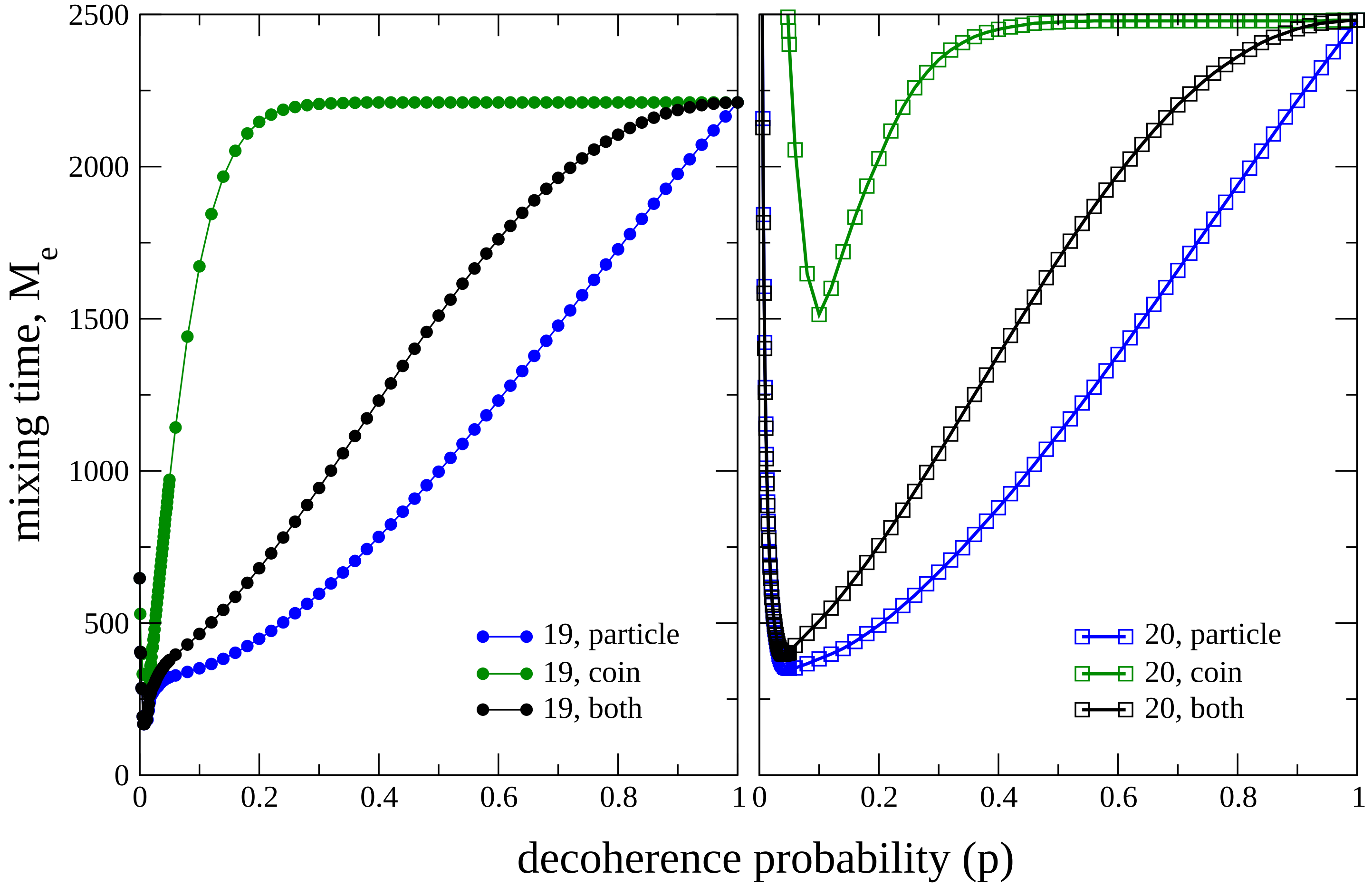
and P_u is the limiting (uniform) distribution over the cycle.
Mixing time quantifies how long before the (time-averaged) probability distribution of the particle position becomes uniform.

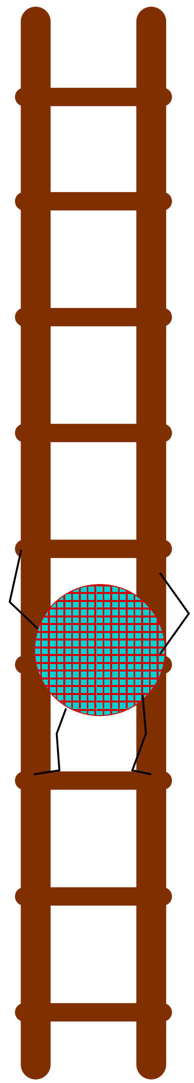
Classical value: $M_\epsilon^{(C)} = N^2/16\epsilon$

(note: <u>not</u> $\log(1/\epsilon)$ because averaged)

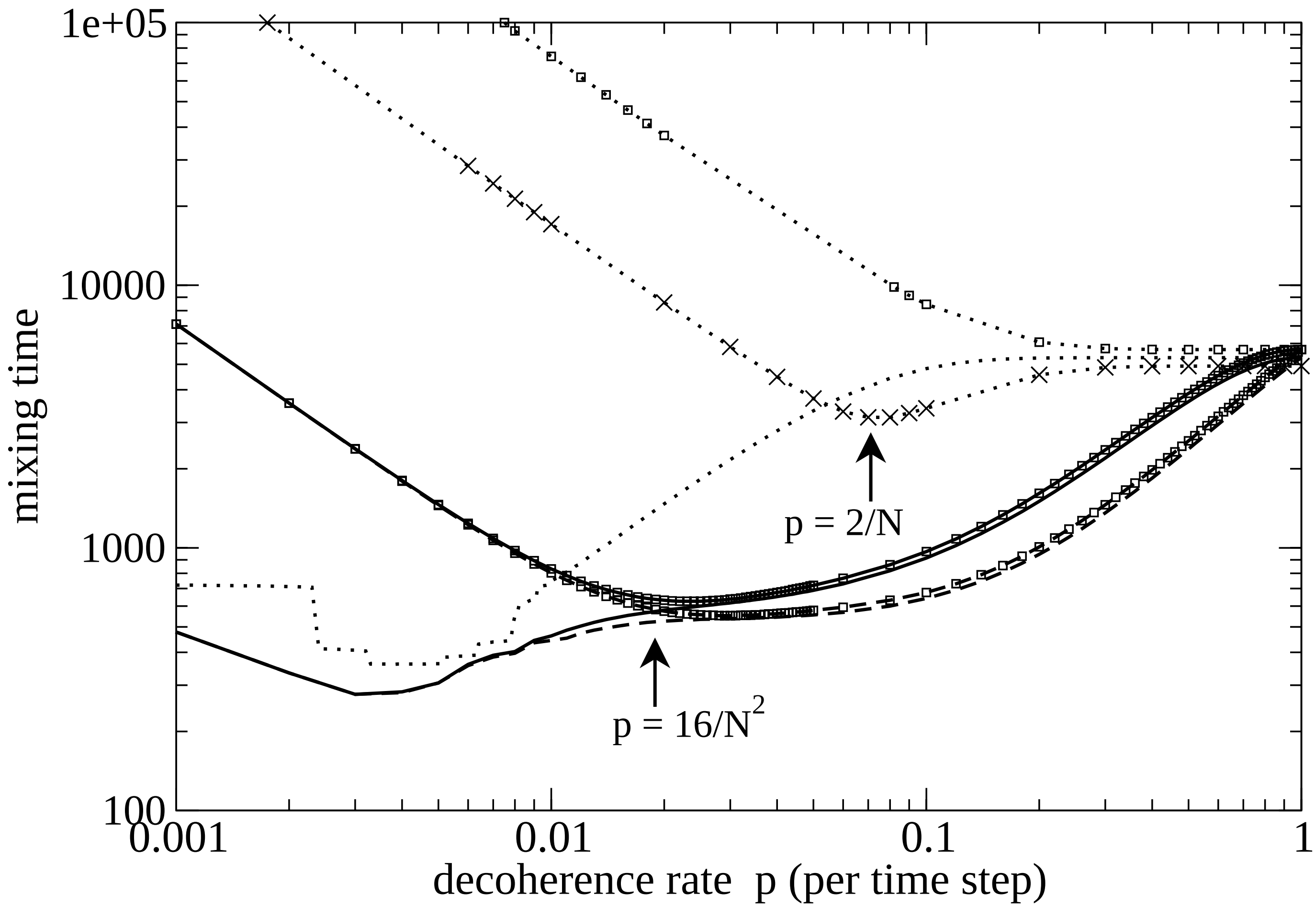
Mixing times for $N = 19$ and $N = 20$

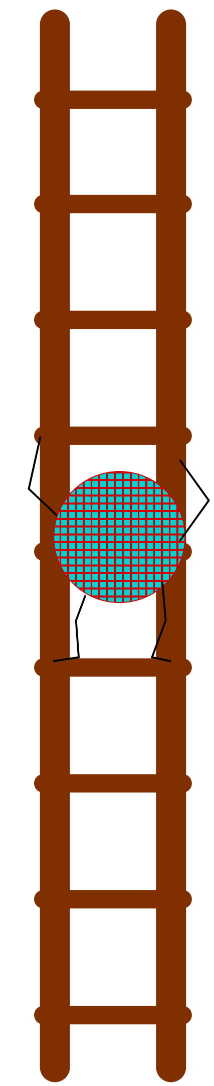
$(\epsilon = 0.01)$





Mixing times for $N = 29$ and $N = 30$





Mixing times on cycle

Decoherence on cyclic quantum walks causes:

Even-N:

- mixes to uniform distribution (pure quantum does not)
- noise on coin, no quantum speed up
- noise on particle, quantum speed up $M_\epsilon^{(\mathrm{min})} \simeq \alpha N/\epsilon = M_\epsilon^Q$

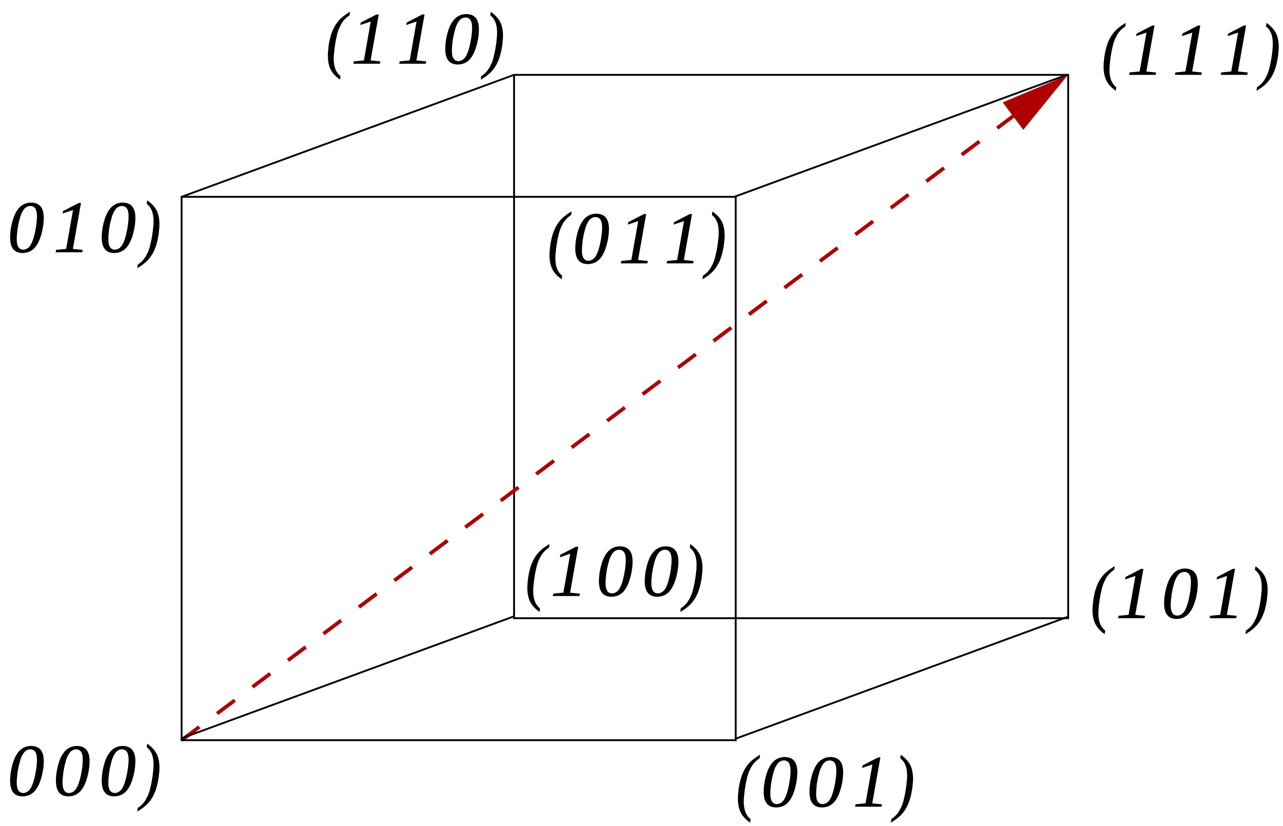
Odd-N:

- mixes even faster than pure quantum, min at $p \simeq 2/N^2$
- for $p \lesssim 16/N^2$, "quantum window" $M_\epsilon \lesssim M_\epsilon^Q$

Hypercube

Kempe, quant-ph/0205083

Quantum random walks hit exponentially faster

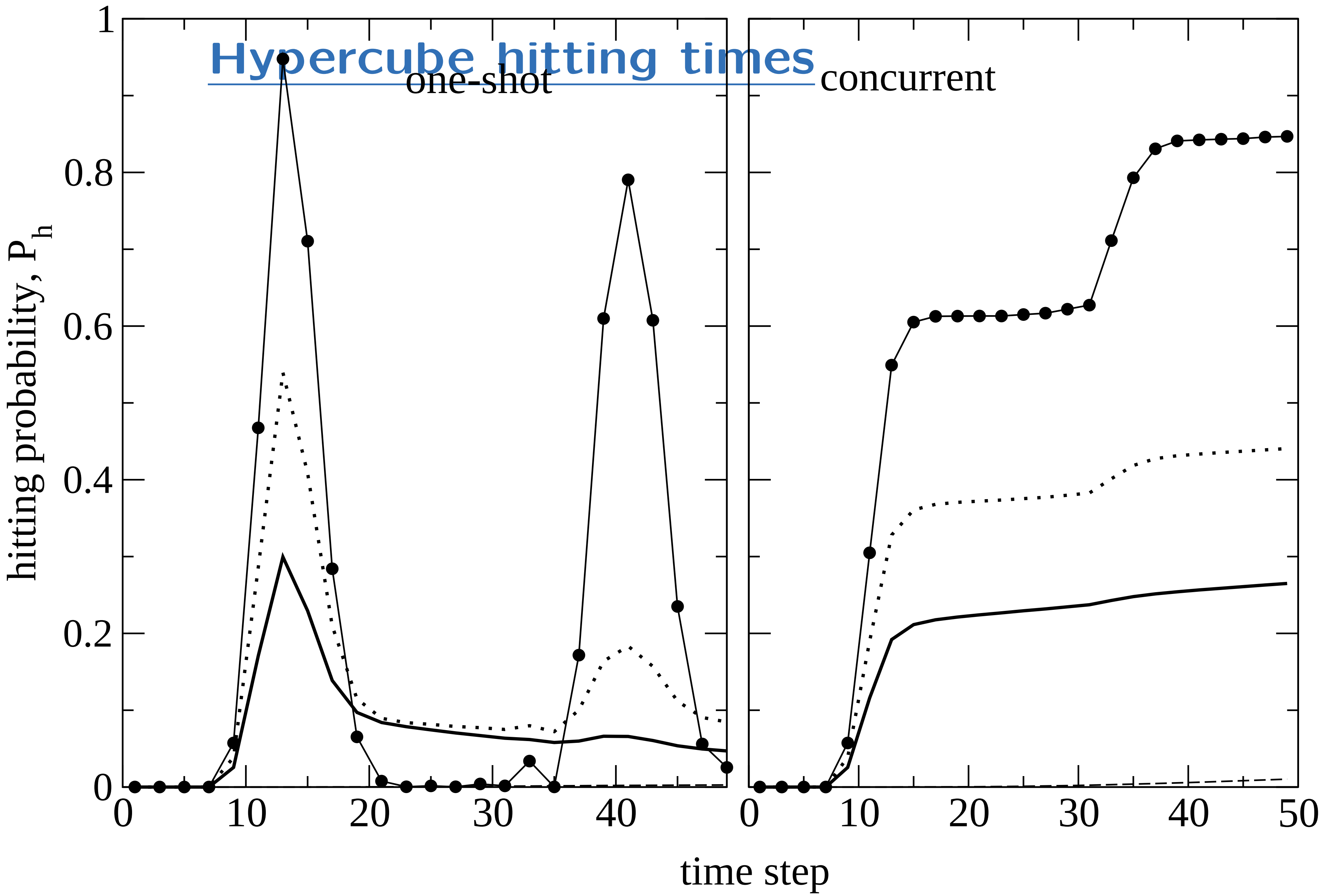


N-dimensional coin

2^N nodes

$P_h(T)$ hitting probability

Hypercube hitting times



Hypercube hitting times

peak height scales as

$$P_h(p) = P_h(0)\,\mathrm{exp}\{-(N+\alpha)p\}$$

($0 \lesssim \alpha \lesssim 2$ depending on coin, particle or both decohered)

So exponential in p, but still quantum window:

$p \simeq 1/N$ only lowers P_h by a factor of $1/e$
still exponentially better than classical.

(Note $p \simeq N$ is sort of critical damping...)

Summary

- interesting regime of low decoherence where quantum things continue to happen
- for algorithms, optimal decoherence rates are > 0

Open questions (partial list...)

- are these effects significant or useful for algorithms?
- do continuous time quantum walks show similar behaviour under decoherence?
- are coined and continuous time quantum walks fundamentally different?
- how can we make more use of the coin to control the walk?

Much more work to do!

Story so far in `quant-ph/0209005 (Kendon, Tregenna)`

Also: `Todd Brun, Hilary Carteret and Andris Ambainis, quant-ph/0208195`